Page III

Vera Effigies Venteri Mandey.
Ætatis Suæ 37 Anno 1682

MELLIFICIUM MENSIONIS:

OR, THE

Marrow *of* Meaſuring.

WHEREIN

A New and Ready Way is ſhewn how to Meaſure *Glazing*, *Painting*, *Plaiſtering*, *Maſonry*, *Joyners*, *Carpenters*, and *Bricklayers* Works.

AS ALSO,

The *Meaſuring* of LAND, and all other *Superficies*, and *Solids*, by *Vulgar Arithmetick*, without reducing the *Integers* into the leaſt Denomination; giving the Content of any *Superficie* or *Solid*, conſiſting of *Feet*, *Inches*, and *Parts of Inches*, in a Fourth Part of the *Time* and *Labour* required by the uſual Way in *Vulgar Arithmetick*.

TOGETHER

With ſome CHOICE PRINCIPLES and PROBLEMS of *GEOMETRY* conducing thereto.

To which is Added,

An APPENDIX, left under the AUTHOR's own Hand.

The CONTENTS *of which, ſee in the next Page.*

Illuſtrated with Copper CUTS.

The like not heretofore Publiſhed.

The Fourth Edition, with Additions.

By VENTERUS MANDEY.

Non Quot, ſed Quales.

LONDON:

Printed for THO. TEBB, in *Little-Britain*; SAM. ILLIDGE, under *Serle*'s Gate, *Lincoln's-Inn New-Square*; and THO. KING, in *Petty-France*. M.DCC.XXVII.

IN this *Edition* is added an APPENDIX. Wherein,

I. The Errors and Diſagreements of ſeveral Meaſurers, in meaſuring the Bricklayers Work of Chimneys of all Sorts, are reformed: Which Miſtakes and Errors are the more pardonable, if we conſider, that the moſt Part of the Work relating to Chimneys, is inviſible, or hid.

II. Rules are laid down for the Meaſuring of Arches and Vaults, whether they are Semicircles, Segments of Circles, or Ellipſes, together with their Butments.

III. The Menſuration of ſome Surfaces and Solids, which were not inſerted in the firſt and ſecond Editions.

IV. Two TABLES, one whereby to determine the Price of one, or any Number of Feet of Brickwork, at any Price *per* Rod: The other, to find the Perpendicular of any Gable End, by having its Baſe.

V. Some Alterations in the Method of Meaſuring are propoſed, as being more agreeable to Truth than the Ways heretofore practiſed.

VI. The whole Work is corrected and amended, by omitting ſome Things, and adding others, where Neceſſity required it.

THE

AUTHOR'S EPISTLE

TO THE

READER.

Courteous Reader,

THE Subject of the ensuing Treatise, is the Science of Measuring in a new and brief Method by vulgar Arithmetick, wherein the laborious and troublesome Part of reducing the Feet and Inches into the least Denomination, *i. e.* into Parts of Inches, is omitted, and a brief and speedy Way directed for the casting up any Dimension whatsoever.

Which Way of Measuring, considering that Workmens Rulers are not divided decimally, and that most Dimensions are taken by a Ruler divided into Feet, Inches, Half Inches, and Quarters, and for the most Part, the Contents are required in

the ſame Kind; I ſay, Conſidering this, it cometh very little ſhort of the Decimal Way of Meaſuring.

For if Dimenſions be given in Feet, Inches, and Halfs, or Quarters of Inches (as moſt uſually they are) and the Contents required in the ſame Method, then I look upon the reducing thoſe Dimenſions into Decimals, and after the Contents are found in Decimals, the reducing them again into Feet, Inches, and Half Inches, *&c.* to take up more Time than this Way which is taught in the enſuing Treatiſe.

Beſides, many Men can multiply and divide by vulgar Arithmetick, which do not underſtand Decimals; for whoſe Sakes chiefly this is written.

I conceive every Book meets with many critical Cenſures, and I doubt not, but this will partake with the reſt; and therefore it might (perchance) be expected, that I ſhould excuſe myſelf for whatſoever any Man ſhall be pleaſed to object againſt in it, which I ſhall neglect, only deſiring the judicious Reader to paſs by ſome ſmall Overſights, which, perhaps, there may be crept into the enſuing Work, as knowing that all Mens Doings are ſubject to Error: But as for groſs Faults, I think there are none; for I have been as careful as I could, both in writing, and likewiſe in correcting the Printed Sheets in this Edition, conſidering my want of Time, occaſioned by my other Employment.

To perſuade any one to the Study of this Science, would be a Folly, ſince the Ignorant (which are blind) cannot judge truly; and to him that already underſtands it, the Labour would be uſeleſs and unprofitable; and to the Averſe and Careleſs, it would be like the caſting of Pearls before Swine; the exquiſite Knowledge whereof, cannot be attained by ſuch prejudicated Perſons, altho' to many that are judicious, the Uſefulneſs hereof is already ſufficiently known; and to thoſe induſtrious Perſons which are yet to ſeek in the Knowledge of this Subject, and deſire to be informed, this Treatiſe will fully anſwer their Expectations.

In the following Treatiſe, I have endeavoured to proceed methodically, and have, to my Knowledge, omitted nothing, which might tend to the making of a Man an expert Meaſurer; in order to which, there are three Books of *Geometry* inſerted, the firſt containing the Rudiments thereof, and the ſecond and third containing choice Problems; which two laſt, I tranſlated from *Latin* Copies, wherein I have endeavoured to render the Meaning of the Author as plain as the Work would permit, and have amended ſome Miſtakes (I ſuppoſe) of the Printer of the *Latin* Copies: I have likewiſe explained the Meaning of ſome difficult Terms, and alſo added a new Diagram belonging to *Chap.* 6. *Lib.* 3. *Fig.* 1. which was wanting in the Copy.

So that the whole Treatiſe conſiſts of ſix Books, a general Account whereof follows.

The firſt Book of this Treatiſe contains the Rudiments of *Geometry*, conſiſting of ſuch Definitions and Propoſitions, as are meet to be known to any Man that intends the Science of Meaſuring, and is an Introduction to the other five Books; in which firſt Book (perhaps) ſome few of the Definitions may ſeem ſtrange, as not agreeing with the Definitions of ſome others, yet, in my Opinion, agreeable both to Reaſon and Truth. There is likewiſe added, the Deſcription of Ovals to any Length and Breadth, with a Pair of Compaſſes; and alſo a Digreſſion concerning Ovaller Arches.

The ſecond Book, being tranſlated from a *Latin* Author, contains a Garden of *Geometrical Roſes*, conſiſting of choice Propoſitions in *Geometry*, wherein a new Way is ſhewn of cutting right Lines in extream and mean Proportion; alſo the Diviſion of Angles, and finding a ſtrait Line, in Length equal to a Circular; as alſo to find the Centre of Gravity of a Semicircle or Quadrant, with ſeveral other Things not heretofore publiſhed in *Engliſh*, that I know of.

The third Book, being likewiſe tranſlated from the ſame Author, contains ſome Principles and Problems in *Geometry*, formerly thought deſperate, now briefly explained in *Engliſh*, conſiſting of the Subject, Principles, and Method of the *Mathematicks*, and of *Algebraical* Operations; likewiſe of a new Method of treating of Solids and their Superficies, by the efficient Cauſes, with ſeveral other Things.

The

The fourth Book contains the Science of *Menſuration*, ſhewing how to meaſure all kinds of Works relating to Building, to wit, *Carpenters*, *Glaſiers*, *Painters*, *Plaiſterers*, *Maſons*, and *Bricklayers* Works, *&c.* the Contents whereof may be ſeen more at large in the following Table.

The fifth Book contains the meaſuring of *ſuperficial Plains*, wherein is ſhewn how to meaſure *Triangles* of all kinds, *Rectangled Figures*, whoſe Sides are equal or unequal, *Circles*, *Ovals*, *Pyramids*, *Cones*, *&c.* In brief, it ſhews how to find the Content of moſt kind of ſuperficial Figures in uſe, whether they be regular or irregular, each Propoſition having a Figure or Diagram belonging to it, for the more eaſier underſtanding how to reſolve it.

This fifth Book alſo ſhews how to meaſure Land lying in any Form, and how to reduce Cuſtomary Meaſure into Statute Meaſure, and the contrary, *&c.*

The ſixth Book contains the Meaſuring of *ſolid Bodies*, ſhewing how to meaſure *Timber*, and *Stone*, &c. alſo how to find the ſolid Contents of *Pyramids*, *Cones*, *Cylinders*, *Spheres*, and other ſuch like Solids, each Propoſition having a Diagram (or Scheme) belonging to it, for the readier attaining the Reſolution of the Propoſition. In it is alſo contained the Science of *Gauging*, or meaſuring *Liquid Veſſels*, and how to find their Contents in Wine or Ale Gallons; and likewiſe

wiſe how to find the Contents of *Brewers* Veſſels, Tuns, or Fats, in Gallons, and to reduce them into Barrels: All which, I have drawn into a Pocket Volume.

This Treatiſe thus finiſhed, I preſent thee with, deſiring thy friendly Cenſure and Acceptance of theſe my Labours; as alſo to paſs by ſuch Faults as may poſſibly have eſcaped the Preſs, or myſelf, which, I am certain, are but few: And in ſo doing, thou wilt oblige him, who is

A Friend to All, but more eſpecially to thoſe that are Mathematically inclined,

V. M.

A TABLE

A

TABLE

To the ſeveral

CHAPTERS, PROPOSITIONS, *and* SERIES *of the whole* TREATISE.

The CONTENTS of the Firſt BOOK, being the Rudiments of GEOMETRY.

CHAP. I.

CHAP. II.

CHAP. III.

The CONTENTS of the Second BOOK, being a Garden of Geometrical Roses.

The Contents of the Third BOOK, conſiſting of ſome Principles and Problems in *Geometry*, formerly thought deſperate.

Chap. 10.

The CONTENTS of the Fourth BOOK, consisting of *Measuring*.

The Contents of the Fifth BOOK, consisting of the Measuring of *Superficial Plains*.

Measuring of LAND.

The Contents of the Sixth BOOK, consisting of the Measuring of *Solid Bodies.*

Of Round Solid Bodies.

The CONTENTS of the APPENDIX.

OF

GEOMETRY *in General.*

GEOMETRY is a *Greek* Word, and imports only the Meafuring of Land; and fince feveral Parcels of Lands are divided into various Forms, it behoves him, that would be efteemed a Land-Meafurer, to know how to meafure all Kinds of fuperficial Figures. But though the Word import no more but Land (or fuperficial) Meafuring; yet under the Name of GEOMETRY is comprifed the Meafuring of all Kinds of Solids. This Science (according to Hiftory) was firft invented by the *Egyptians*; and the Caufe which put them upon inventing it, was the Overflowing of the River *Nilus*, the greateft and longeft River in the World, which, when it overflowed, wafhed away

their Banks and Land-Marks; and when the Waters were diſperſed, it was a difficult Matter for every Man to know his own Quantity of Land; inſomuch that it cauſed Quarrelling and Strife amongſt them, till at length, through the Induſtry of ſome ingenious Perſon or Perſons, this Science of GEOMETRY was found out, which put an End to their Quarrellings, and reſtored to every Man the ſame Quantity of Land after the Flood, that he had before.

IN brief, GEOMETRY is a Science, whereby the Quantities of Things not meaſured, are determined, by comparing them with other Quantities meaſured.

OF

OF

GEOMETRY.

CHAP. I.

DEFINITIONS.

I. A POINT is a Body whoſe Quantity is not conſidered; if conſidered, is that which is not put to account in Demonſtration; and is made with the Point of the Compaſs, of a Pen or Pencil, or ſuch like, as the Point noted by A.

II. A Line is a Body whoſe Length is conſidered without its Breadth, and is made by the Motion of a Point from one Extreme to another.

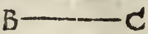
Extreme ſignifies the Beginning or End of a Line.

Of Lines are Three Sorts.

Firſt, Crooked or Circular. Secondly, Streight or Right. Thirdly, Mix'd.

1. Crooked or Circular Lines are thoſe, which have a Poſſibility of diducting or ſetting farther aſunder their Extremes, as the Line

2. Streight

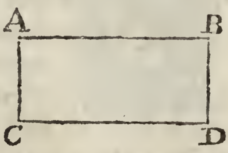
2. Streight or Right Lines are thoſe which have no Poſſibility of diducting or ſetting farther aſunder their Extremes; and therefore is the leaſt Length between the two Extremes. *Plato* defines a Right Line to be that, whoſe Extremes do ſhadow all the middle Parts, and is re- 

preſented by the Line BC.

3. Mix'd Lines are compounded of Streight and Circular Lines, as the Line 

III. A Superficie is that which hath Length and Breadth, the Thickneſs not being conſidered; and as a Line is produced by the Motion of a Point, ſo a Superficie is produced by the Motion of a Line ſuppoſed to move tranſverſly; that is to ſay, the Line AC being ſuppoſed to move ſide-ways to BD, it will produce a Superficies ABCD.

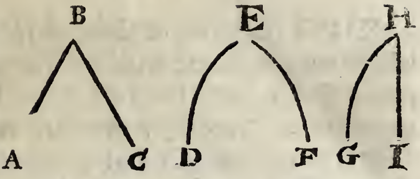
IV. The Extremes of a Superficie are Lines. But of a Circular Superficie, a Line is the Extreme.



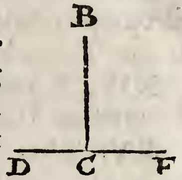
V. An Angle is an Inclination of two Lines, the one to the other, the one touching the other, and not lying ſtreight forth at length. And of Angles there are three Sorts, namely, Right-lined, Curve-lined, and Mix'd.

Note, When an Angle is mentioned by three Letters, the middlemoſt Letter ſignifies the Angle intended.

So

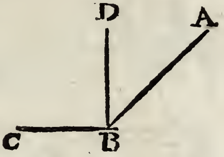


So ABC is a right lined Angle DEF is a curve lined Angle, and GHI is a mix'd Angle.



VI. When a right Line BC ſtanding upon a right Line DE, making the Angles on either ſide BCD, and BCE equal, they are called right Angles; and the right Line BC, is called a perpendicular Line to that upon which it is erected, *viz.* DE.

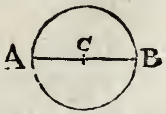
VII. An Angle is ſaid to be obtuſe or blunt, when it is greater than a right Angle; and acute or ſharp, when it is leſs than a right Angle: So the Angle ABC is an obtuſe Angle, and the Angle ABD is an acute Angle, and the Angle DBC is a right Angle.



VIII. A Circle is a plain Figure, comprehended by one Line, being generated by the Motion of a Compaſs, or other equivalent Means, wherein all right Lines drawn from the Centre to the Circumference, are of equal length. The Centre is a Point exactly in the Middle of the Circle.

IX. The Diameter of a Circle is a right Line, as AB drawn by the Centre C, and being terminated by the Circumference on either ſide, divides the Circle into two equal Parts.

X. The Semi-diameter is half the Diameter, as AC or CB.



XI. A Semicircle is one half of the whole Circle.

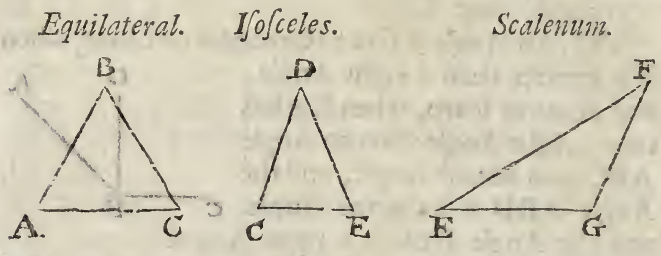
XII. Tri-

XII. Trilateral, or three-ſided Figures, are thoſe which are contained within three right Lines, and are called Triangles; becauſe three Lines being joined together at Angles, conſtitute three Angles.

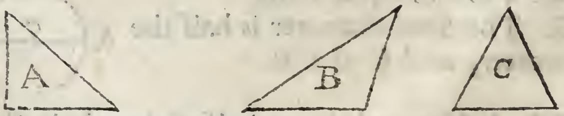
XIII. Of three-ſided Figures, that which hath three equal Sides, is called an Equilateral Triangle, as the Triangle ABC.

XIV. But that which hath two Sides alike equal, is called an Iſoſceles Triangle, as CDE.

XV. That Triangle, whoſe three Sides are unequal, is called a Scalenum, as EFG.



XVI. There are alſo other Triangles, and are named according to the Quality of their Angles. A right angled Triangle, is that which hath one right Angle, as the Triangle A. An Ambligonium, or obtuſe-angled Triangle, is that which hath one Angle obtuſe, as the Triangle B. An Oxigonium, or acute angled Triangle, is that which hath three acute Angles, as the Triangle C.



XVII.

XVII. In every Triangle, two of the Lines being taken for two ſides, the third Line remaining is called the Baſe, whether it be the lowermoſt Line of the Triangle, or not; ſo that any one of the three Lines, which incloſe a Triangle, may be taken for the Baſe.

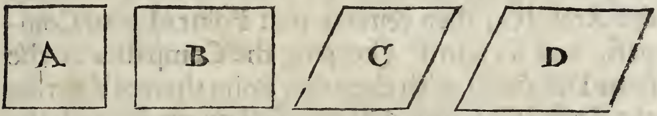
XVIII. Of Quadrilateral, or four-ſided Figures, there are ſeveral Sorts.

XIX. A Quadrate, or Square, is that which hath equal ſides, and right Angles, as the Figure A.

XX. An Oblong hath the oppoſite Sides alike, and right Angles, as the Figure B.

XXI. A Rhombus, or Diamond Figure, hath four equal Sides, and two Angles acute, and two obtuſe, as C.

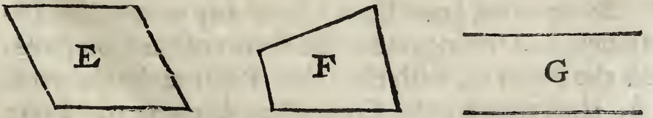
XXII. A Rhomboides, hath oppoſite Sides, and oppoſite Angles alike, but it is neither equilateral nor right angled, as the Figure D.



XXIII. A Paralallelogram is a four-ſided Figure, whoſe oppoſite Sides are parallel, as the Figure E.

XXIV. All other four-ſided Figures are called Trapezia's or Tables, as F, *&c.*

XXV. Parallel Lines are ſuch, which being in the ſame Superficie, if produced, will not meet, as G.



Thus much may ſerve for Definition; I proceed now to Practice.

C H A P.

CHAP. II.

Note, *That the Figures belonging to the following Propositions are in the next folded Page.*

PROP. I. Fig. I.

From a Point given in a ſtreight Line, to raiſe a Perpendicular, that ſhall cut the ſtreight Line at right Angles.

LET C be the Point propoſed in the Line AB, from which a Perpendicular is to be raiſed.

From the Point given C, draw at pleaſure any Semicircle EF; then from the Points EF, you muſt make a Section thus: Open your Compaſſes to the Diſtance EF, ſetting one Point in E, with the other deſcribe the Arch IG; then remove that Point of your Compaſs, and ſet it in F (keeping the Compaſſes at the ſame Diſtance) with the other Point thereof deſcribe the Arch IH, which will interſect or cut through the Arch IG; from which Interſection at I, a Line being drawn to the Point C, will be perpendicular, and cut the Line AB at right Angles.

This Prop. *may be performed, as in* Fig. II.

By opening your Compaſs to any convenient Diſtance, and ſetting one of the Points of the Compaſſes in the Point C, with the other Point make the prick A, then turning the Compaſſes, keeping one Point ſtill at C, with the other Point make the prick B in the Line given; then ſetting one Foot in A, and opening

opening the Compaſſes to B, deſcribe the Arch HI; alſo remove the Compaſſes, and ſetting one Point in B, deſcribe another Arch HK, from which Interſection, draw the Line HC, which will cut the Line AB at right Angles.

PROP. II. Fig. III.

To raiſe a Perpendicular at the End of a Line.

LET the Line give be AB, and on the End B, it is requir'd to raiſe a Perpendicular.

Set one Foot of your Compaſſes above the Line AB at pleaſure, as ſuppoſe at D, and opening the other Point till it ſtay in B, with this Diſtance keeping the Point at D, deſcribe a Semicircle FBE; by the Points F and D, draw the ſtreight Line FDE; and where the ſtreight Line cuts the Circle as at E, from thence draw the Perpendicular EB.

To perform this Prop. *another way.* Fig. IV.

LET the Point B, at the End of the Line AB, be the Point from whence a Perpendicular is to be raiſed.

The Compaſſes being opened at Pleaſure, ſet one Foot in the Point B, and with the other Foot deſcribe the Arch of a Circle CDG, keeping the Compaſſes at the ſame Diſtance, ſetting one Foot in C, deſcribe the Arch BD, then ſetting one Foot in D, deſcribe the Arch FEB, and from the Point E, draw the Arch ID, and from the Interſection of this Arch, with the Arch FEB, draw the Perpendicular IB.

PROP.

PROP. III. Fig. V.

To raiſe a Perpendicular at the End of a Line, having little or no Paper beyond that End of the Line, whereon you are to raiſe the Perpendicular.

LET AB, be the Line given, and B the End. Opening the Compaſſes at Pleaſure, ſetting one Foot in B, with the other Foot deſcribe the Arch CD; then remove one Foot of the Compaſs, and ſet it in C, and interſect the Arch CD at E; then lay a Ruler from C to E, and ſtrike a Line as EF; then ſet one Foot of your Compaſſes (being at their firſt Diſtance) in E, and with the other Foot make a Point or Prick in the Line EF at F, from whence draw the Perpendicular FB.

PROP. IV. Fig. VI.

Upon an Angle given to raiſe a Perpendicular.

LET ABC be the Angle given; ſetting one Foot of your Compaſſes in B, deſcribe the Arch AC; then opening the Compaſſes a little wider, and ſetting one Point in C, deſcribe the Arch DE; then removing the Point to A, interſect the Arch DE at F, from whence draw the Perpendicular FB.

PROP.

PROP. V. Fig. VII.

To let fall a Perpendicular upon a Line given, and from a Point above the Line.

LET the Line given be AB, and the Point given be C; ſetting one Foot of the Compaſſes in C, deſcribe the Arch DE, and from D and E make the Interſection at F, from whence draw the Line CF.

PROP. VI. Fig. VIII.

By a Point given, to draw a Line parallel to a Right Line given.

LET A be the Point, by which we muſt draw a Line, which may be parallel to the Line BC. Draw at Pleaſure the Oblique (or Diagonal) Line AD; from the Point A draw the Arch DE, and from the Point D deſcribe the Arch AF, then ſetting one Point of the Compaſſes in the given Point A, extend the other Point to that Part of the given Line which is interſected by the Arch AF, with this Diſtance, ſetting one Point of the Compaſſes at the Interſection D, extend the other upon the Arch DE, and where that Point falls upon the Arch as at G, draw a Line from the Point A through the Point at G, and it will be parallel to the Line BC.

Another Way to draw a Parallel to the Line B C. Fig. IX.

THE Compaſſes being ſet to the Space you intend ſhall be between the two Lines (otherwiſe any Diſtance will ſerve) ſetting one Foot on the End of the Line B, with the other Foot deſcribe the Arch DE; this being done, keeping the Compaſſes at the ſame Diſtance, ſet one Point on the End of the Line C, and deſcribe the Arch FG, then draw the Line HI, juſt to touch the uppermoſt Part of theſe two Arches, and it will be parallel to the Line BC.

PROP. VII. Fig. X.

To cut a right Line AB *equally in the Middle.*

THE Compaſſes being opened to any Diſtance ſhorter than the whole Line, and longer than half the Line; as ſuppoſe them opened from A to C, ſetting one Point of the Compaſs in A, with the other deſcribe the Arch DE, keeping the Compaſſes at the ſame Diſtance, ſetting one Foot in B, with the other deſcribe the Arch FG, and from the Interſection (or cutting thro') of theſe two Arches, draw the Line HI, it will divide the Line AB equally in the Middle.

The Figures of the Propoſitions following, you will find in the next folded Page to this.

PROP.

Fig: I.

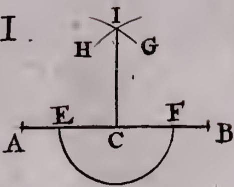


Fig. II.

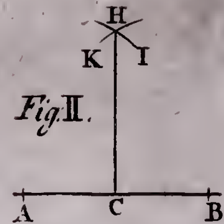


Fig. III

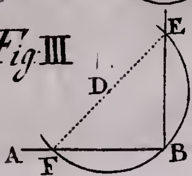


Fig. IIII

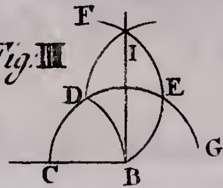


Fig. V.

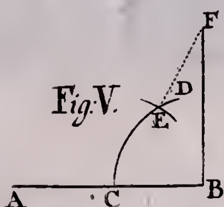
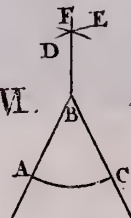


Fig. VI.



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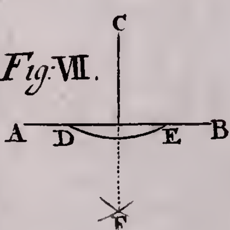


Fig. VIII.

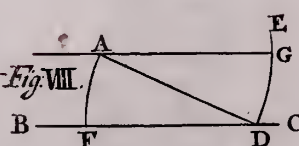


Fig. IX.

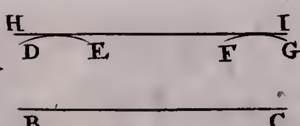
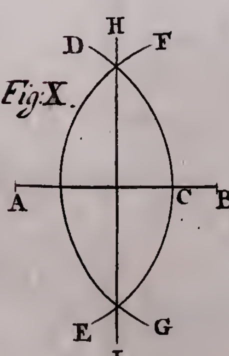


Fig. X.



The Figures of the Propositions following, you will find in the next folded Page to this.

PROP.

PROP. VIII. Fig. XI.

To cut a right lined Angle B A C, *into two equal Parts.*

OPening the Compaſſes at Pleaſure (*viz.* opening them to any Diſtance) ſetting one Point of them on the Angle A, with the other Point deſcribe the Arch D E, then from D and E, make the Interſection at G, through which Interſection, and the Angle A, draw the Line A G, which divides the Angles BAC into two equal Parts.

PROP. IX. Fig. XII.

To make a Triangle ABC, *of three right Lines* (viz. AB, BC, CA) *equal to three Lines given* D. E. F. *of which three Lines, any two being taken and added together, muſt be longer than the third Line remaining, otherwiſe you cannot make a Triangle of them.*

FIrſt draw the ſtrait Line G H, then upon that Line take G A, equal to the given Line D; alſo upon the ſame Line G H, ſet the Line E, from A to C; alſo ſet the given Line F, from C to D, then opening the Compaſſes to G A, with one Point in A, deſcribe the Circumference GBC; this being done, ſet one Point of the Compaſs at C, and the other being opened to D, deſcribe the Circumference DBI, then join the Triangle

where theſe two Circles interſect, which is at B, and make the Triangle ABC, which is equal to the three Lines given D. E. F.

PROP. X. Fig. XIII.

At a Point A, *in a right Line given* AB, *to make a right lined Angle* A, *equal to a right lined Angle given* D.

SET one Point of the Compaſſes at D, and with the other deſcribe the Arch EF, then with the Compaſſes being at the ſame Diſtance, ſetting one Point at A, with the other Point deſcribe the Arch BC, then take the Chord Line of the Arch EF (*viz.* the ſtrait Line EF) between the Points of your Compaſſes, and ſet one Point at B, and where the other Point toucheth the Arch BC as at C, by that Point draw the Line AC, and the Angle A, will be equal to the Angle D.

PROP. XI. Fig. XIV.

To make a Parallelogram ABCD, *equal to a Triangle given* EFB, *in an Angle equal to a right lined Angle given* G.

THrough the Point E, draw the Line ED, parallel to the Line FB (by the 6 Prop.) then upon the Point B raiſe the Line BD, making the Angle at B, equal to the given Angle G, (as

(as you were taught in the 10th Prop.) then Biſect (*viz.* divide in the Middle) the Baſe FB, as at A, and draw the Line AC parallel to BD, and the Parallelogram ABCD, will be equal to the Triangle given EFB, and like the Angle given G.

PROP. XII. Fig. XV.

Upon a right Line given A, *to make a Parallelogram* FL, *at a right lined Angle given* C, *equal to a Triangle given* B.

BY the foregoing Prop. make a Parallelogram FD, equal to the Triangle B, ſo that the Angle GFE may be equal to the Angle given C; continue the Line GF, 'till FH be produced equal to the given Line A; then by the Point H, draw the Line IL, parallel to EF, alſo continue the Line DE 'till it touch the Line HI, then draw the Diagonal Line IK, 'till it meet with the Line DG being continued, then through the Point K, draw the Line KL parallel to GH, then extend or continue the Line EF unto M, and the Line IH unto L, then ſhall FL be the Parallelogram required; for the Parallelogram FL is equal to the Parallelogram FD, and FD is equal to the Triangle given B, and the Angle MFH, is equal to GFE, and GFE is equal to the given Angle C.

PROP. XIII. Fig. XVI.

Parallelograms BCDA, GHFE, *ſtanding upon equal Baſes* BC, GH, *and betwixt the ſame Parallels* AF, BH, *are equal one to the other.*

DRaw BE and CF, becauſe BC is equal to GH, and GH equal to EF, therefore is BCFE a Parallelogram. Whence the Parallelogram BCDA is equal to BCFE, and that equal to GHFE, which was to be demonſtrated.

PROP. XIV. Fig. XVII.

Triangles BCA, BCD, *ſtanding upon the ſame Baſe* BC, *and between the ſame Parallels* BC, EF, *are equal one to the other.*

DRaw BE parallel to CA, and CF parallel to BD, then is the Triangle BCA, equal to half the Parallelogram BCAE, and alſo equal to half BDFC, and that equal to the Triangle BCD, which was to be demonſtrated.

PROP.

PROP. XV. Fig. XVIII.

If a Parallelogram ABCD, *have the ſame Baſe* BC, *with the Triangle* BCE, *and be between the ſame Parallels* AE, BC, *then is the Parallelogram* ABCD, *double to the Triangle* BCE.

LET the Line AC be drawn. Then is the Triangle BCA equal to BCE: Therefore is the Parallelogram ABCD, equal to two ſuch Triangles as BCA, and likewiſe alſo equal to two ſuch Triangles as BCE, which was to be demonſtrated.

From hence may the Area (or Content) of any Triangle as BCE be found. For whereas the Area of the Parallelogram ABCD is produced by the Altitude drawn into the Baſe, therefore ſhall the Area of a Triangle be produced by Half of the Altitude drawn into its Baſe, or Half its Baſe drawn into its Altitude; as if ſo be, the Baſe BC be 8, and the Altitude 7, taking the Half of the Baſe 4, and multiplying it by the Altitude 7, it produceth 28, which is the Area or Content of the Triangle BCE; or otherwiſe, if you take the whole Baſe 8, and half the Altitude, 3 and an Half, and multiply them, they produce 28 (as before) for the Content.

PROP. XVI. Fig. XIX.

Upon a right Line given FG, *to make a Parallelogram* FL, *equal to a right lined Figure given* ABCD, *at a right lined Angle given* E.

RESolve the right lined Figure given into two Triangles BAD, BCD; then make a Parallelogram FH equal to BAD, ſo that the Angle F may be equal to the Angle E (as you were taught at the 12th Prop.) FI being produced to K, make the Parallelogram IL equal to the Triangle BCD. Then is the Parallelogram FL equal to FH more IL, and therefore equal to the Figure given ABCD, which was to be done.

SCHOL. Fig. XX.

HEnce is eaſily found the Exceſs HE, whereby any right lined Figure A, exceeds a leſs right lined Figure B; namely, If to ſome right Line CD both be applied, and both the Trapezia's, each of them being divided into two Triangles, and working as before is taught, you will find the Parallelogram DF equal to the Trapezia A, and the Parallelogram DH equal to the Trapezia B, ſo that the Figure A exceeds the Figure B by ſo much as the Parallelogram GEFH contains.

PROP.

PROP. XVII. Fig. XXI.

In right Angled Triangles BAC, *the Square* BE, *which is made of the Side* BC *that ſubtends the right Angle* BAC, *is equal to both the Squares* BG *and* CH, *which are made of the Sides* AB, AC, *containing the right Angle.*

JOin AE and AD, and draw AM parallel to CE, becauſe the Angle DBC is equal to FBA, add the Angle ABC common to them both, then is the Angle ABD equal to FBC. Moreover AB is equal to FB, and BD equal to BC; therefore is the Triangle ABD equal to FBC. But the Parallelogram BM, is equal to two ſuch Triangles as ABD, and the Parallelogram or Square BG, is equal to two ſuch Triangles as FBC (for GAC is one right Line by the Hypotheſis) therefore is the Parallelogram BM equal to the Quadrate BG. By the ſame Way of Argument, is the Parallelogram CM equal to the Quadrate CH; therefore is the whole Parallelogram (or Square) BDEC equal to the two Quadrates BAGF and ACIH, which was to be done.

PROP.

PROP. XVIII. Fig. XXII.

There are Three Quadrates or Squares given, whereof the Sides are AB, BC, CE, *and it is required to make one Square, whose Area or Content shall be equal to the Area of those Three Squares.*

Make the right Angle FBZ, having the Sides infinite (the Meaning of infinite is to draw the Sides long enough, and of what Length there is no Determination) and on these two Sides transfer AB and BC; that is to say, Take the given Line AB between your Compasses, and place it from the Angle B to A; also take the Line given BC, and place it from the Angle B to C; join AC (*viz.* draw the Line AC) then is a Square, whose Side is AC, equal to two Squares made of the two Lines AB and BC, then take the Line AC and place it from B to X; also take the third given Line or Side CE, and place it from B to E, then draw the Line EX, then a Square being made whose Side is EX, is equal to three Squares, being made of the three Lines or Sides given, AB, BC, CE, which was to be done. The Truth whereof is manifested by Arithmetick; for let the Line AB be 8 Feet in Length, then the Line BC will be 5 Feet, and the Line CE will be 4 Feet, and the Line EX will be 10 Feet and 6 Inches: Now the Square of 8 is 64, and the Square of 5 is 25, and the Square of 4 is 16, which three Squares being added together, produce

duce 105, for the Area of the three given Squares; therefore the Square of the Line EX being 10 Feet and 6 Inches, is 105 Feet, and equal to the three Squares given.

PROP. XIX. Fig. XXIII.

Two unequal right Lines being given AB, BC; *to make a Square equal to the Difference of the two Squares of the given Lines* AB, BC.

FRom the Centre B, with the Diſtance BA deſcribe a Semicircle, and from the Point C erect a Perpendicular CE, meeting with the Circumference in E, and draw BE. Then is the Square of BE (or BA) equal to the Square of BC and CE. Therefore when the Square of BC is taken out of the Square of BA, the remaining Part of the Square of BA will be equal to the Square of CE, which was to be done.

PROP. XX. Fig. XXIV.

Any two Sides of a right Angled Triangle ABC, *being known, to find out the third.*

LET the Sides AB, AC, encompaſſing the right Angle, be the one 6 Feet, the other 8 Feet: Therefore, whereas the Square of AB is 36, and the Square of AC is 64, which being added, make 100, therefore (as you were taught at the 17th Prop.) the other Side ſought for, muſt be equal in Power (*viz.* being ſquared) to the two

two given Sides being ſquared, which contain 100, whoſe Square Root is 10, the Length of the Side ſought B C, which was to be done.

PROP. XXI. Fig. I.

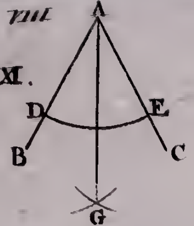
To deſcribe a Circumference that ſhall touch any three Points given, provided they are not in a right Line; ſuppoſe the Points given, to be A. B. C. *(The Figures of this, and the following Propoſitions, you will find in the next folded Page.)*

TAke the Diſtance between A and B with your Compaſſes, and ſetting one Point of the Compaſs on the Point by A, deſcribe the Arch D E; then with the ſame Diſtance ſetting one Point of the Compaſs on the Point by B, deſcribe another Arch, which will cut the former Arch in the Points by 1; then laying a Ruler to the Points by 1, where the Arches interſect, draw a ſtrait Line F G. This being done, take with your Compaſſes the Diſtance between the other Point C and B, and with this Diſtance, ſetting one Point in B, deſcribe the Arch HI; then with the Compaſſes at the ſame Diſtance, ſetting one Point on the Prick by C, deſcribe another Arch, cutting the former in the Point, and by 2, thro' which Points draw another ſtrait Line, 'till it cut through the firſt ſtrait Line, as at G; I ſay G is the Centre, from whence the Circumference A. B. C. is deſcribed, which toucheth the three given Points.

PROP.

Prop. VIII.

Fig: XI.



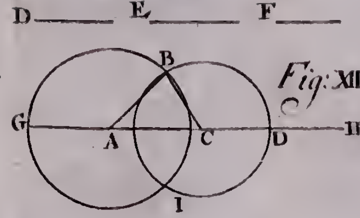


Fig: XII.

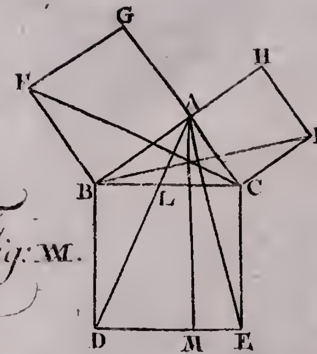
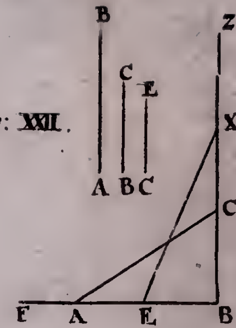


Fig: XXI.

Fig: XXII.



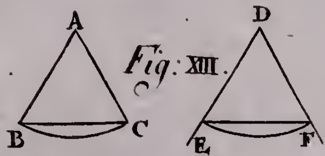


Fig: XIII.

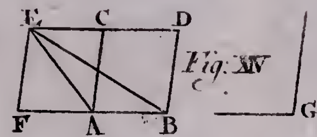


Fig: XIV.

Fig: XV.

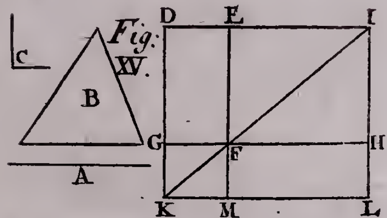


Fig: XVI.

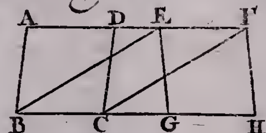


Fig: XXIII.

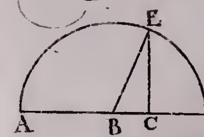


Fig. XXIIII.

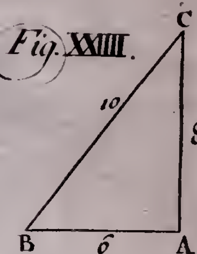
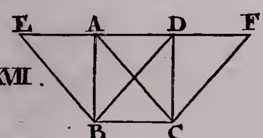


Fig: XVII.



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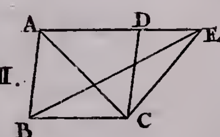

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Fig: XIX.

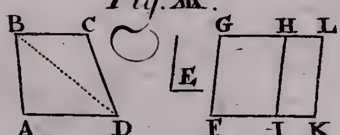
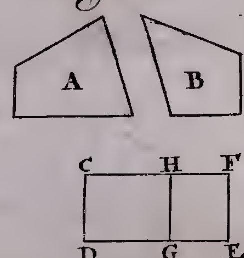


Fig: XX



PROP. XXII. Fig. II.

To describe an Oval upon a Length given AB.

DIvide the given Line into three equal Parts (as the next Proposition following will teach) ACDB, setting one Point of the Compasses at the Point by C, with the Distance CA, describe the Circle AEF, then with the same Distance setting one Point by D, describe the Circle BEF, then draw four strait Lines thro' the Centres C and D, and the Intersection of the two Circles E and F; then setting one Point of the Compasses in E, and extending the other Point to I, describe the Arch IH, then with the Compasses at the same Distance, setting one Point on the Intersection by F, describe the Arch OP, which concludes the Oval. *Note*, That IAO and HBP, are vulgarly called Hanses, and IH and OP, are called Schemes.

PROP. XXIII. Fig. III.

To divide a strait Line given AB, *into three equal Parts.*

FRom the End A, draw at Pleasure the Line AC, making what Angle you will, then from the other End of the Line B, draw the Line BD, parallel to the Line AC; then opening your Compasses at Pleasure, setting one Point in A, turn them three Times over the Line AC, which will make three Divisions, *viz.* EFG, then with the Compasses continuing at the same Distance, setting one Foot in B, make three Divisions on the

Line

Line BD, *viz.* HIK, then draw with a Ruler, and the Point of the Compaſs, a ſtrait Line from A to K, another from E to I, alſo another from F to H, and another from G to B; and they being drawn, the given Line AB is divided into three equal Parts, which was to be done. You may, if you pleaſe, divide the ſame Line, or any other, into what Number of equal Parts you pleaſe, by dividing the two parallel Lines AC and BD, into ſo many equal Parts as you would have the given Line divided into.

PROP. XXIV. Fig. IV.

To deſcribe an Oval equal in Length to the firſt Oval not riſing ſo high.

DIvide the given Line AO into four equal Parts (by the foregoing Prop.) in BCD, then taking one of thoſe Parts between the Compaſſes, upon the Centres BCD, deſcribe three Circles, and thoſe 2 Parts of the middlemoſt Circle, that is without the 2 other Circles, divide in the Middle at E and F, then from E to the Centre D, draw a ſtrait Line, and contine it to the Circumference at 4, alſo draw another ſtrait Line from E, thro' the Centre by B to the Circumference, which will cut it at 3; likewiſe from F thro' the ſame Centres draw right Lines, which will cut the two Circumferences, the one in 2, the other in 1. Then from the Centre F with the Radius (or Diſtance) F, 1, deſcribe the Arch 1, 2; alſo from the Centre E with the ſame Radius, deſcribe the Arch 3, 4, which concludes the Oval.

PROP.

PROP. XXV. Fig. V.

Another Way to deſcribe Ovals.

UPon the Line given, deſcribe two equilateral Triangles, join them together with one common Baſe, ſo that they make a Rhombus; then continue (or draw) the Line A C to 3, ſo that C 3 may be 6, ſuch Parts whereof A C is 5, *viz.* It muſt be the Length of A C, and one 5th Part more of it. Alſo draw the Line B D to 2, that it may be the ſame Length with A 3; alſo draw B C to 1, and A D to 4, being all of one Length: Then from the Centre A, with the Radius A 3, deſcribe the Arch 3, 4; alſo from the Centre B, with the ſame Radius, deſcribe the Arch 1, 2; then from the Centre C, with the Radius C 1, deſcribe the Arch 1, 3; likewiſe from the Centre D, with the ſame Radius, deſcribe the Arch 2, 4, which will incloſe the Oval. From theſe 4 Centres you may deſcribe Ovals, greater or leſſer as you pleaſe.

PROP. XXVI. Fig. VI.

To deſcribe an Oval according to any Length and Breadth given.

LET the Length given be AB, and the Breadth CD.

Apply the 2 given Lines together, ſo that they may cut each other into 2 equal Parts, and at right Angles in the Point E; then take half the Line AB between your Compaſſes, and ſetting one Point of the Compaſſes in C, extend the other till it touch the

the Line A B, in K and L, which 2 Points are called the burning Points, or Focus's. In which Points, drive 2 Nails if you deſcribe it upon Boards, but upon Paper, as here, 2 Pins will do; the Pins being ſtuck firm in the Points K and L, ſtick alſo another Pin in the Point C; then take a Thread and encompaſs theſe 3 Pins in Form of a Triangle, pulling the Thread tight, tie the 2 Ends of the Thread together by a Knot at C; then taking out the Pin at C, take a Pencil, holding it cloſe to the Inſide of the Thread, and carrying the Pencil round upon the Paper, about the Pins, with the Thread always ſtrait, the Ellipſis or Oval A C B D ſhall be thereby deſcribed.

PROP. XXVII. Fig. VII.

To find the Centre and the two Diameters of an Oval.

LEt S E T D be the Oval whereof the Centre and the Diameters are to be found.

Within the Oval, draw at Diſcretion the Parallel Lines E F, G H; cut theſe Lines into 2 equally in I and K, draw the Line I K, cut it into 2 equally in L, which is the middle Centre of the Oval; upon this Centre L, deſcribe at Pleaſure the Circle M N O, cutting the Oval in P and Q, from which Sections, draw the right Line P Q, cut it in the Middle in R, from which, through the Centre L, draw the greater Diameter S T, and from the Centre L, draw the leſſer Diameter E L D parallel to the Line P Q, which was to be done.

PROP.

PROP. XXVIII. Fig. VIII.

To deſcribe an Oval with a pair of Compaſſes, to any Length and Breadth given.

I Shall only deſcribe a Semi-oval, and according to the ſame Rules, if you will, you may deſcribe the whole Oval.

Let the Length given be A B, and one half of the Breadth C D; divide A B into ſeven equal Parts; then upon one ſeventh Part from A, as at E, raiſe a Perpendicular from the Line A B (*viz.* E G.) Alſo at one ſeventh Part from B, as at F, raiſe another Perpendicular F H; then divide the half Breadth given C D, into fifteen equal Parts, and take eleven of thoſe Parts and ſet upon the Perpendicular from E to G, and likewiſe from F to H; then taking the Space between A and G, ſetting one Point of the Compaſſes in A, deſcribe the Arch G i; keeping the Compaſſes at the ſame Diſtance, ſet one Point in G, and deſcribe another Arch, which will cut the former in the Point by i; from which Point, with the Radius A G, deſcribe the Semi-hanſe A G. This being done, take between your Compaſſes the Space B H, and ſetting one Point in B, deſcribe the Arch I i; then remove your Compaſſes to H, and interſect that Arch in the Point by i; then ſetting your Compaſſes on the Point i, with the ſame Diſtance, deſcribe a Part of the Oval B H, which part, as alſo the other part A G, are vulgarly called Semi-hanſes, becauſe it is but a Semi-oval (Semi ſignifies half) but if it had been an whole Oval, then the Semi-hanſe above the Line A, and another Semi-hanſe below the

Line A, being joined, is called an Hanſe, from the *Latin* Word *Hanus*, ſignifying a great-bellied thing. The other part to be deſcribed, from G to H, is called the Scheme, which to deſcribe, continue or draw longer the half Breadth D C, and in that Line find a Centre, whereon ſetting one Point of the Compaſſes, the other Point may touch the three Points G, D, H, as on the Centre I, whereby deſcribe the Scheme G D H, which was to be done.

CHAP. III.

A Digreſſion concerning Ellipſis Arches.

AND ſince Ellipſis or Semi-oval Arches, being neatly wrought in Brick, ſhew very pleaſant; and are ſometimes uſed over Gate-ways, and ſometimes over Kitchen Chimneys, inſtead of Mantle-trees; I think fit to write ſomething concerning them, relating to Bricklayers making the Moulds, and dividing the Courſes.

The Ellipſis you may deſcribe to what Length and Heighth you pleaſe, either by the laſt Propoſition, or by the 26th.

We will ſuppoſe an Ellipſis Arch to be made over a Chimney, whoſe Diameter between the Jaums is 8 Feet, and the under Side of the Arch, at the Key, to riſe in Height 18 Inches, from the Level of the Place whence you begin to ſpring the Arch. The Height or Depth of the Arch, we will ſuppoſe to be made of the Length of two Bricks, which when they are cut to the Sweep of the Arch,

will

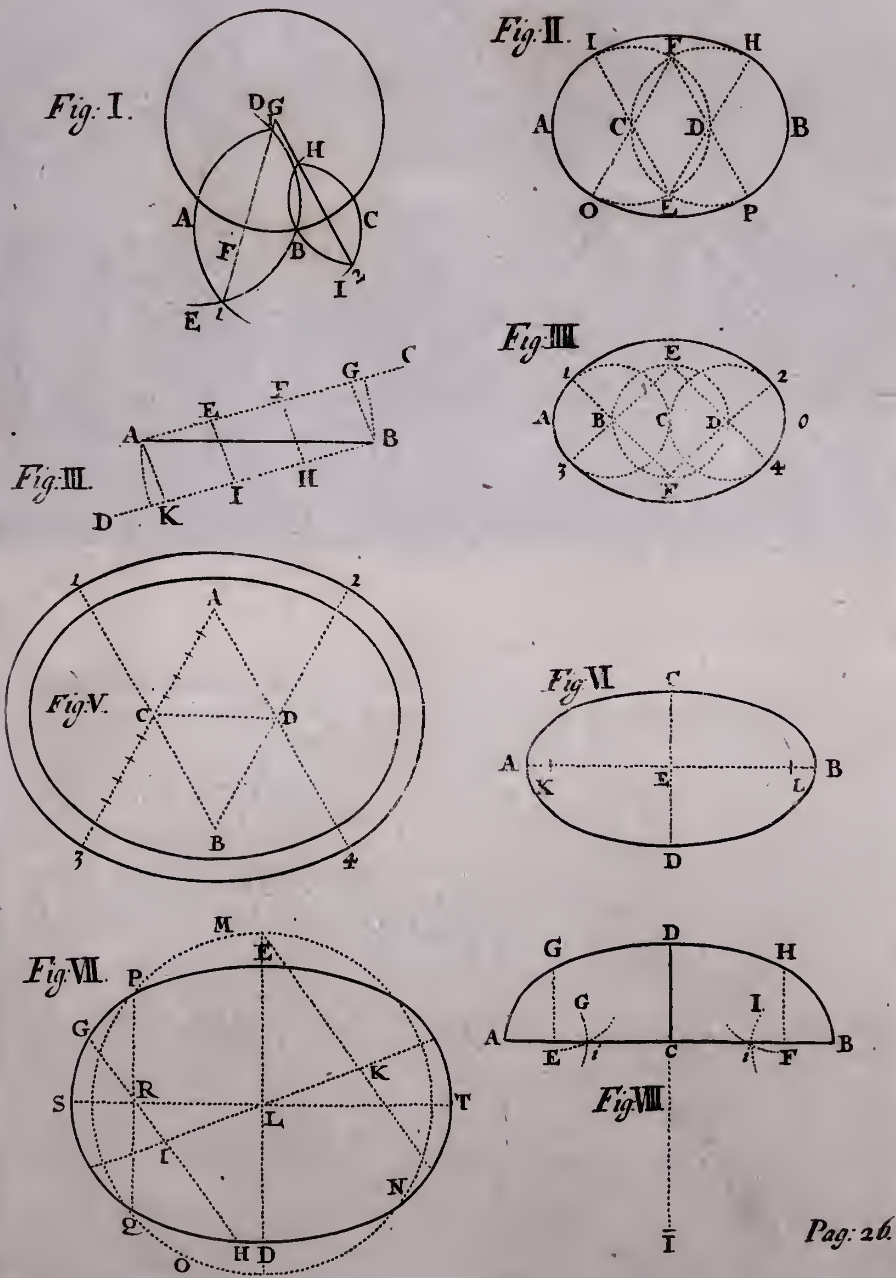


Fig: I.
Fig: II.
Fig: III.
Fig: IIII
Fig: V.
Fig: VI
Fig: VII.
Fig: VIII

will

will not contain above 14 Inches; and perhaps you muſt cement Pieces to many of the Courſes in the Hanſe, to make them long enough to contain or hold 14 Inches, eſpecially if you intend to make the Courſes of the Hanſe, and the Courſes of the Scheme to ſeem alike in Greatneſs on the under Side of the Arch. For if you make the Hanſe to come to a true Sommering for the Scheme, by that time that you have ended the Hanſe, and are ready to ſet the firſt Courſes of the Scheme, the Mould, and ſo likewiſe each Courſe in the Hanſe, will be much leſs at the lower Part, or under Side of the Arch, than the Mould or Courſes of the Scheme; as you may perceive by the Hanſe B K, in the IX *Fig.* which Way of working theſe kind of Arches is ſtronger, than to make the Courſes ſeem alike in Bigneſs in Hanſe and Scheme, altho' it be not ſo pleaſing to the Eye. In the IX *Fig.* I will ſhew how to make one Half of the Arch this Way, and in the other Half ſhew how to make the Courſes in Hanſe and Scheme of a Bigneſs.

Firſt, Deſcribe the under Side of the Arch (*viz.* the Ellipſis A D B, whoſe Diameter A B is 8 Feet, and the Height C D 18 Inches) upon ſome ſmooth Floor, or ſtrait plaiſtered Wall, or ſuch like; then continue (*viz:* draw longer) both the Lines A B, C D, cutting each other at right Angles; then from A to E; alſo from B to F; likewiſe from D to G, ſet 14 Inches, the intended Height of your Arch. Then deſcribe another Ellipſis to that Length and Height, after this manner: Lay a ſtrait Ruler on the Centre by I, and on the joining of the Hanſe and the Scheme togethet, as at K, and draw the Line K L; then ſet one Point of your

Compaſſes in the Centre of the Hanſe at M, and open the other Point of the Compaſſes to F, and deſcribe the upper Hanſe F L; likewiſe ſetting one Point of the Compaſſes in the Centre by I, with the other extended to G, deſcribe the Scheme G L; (altho' I ſpeak here of Compaſſes, yet when you deſcribe an Arch to its full Bigneſs, you muſt make uſe of Centre Lines, or Rules; the laſt are beſt, becauſe Lines are ſubject to ſtretch) then taking between your Compaſſes the Thickneſs of a Brick, abating ſome ſmall Matter, which will be rubb'd off from both Beds of the Brick, with the Compaſſes, at this Diſtance, divide the upper Hanſe from L to F into equal Parts, and if they happen not to divide it into equal Parts, then open them a ſmall matter wider, or ſhut them a ſmall matter cloſer, till it doth divide it into equal Parts, and look how many equal Parts you divide the upper Hanſe into, ſo many equal Parts you muſt divide the lower Hanſe from K to B into likewiſe (or you may divide the upper Hanſe from the Centre O, making a right Angle from each Sommering Line to the Ellipſis, as is ſhewn in deſcribing the ſtreight Arches following; and from the Centre O, and the Diviſions in the upper Hanſe being thus divided, you may draw the ſtrait Lines to the lower Hanſe, and not divide it with the Compaſſes) thro' each of which Diviſions, with a Rule and Pencil, draw ſtrait Lines; then get a Piece of thin Wainſcot, and make it to fit between two of theſe Lines, allowing what thickneſs for Mortar you intend, this will be the Sommering Mould for the Hanſe; then divide the upper Scheme likewiſe, with the Compaſſes, at the ſame Diſtance, into equal Parts, and laying

laying a Ruler on the Centre I, from each Division in the Scheme G L, draw ſtrait Lines to the lower Scheme D K, then make another Sommering Mould to fit between two of theſe Lines, abating ſo much as you intend the Thickneſs of your Joints of Mortar to be, which, if you ſet very cloſe Mortars, the Breadth of the Line will be enough to allow: then laying the inner Edge of a Bevil ſtrait on the Line K L, bring the Tongue to touch the under Side of the firſt Courſe of the Scheme; then take up the Bevil, and ſet that Bevil Line upon the Sommering Mould of the Scheme, which Bevil Line ſerves for each Courſe in the Scheme: but you muſt take the Bevil of each Courſe in the Hanſe, and ſet them upon your Sommering Mould, and number them with 1, 2, 3, 4, *&c.* becauſe each Courſe varies.

Thus having made your Sommering Moulds, in the next place you muſt make the Moulds for the Length of your Stretchers, and for the Breadth of the Headers and the Cloſiers. A Piece of Wainſcot 7 Inches long, and 3 Inches and an half broad, wil ſerve for the Length of the Stretchers, and the Breadth of the Headers; the Cloſiers will be 1 Inch $\frac{3}{4}$ broad.

So the Cloſier will be half the Breadth of the Header, and the Header half the Length of the Stretcher, which will look well.

It remains now to ſpeak ſomething to the other part of the Arch, to wit, A D, whoſe Courſes both in Hanſe and Scheme run alike upon the Ellipſis Lines, and ſeem of one Bigneſs (altho' perhaps there may be ſome ſmall matter of Difference, by reaſon I have not divided the Courſes in this Fi-

gure from a right Angle, but every Courſe from the Angle, which it makes with the Ellipſis, which I choſe rather to do, that ſo the Bevil of one Courſe might not ſeem to run more upon the Ellipſis than the Bevil of another, and the Difference of the Thickneſſes being ſo inconſiderate, is not diſcerned.)

Having deſcribed both the Ellipſis Lines A D, E G, divide each of them into a like Number of equal Parts, always remembring to make each Diviſion on the upper Ellipſis Line no greater than the Thickneſs of the Brick will contain, when it is wrought; then through each Diviſion, in both the Ellipſes, draw ſtrait Lines, continuing them 4 or 5 Inches above the upper Ellipſis Line, and as much below the lower Ellipſis Line: Then having provided ſome thin Sheets of fine Paſtboard about 20 Inches ſquare, cutting one Edge ſtrait, take one Sheet, and lay the ſtrait Edge even upon the Line A E, ſo that it may cover both the Ellipſis Lines, and being cut to Advantage, it may cover 8 Courſes (or 9 of the ſtrait Lines.) Having laid it thus upon the Figure of the Arch, ſtick a Pin or two through it, to keep it in its Place; then lay a Ruler upon the Paſtboard to the 7th, 8th, or 9th ſtrait Line of the Arch, according as the Paſtboard be in Bigneſs to cover them, and take a ſharp Penknife, laying the Ruler upon the Paſtboard true to the ſtrait Line (whoſe Ends being continued longer than the Arch is deep, as I directed before, will be ſeen beyond the Paſtboard) and cut the Paſtboard true to the Line; then take another Sheet and join to it, and cut it as you did the firſt, and ſo continue till you

you have covered the Arch from A E juſt to the Line D G, ſticking Pins in each Sheet to keep them in the Places where you lay them: Then deſcribe both the Ellipſis Lines upon the Paſtboard, from the ſame Centers and Radii that you deſcribed the Ellipſes under the Paſtboard, and either divide the Ellipſis Lines with the Compaſſes on the Paſtboard, or elſe draw Lines upon the Paſtboard from or by the ſtrait Lines underneath whoſe Ends you ſee; but the ſurer Way is to divide the Ellipſes on the Paſtboard, and draw Lines through thoſe Diviſions as you did beneath the Paſtboard; then ſet 7 Inches, being the Length of each Stretcher, from A towards E, and from D towards G, and deſcribe, from the former Centres, the Ellipſis *o o* thro' each other Courſe on the Paſtboard, as you may ſee in the Fig. alſo ſet 3 Inches and an Half, being the Breadth of the Header from A towards E, and likewiſe from D towards G; alſo ſet the ſame 3 Inches and an Half from E towards A, and from G towards D, and deſcribe theſe two Ellipſis Lines from the ſame Centres thro' each Courſe which the Ellipſis Line of the Stretchers miſs'd; likewiſe draw in the ſame Courſes, two other Ellipſis Lines one Inch and $\frac{3}{4}$ from each of thoſe two Lines you drew laſt; which is the Breadth of the Cloſiers; thus one Courſe of the Arch will be divided into two Stretchers, and the next to it into three Headers, and two Cloſiers through the whole Arch. This being done, cut the Paſtboard according to the Lines into ſeveral Courſes, and each other Courſe into two Stretchers, and the Heading Courſe into three Headers, and two Cloſiers, exactly according

to the Sweep of the black Lead Lines, and mark each Courſe with Figures, marking the firſt Courſe of the Hanſe with 1, the next with 2, the third with 3, and ſo continue till you have marked all the Courſes to the Key or Middle, for every Courſe differs; you were beſt to mark the lower Cloſier in each Courſe with a Cypher on the left Hand of its own Number, that you may know it readily from the upper Cloſier, and make no Miſtakes when you come to ſet them; alſo the middle Headers in each Courſe ſhould be marked beſides it own Number, the Thickneſs of the upper Header being eaſily diſcern'd from the lower Header needs no marking beſides its own Number: The croſs Joints, and likewiſe the under ſide and upper ſide of each Courſe muſt be cut circular, as the Paſtboards, which are your Moulds, direct you.

If you will add a Keyſtone and Chaptrels to the Arch, as in the Figure; let the Breadth of the upper part of the Keyſtone be the Height of the Arch, *viz.* 14 Inches, and Sommer, from the Center at I, then make your Chaptrels the ſame Thickneſs that your lower part of the Keyſtone is, and let the Keyſtone break without the Arch, ſo much as you project or ſail over the Jaums with the Chaptrels.

Other kind of Circular Arches, as half Rounds and Schemes, being deſcribed from one Centre, are ſo plain and eaſy, that I need ſay nothing concerning them: But ſince ſtrait Arches are much uſed, and many Workmen know not the true Way of deſcribing them, I ſhall write ſomething briefly concerning them.

Strait Arches are uſed generally over Windows and Doors, and according to the Breadth of the

Piers

Piers between the Windows, ſo ought the Skew-back, or Sommering of the Arch to be; for if the Piers be of a good Breath, as 3 or 4 Bricks in Length, then the ſtrait Arch may be deſcrib'd (as it's vulgarly called) from the *Oxi*, which being but part of a Word is taken from the Word *Oxigonium*, ſignifying an equilateral Triangle with three ſharp Angles; but if the Piers are ſmall, as ſometimes they are but the Length of two Bricks, and ſometimes but one Brick and an Half, then the Breadth of the Window, or more, may be ſet down upon the middle Line for the Centre, which will give a leſs Skew-back or Sommering than the Centre from an *Oxi*. I will ſhew how to deſcribe them both Ways, and firſt from the *Oxi*.

Suppoſe a ſtrait Arch 1 Brick and an Half in Height, to be made over a Window, 4 Feet in Width. [See *Fig* X.] wherein one Half of the Arch is deſcribed from the Oxi, and the other Half from the Width of the Window. Let the Width of the Window be A B; taking the Width between the Compaſſes, from A and B as two Centres, deſcribe the two Arches, interſecting each other at P, (though I ſpeak here of Compaſſes, yet when you deſcribe the Arch to its full Bigneſs, you muſt uſe a Ruler or a Line, ſcarce any Compaſſes being to be got large enough) then draw another Line above the Line A B, as the Line C D, being parallel to it, at ſuch a Height as you intend your Arch to be, as in this *Fig*. at 12 Inches; but moſt commonly theſe ſort of Arches are but 11 Inches in the Height, or thereabouts, which anſwers to 4 Courſes of Bricks, but you may make them more or leſs in Height according as Occaſion requires; then

then laying a Ruler on the Centre P, and on the End of the Line A, draw the Line A C, which is vulgarly called the Skew-back, for the Arch.

The next thing to be done, is to divide thoſe two Lines A B and C D into ſo many Courſes as the Arch will contain, the Thickneſs of a Brick being one of them, which ſome do by dividing the upper Line into ſo many equal Parts, and from thoſe Parts, and from the Centre P, draw the Sommering Lines or Courſes; others divide both the upper and lower Line into ſo many equal Parts, and make no uſe of a Centre, but draw the Courſes by a Ruler, being laid from the Diviſions on the upper Line, to the Diviſions on the lower Line, both which Ways are falſe and erroneous; [but this by Way of Caution.]

Having drawn the Skew-back A C, take between your Compaſſes the Thickneſs that a Brick will contain, which I ſuppoſe to be two Inches when it is rubb'd, and ſetting one Point of the Compaſſes on the Line C D, ſo that when you turn the other Point about, it may juſt touch the Line A C in one place, and there make a Prick in the Line C D, but do not draw the Sommering Lines until you have gone over half the Arch, to ſee how you come to the Key or Middle; and if you happen to come juſt to the middle Line, or want an Inch of it, then you may draw the Lines, but if not, then you muſt open or ſhut the Compaſſes a little till you do.

Then keeping one End of the Rule cloſe to the Centre at P, (the ſureſt Way is to ſtrike a ſmall Nail in the Centre P, and keep the Rule cloſe to the Nail) lay the other End of the Rule cloſe to the

the Prick that you made on the Line C D, keeping the Compaſſes at the ſame Width (*viz.* 2 Inches) ſet one Point of the Compaſſes on the Line C D as before, ſo that the other Point being turned about may juſt paſs by the Rule, and, as it were, touch it in one Place (you muſt remove the Point of the Compaſſes upon the Line C D, farther or nearer to the Rule, until it juſt touch the Rule in one Place) and ſo continue with the Rule and Compaſſes until you come to the middle Line, and if it happen that your laſt Space want an Inch of the Middle, then the Middle of the Key-courſe will be the Middle of the Arch, and the Number of the Courſes in the whole Arch will be odd; but if the laſt Space happen to fall juſt upon the middle Line E F, as it doth in the Fig. then the Joint is the Middle of the Arch, (but if it ſhould happen neither to come even to the Line, nor want an Inch of it, then you muſt open or ſhut the Compaſſes a ſmall matter, and begin again till it doth come right) and the Number of the Courſes in the whole Arch is an even Number.

Note, When the Number of all the Courſes in the Arch is an even Number, then you muſt begin the two Sides contrary, *viz.* A Header to be the lower Brick of the firſt Courſe on one Side (or half) of the Arch, and a Stretcher the lower Brick of the firſt Courſe on the other Side (or Half) of the Arch; And contrariwiſe, if it happen that the Number of the Courſes be an odd Number, as 25 or 27, or ſuch like, then the firſt Courſes of each Half of the Arch muſt be alike, that is, either both Headers or both Stretchers, at the Bottom.

Thus having deſcribed the Arch, the next thing to be done is to make the Sommering Mould, which

to

to do, get a Piece of thin Wainſcot (being ſtrait on one Edge, and having one Side plained ſmooth to ſet the Bevil Strokes upon) about 14 Inches long, and any Breadth above 2 Inches; then laying your Ruler, one End at the Centre P, and the other End even in the Skew-back Line, clap the ſtrait Edge of the Wainſcot cloſe to the Rule, ſo that the lower End of the Wainſcot may lie a little below the Line A B; then take away the Centre Rule, but ſtir not the Wainſcot, and laying a Ruler upon the Wainſcot, juſt over the Line C D, ſtrike a Line upon the Wainſcot; then ſet one Point of the Compaſſes, being at the Width of a Courſe (*viz.* 2 Inches) upon that Line, ſo that the other Point being turned about, may juſt touch the ſtrait Edge of the Wainſcot (as you did before in dividing the Courſes) then make a Prick on the Line on the Wainſcot, and laying your Centre Rule upon it, and on the Centre P, draw a Line upon the Wainſcot by the Ruler with a Pencil, or the Point of a Compaſs, and cut the Wainſcot to that Line, and make it ſtrait by ſhooting it with a Plane; then your Wainſcot will fit exactly between any two Lines of the Arch. You may let it want the Thickneſs of one of the Lines, or ſome ſmall Matter more, which is enough for the Thickneſs of a Mortar. The Length of your Stretcher in this Arch may be 8 Inches and $\frac{1}{4}$, and the Header 3 Inches and $\frac{3}{4}$, but if your Arch be but 11 Inches in Height, then make your Stretcher 7 Inches and an Half long, and the Header 3 Inches $\frac{1}{2}$. One Piece of Wainſcot will ſerve both for the Length of the Scretcher, and the Length of the Header, making it like a long Square, or Oblong, whoſe Sides are 8 Inches $\frac{1}{4}$, and 3 Inches and $\frac{3}{4}$.

Then

Then take a Bevil, and laying the inner Edge of it ſtrait with the Line A B, and the Angle of the Bevil juſt over the Angle at A; take off the Angle that the Skew-back Line A C makes with the Line A B, and ſet it upon the ſmoothed Side of your Sommering Mould for the Bevil Stroke of your firſt Courſe; then drawing your Bevil towards E, ſtrait in the Line, until the Angle of the Bevil be juſt over the Angle that the ſecond Sommering Line makes with the Line A B. When it is ſo, draw the Tongue of the Bevil to lie even upon the ſecond Sommering Line, (in brief, cauſe the Bevil to lie exactly on the Line A B, and on the ſecond Sommering Line) then take up your Bevil, and lay it on the Mould, and ſtrike that Bevil Line on the Mould with the Point of the Compaſſes about half a quarter of an Inch diſtant from the firſt, and that is the Bevil of the Under-ſide of the ſecond Courſe. Proceed thus, until you come to the middle Line E F, but after you have ſet 3 Bevil Lines upon your Sommering Mould, leave about $\frac{1}{4}$ of an Inch between the 3d and 4th, and ſo likewiſe between the 6th and 7th, and the 9th and 10th; which will be a great Help to you in knowing the Number of each Line on the Mould.

The Moulds for the other half of the Arch, namely E B, are made after the ſame Manner; but the Arch is deſcribed from a Centre beneath P, as Q; which cauſeth a leſs Skew-back (*viz.* B D.)

The diminiſhing of the Sommering Mould to any Skew-back may be found by the Rule of Three, by dividing a Foot into 10 equal Parts, and each of thoſe into 10 Parts, ſo that the whole Foot may contain 100 Parts. Then proceed thus: The upper Line

Line C F, will be 309, that is 3 Feet and almost 1 Inch, and the lower Line A E will be 252, that is, 2 Feet and an Half and $\frac{2}{100}$; and the upper Part of the Sommering Mould will be 17 almost, that is, two Inches of such whereof there are 12 in a Foot; having these three Numbers (*viz.* 309, 252, 17) work according to the Rule of Three, and you will find 13 and ^{6} of 100 Parts, that is almost 14 (such Parts whereof there are 100 in a Foot, Line-measure) for the Breadth of the lower Part of the Mould.

You may likewise find it Geometrically thus:

Having drawn the upper Line and under Line of the Arch, as C F and A E, and drawn any Skew-back, as suppose A C in [*Fig.* X.] make at Discretion the Angle G C H in [*Fig.* XI.] then take the upper Line C F, and set it from C to F; also take the lower Line A E, and set it from C to E, and draw the Line E F; then take the Thickness of your Brick, which suppose to be 2 Inches, and set it from F to G, and draw G H parallel to F E; I say, F G is the Breadth of the upper Part of the Sommering Mould, and E H the Breadth of the lower Part. Then make your Sommering Mould true to those two Lines, and beginning in the middle Line A C, describe the strait Lines by the Mould from the Key F E, until you come to the Skew-back A C, and then take off the Bevil Lines, and set them on your Sommering Mould: With which I conclude this first Book of Geometry, being, as it were, an Introduction to that which follows.

A

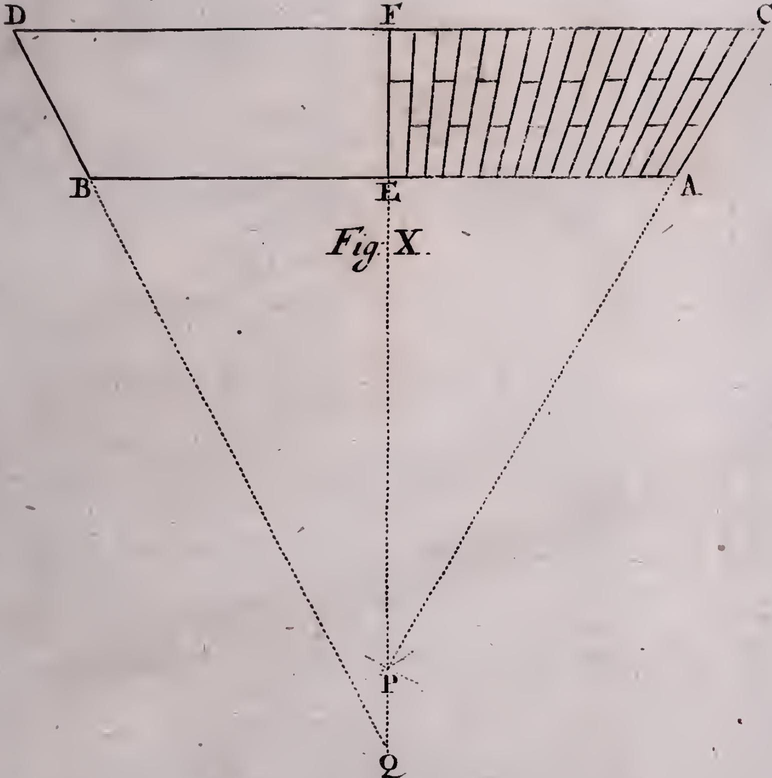


Fig: X.

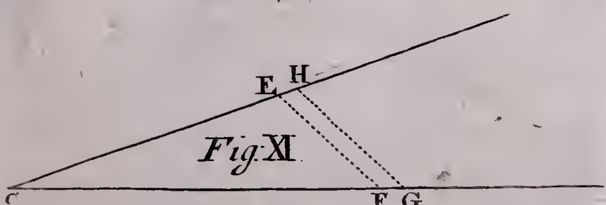


Fig: XI.

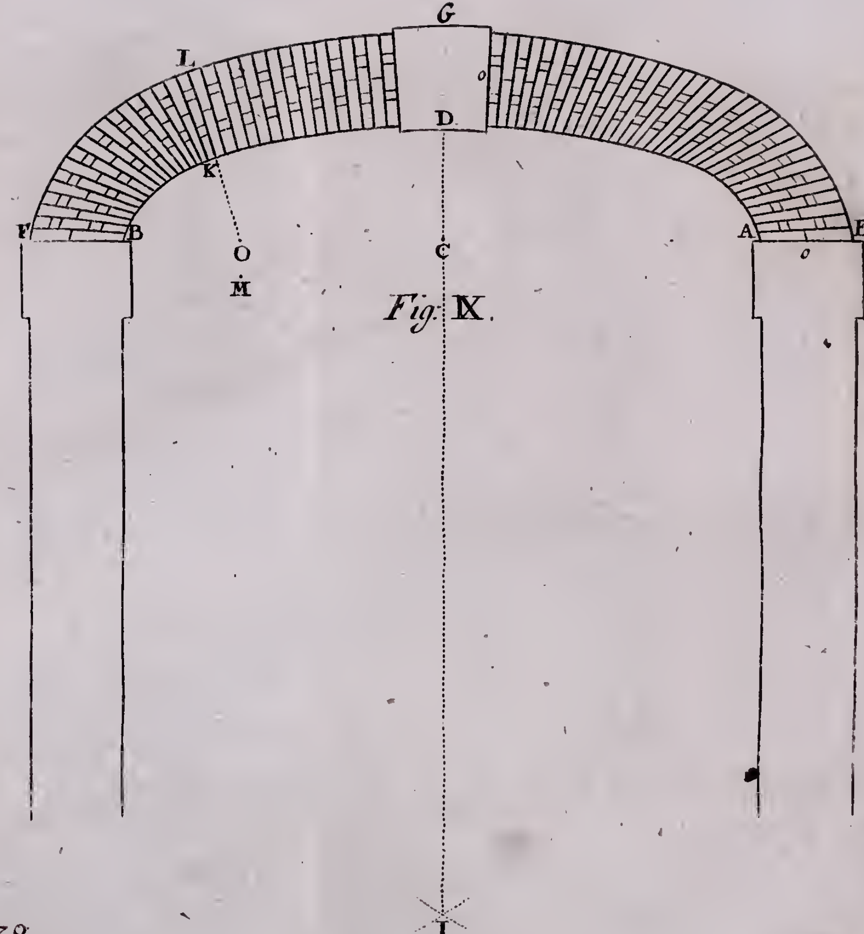


Fig: IX.

Pag: 38

A

GARDEN

OF

Geometrical Roſes:

OR, SOME

PROPOSITIONS

Being hitherto hid, are now made known.

The SECOND BOOK.

Written in *Latin*

By *THOMAS HOBBES*:

And done into *Engliſh*

By *VEN. MANDEY.*

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Geometrical ROSES.

PROP. I.

Of cutting a Right Line in extream and mean Proportion.

LET there be deſcribed a Square A B C D, and let each Side be divided in the Middle, in E, F, G, H; and F E, G H, being joined, they will cut each other in the Centre of the Square at I. Likewiſe from the Centre D, let there be deſcribed a Quadrant D A C, cutting F E and G H in K and X.

Laſtly, Let E X be drawn; I ſay E X is equal to the greater Segment of the right Line E F, or of the Side A B, being divided in extream and mean Proportion.

Let F X be drawn, and with that Semidiameter deſcribe an Arch of a Circle X z, cutting F E in z. Likewiſe with the Semidiameter E X, deſcribe an Arch of a Circle X y, cutting the ſame F E in y, and let the right Lines X z, X y be drawn.

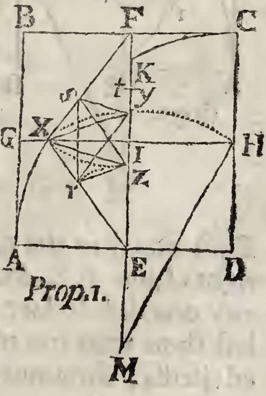
Now the Angle E z X is equal to thoſe 2 Angles z F X, F X z, becauſe theſe two Angles are within,

within, the other is without the Triangle z F X. Again, the same Angle E z X for the same Cause is equal to the 2 Angles X y z, y X z. And X z y and F X z are equal. Wherefore also the 2 Angles X F z, and z X y, are equal to one another.

Therefore the 2 Triangles F X z, y X z, have 2 Angles equal to 2 Angles of the Triangle z X y, and by consequence 3 to 3: Therefore they are Equiangled. Wherefore, as X F or E X, to X z; so is X z or X y, to z y. Therefore X z (or X y) is a mean Proportional between E X (or E y) and z y.

In the right Lines E X, F X, let there be taken E r, F s, and either of them equal to E z, or F y. Let r s, and s z be joined. Then r X, X s will be equal. Also r s will be parallel to the right Line E F; and therefore the Altern Angles r s y, F y s, and likewise the Altern Angles s r y, E z r, are all equal to one another; and F s y r, and E z s r, will be Rhombus's both alike. Therefore the right Lines F s, s z, r y, r E, E z, X y, are equal to one another.

And because it is manifest, that X y is a mean Proportional betwen E y and z y; also F y (equal to X y) will be a mean Proportional between E y and z y.



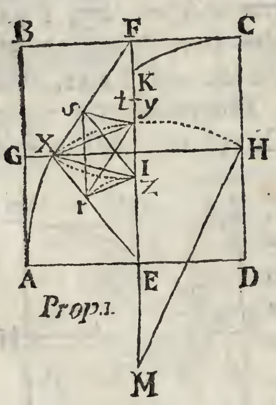
Prop. 1.

Therefore it will be, as E y to F y, so F y to z y; and being compounded, as E y more y F (that is E F) to F y more y z (that is, F z, or E y) so likewise is E y to E z. There-

fore EF is divided in z and y, ſo that the whole Line EF, a Part Ey, and the remaining Part Ez; likewiſe FE, a Part Fz, and the remaining Part Fy, are continual Proportionals. Therefore Ey, that is EX, is the greater Segment of the right Line EF, being divided in extream and mean Proportion, which was to be demonſtrated. Let him reprehend it that can.

ANIMADVERS.

To him that rightly conſiders, it is a Paradox: Neither *Euclid*, nor any other, hath taught this. Who ever thought that right Line which joins half one Side of the Square, with one third Part of the Quadrant inſcribed within the Square, to be the greater Segment of the Side cut in extream and mean Proportion? Thoſe Segments, of all hitherto, have been deſigned after this Manner. Produce the right Line IE in M, ſo that IE, EM, may be equal, and IM equal to the Side. Moreover from the Centre M, with the Radius MH or MG, deſcribe an Arch of a Circle, it will cut off a Part from EF (ſuppoſe



*Prop.*1.

Ey) equal to the greater Segment. But that the right Line EX ſhould be equal, they never, nor any one did declare. Therefore, Reader, beware, leſt thou truſt too much in an uncertain Thing unadviſedly, foraſmuch as we take Things near the Truth

Truth for true Things. Examine the Demonſtration, and confer with true or ſurd Numbers, or conſult Algebraiſts.

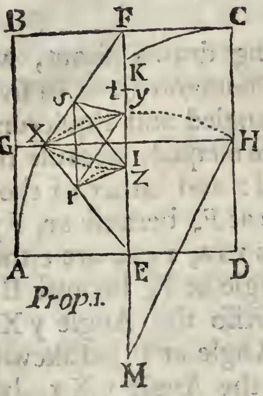
Consect. I.

THE Lines sy, rz, being drawn about, will make the Figure of a Pentagon (*viz.* a five-ſided Figure) Xsyzr, Equiangled and Equicrural. For the Angles Xsr, Xrs, are equal, becauſe the right Lines Xs, Xr are equal: and either of thoſe Angles are equal to the Angle at F, becauſe sr, Fy, are Parallels; and the Angles rsz, rsy, are either of them equal to the ſame Angle at F, becauſe Es, Fr, are Rhombus's alike. Alſo the Angle yXz appears to be equal to the Angle at F. Likewiſe the Angle Xzr, is equal to the Angle yXz, becauſe rz, Xy are Parallels. Laſtly, the Angles Xzs, Xyr, are equal to the ſame Angle yXz, becauſe the Baſes Xs, Xr, yz, are equal.

Consect. II.

THE Square of the greater Segment Ey, is equal to two Squares (to wit) to the Square of the right Sine of 30 Deg. and to half the Square of ſo many Deg. of the Line of Chords. For EI is equal to the right Sine of 30 Deg. And becauſe (the Chord XK being drawn) the Triangle XKI, is made Equicrural and right Angled, the Square of XI will be half the Square of the Chord XK; therefore Ey, that is, EX being ſquared, contains as much as the Square of EI and XI added together.

Consect. III.



Prop.1.

K y is the Half of F I: cut F I in the Middle at t; therefore because F y more y I, and I t more t F are equal; by how much F y is greater than F t, by so much t I is greater than y I; but F y is greater than F t, by so much as t y is.

Wherefore also t I, that is F t, is greater than y I, by the same Quantity t y. And F t is divided in K, as t I in y. For because F y more y I, and I K more K F are equal; by how much F y is greater than I K, so much is F K greater than y I.

Therefore when as F t is made equal to t y more y I; F t will be divided in K, as t I in y.

Wherefore t K is equal to y I; and K y equal to F K more y I. Therefore K y is equal to the Half of F I.

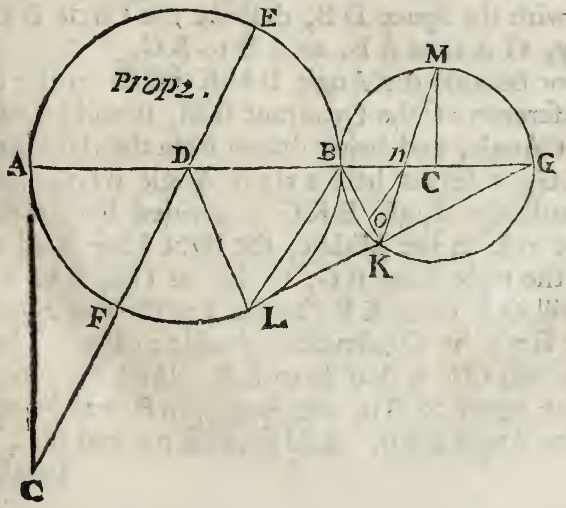
PROP.

PROP. II.

One Segment being given of a right Line divided in extream and mean Proportion, to find the other.

LET the right Line given be A B, being the greater Segment of any right Line. Cut A B in the Middle in D, and from the Centre D, with the Interval (or Semidiameter) A D, deſcribe the Circle A F B E. Then to the Point A, raiſe a Perpendicular A C, equal to the given Line A B; and through the Centre D, draw a Line to the Concave Periphery C E, to which let A G be made equal.

I ſay, the whole Line A G, hath the ſame Proportion to A B, that A B hath to B G. For, becauſe



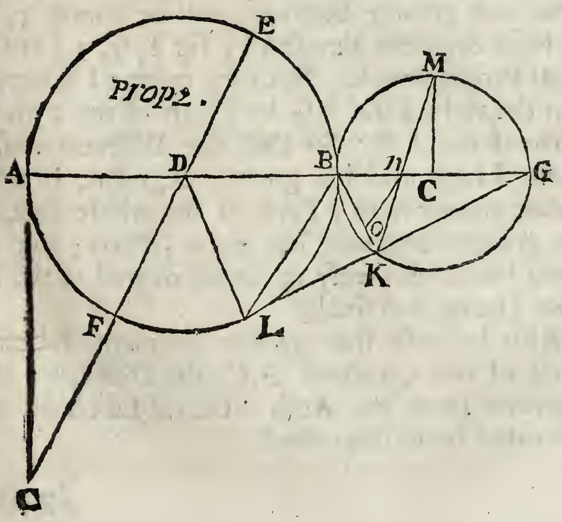
cauſe (by the 36 of the 3d Book of *Euclid*) as E C, is to E F; ſo is E F, to F C. Alſo as A G, together with his Equal E C, is to A B, and his Equal E F; ſo will A B, be to B G, and his Equal F C. Wherefore (by the Definition of extream and mean Proportion) the right Line A G is divided in extream and mean Proportion in the Point B.

Again, let B G be the leſſer Segment of any right Line. Cut B G the given Line in the Middle in C. And from the Centre C, with the Semidiameter CB, deſcribe a Circle B M G. In which Circumference, the Quadrant B M being taken, draw by the Point n, the right Line M K, ſo that B n be a third Part of the given Line B G. Then draw B K and G K, and in the Line B K, take a Part B o, which ſhall be equal to the right Line B n, draw the Line o, n, to which let there be a parallel Line drawn, B L, cutting G K being produced in L. Finally, raiſe a Perpendicular to G L in the Point L, which will cut G B, being produced in D. From the Centre D, with the Space D B, deſcribe the Circle B L A. I ſay, G A is to A B, as A B to B G.

For becauſe the Angle B M K inſiſts in the Circumference of the Quadrant B M, it will be half a right Angle, and being drawn from the right Angle B K G, it leaves half a right Angle n K G. And becauſe the Angle B K G is divided by the right Line n K in the Middle, the right Line K n, will cut the right Line B G, ſo that as G n, is to n B; ſo will G K be to K B (by the 3 of the 6 of *Euclid*). But G n is by Conſtruction double to B n Therefore alſo G K is double to K B. And becauſe B o is put equal to B n, the Angle o n B will be equal to the Angle B o n. And becauſe n o and B L, are Parallels,

Parallels, the Angle L B D will be equal to the Angle o n B. Likewiſe becauſe L D and K B are Parallels and Perpendiculars, the Angles D L B, and B o n, will be equal. Likewiſe B o n, and B n o, are equal. Therefore D L B and B n o are equal. But B n o and D B L are equal, therefore D L B and D B L are equal. Therefore alſo the right Lines D L, D B, being Chord Lines, are equal. Likewiſe the Circle deſcribed with the Space (or Semidiameter) D B will paſs by the Point L. And becauſe D L is perpendicular to G L, G L toucheth the Circle B L A in L. Therefore it will be (by the 36 of the 3 of *Euclid*) as G A to G L, ſo G L to G B. And A B is equal to the ſaid G L. (For whereas the right Line G K is double to the right Line B K; ſo alſo the right Line G L, will be double to the right Line L D). Therefore in the

E 4 ſame



ſame Manner the right Line B A is double, and for that Cauſe equal to the right Line G L. Therefore alſo it will be as G A is to A B; ſo A B to B G, and by conſequence, we have added the other Segment of the right Line given, &c. which was to be done.

ANIMADVERS.

I ſee not wherefore he hath invented this new Way of cutting proportional Lines, unleſs, perhaps, from this he thought to find the Greatneſs of the Circle, which he ſhould ſeek for, by comparing the Power of the greater Segment, with the Power of the Semidiameter, not before known.

The Quantities of theſe Segments, nor the Squares of them, cannot be expreſſed accurately by Numbers. But their Proportion comes very nigh to the Proportion of 5 to 3; for if the Side A B be 8 Parts, the greater Segment will be almoſt 5, and the leſſer Segment almoſt $3\frac{1}{8}$; for 8, 5, $3\frac{1}{8}$ are continual Proportionals. But they make a Line greater than the right Line A G by $\frac{1}{8}$ part of the 25th Part of the Line A B: So that the Difference of the whole Line, and his greater Segment, is a ſmall matter greater than $\frac{3}{8}$ Parts of the whole Side, and the greater Segment leſs than $\frac{5}{8}$ Parts; but how much leſs is not eaſy to be diſcovered in the Diagram (being it is ſmall).

Alſo becauſe that greater Segment ſubtends $\frac{2}{7}$ Parts of the Quadrant A C, the Difference of the Segment from the Arch which it ſubtends, could not eaſily be diſtinguiſhed.

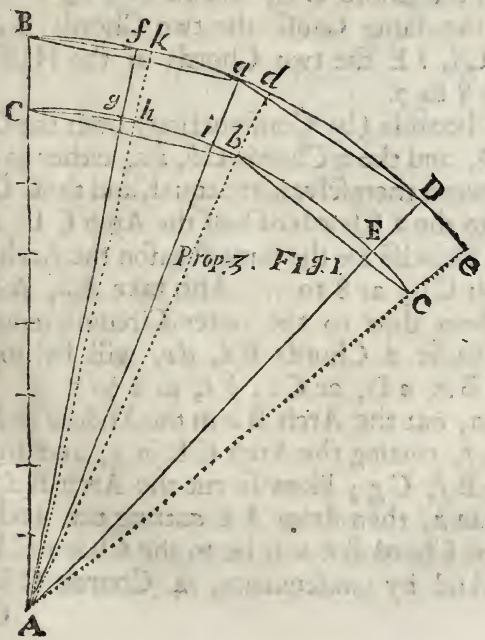
PROP.

PROP. III.

Of Regular POLYGONS.

In a Circle given to describe a Regular Heptagon.

WHatsoever right Line, suppose A B, let it be cut in 8 equal Parts, whereof let A C be 7. Then from the Centre A, with the Semidiameters A B, A C, describe 2 Circles. Moreover take one eight Part of the Perimeter of the outward Circle (to wit) B D (which is easily done) and draw the Line



Prop. 3. Fig. 1.

Line A D, cutting the inner Circle in E, which will cut off the eighth Part of it C E.

Divide the Arch B D in the Middle in *a*, and draw the Chord Lines B *a*, *a* D; likewiſe in the Arch C E apply the right Line C *b* equal to B *a*, and again *b c* equal to the ſame B *a* or *a* D; for they are equal.

I ſay, A right Line C *c* being drawn, is the Side of a Heptagon in the Circle C E. For becauſe A B is to A C, as 8 to 7; ſo alſo the Perimeter of the Circle B D to the Perimeter of the Circle C E, will be as 8 to 7.

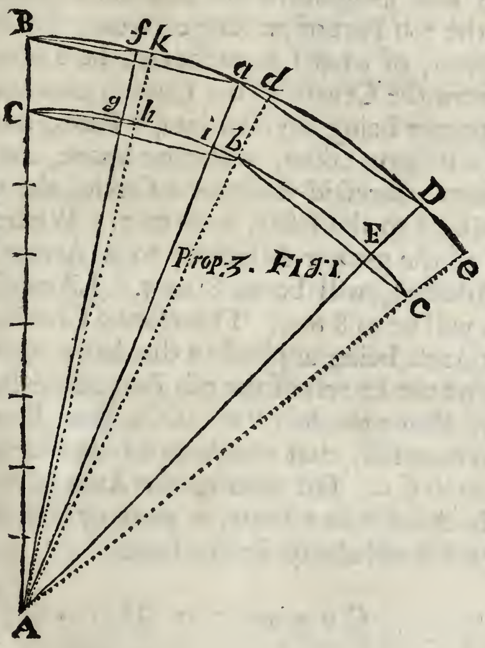
Alſo becauſe the Sectors A B D and A C E are alike, and the Triangles A B D, A C E are alike, both the Arch B D, to the Arch C E, and the Chord B D, to the Chord C E, will be as 8 to 7.

For the ſame Cauſe the two Chords B *a*, *a* D will to C *i*, *i* E the two Chords of the Half Arch C E, as 8 to 7.

Now becauſe (by Conſtruction) both the Chords B *a*, *a* D, and the 2 Chords C *b*, *b c*, either to other, and between themſelves, are equal, and thoſe Chords will be to the 2 Chords of half the Arch C E, as 8 to 7. And likewiſe for the ſame Reaſon the Arch C *c* to the Arch C E, as 8 to 7. Alſo take A *b*, A *c*, and draw them thro' to the outer Circumference in *d* and *e*, thoſe 2 Chords B *d*, *d e*, will be to the 2 Chords B *a*, *a* D, or C *b*, *b c*, as 8 to 7.

Again, cut the Arch B *a* in the Middle in *f*, and draw A *f*, cutting the Arch C E in *g*, and draw the Chords B *f*, C *g*; likewiſe cut the Arch B *d* in the Middle in *k*, then draw A *k* cutting the Arch C E in *h*, the Chord B *k* will be to the Chord C *h*, as 8 to 7. And by conſequence, 4 Chords C *h*, to 4 Chords

Chords C g, as 8 to 7. Likewiſe if 2 Arches C *b* and C g, be cut in the Middle, and their Biſegments cut *ad infinitum*, every Chord of thoſe, to evey Chord of theſe, will be in Proportion, as 8 to 7. But the Chords thus taken infinite in any Arch are (all together) the Chord of the ſame Arch. Wherefore the Arch C *c* is to the Arch C E, as 8 to 7. And the Arch C *c* is the 7th Part of the whole Perimeter by C. Therefore a right Line being drawn C *c*, is the



Prop. 3. Fig. 1.

Side of a Heptagon in the Circle by C. Likewiſe the right Line A *e* being drawn, will cut the 7th Part of the Perimeter by B.

For

For if every one of the Chords to every one of the Arches which they ſubtend, are not equal, thoſe ſeveral Arches, and their Biſegments, may be again biſected, which is contrary to the Suppoſition. Therefore the Side of a Heptagon is found, which was propoſed.

Consect. I.

From this Demonſtration appears a Method of finding the 7th Part of an Angle given. For if to an Arch given, of what Limits ſoever, be drawn ſtrait Lines from the Centre of the Circle; moreover the Semidiameter being divided into 8 Parts, and 7 of thoſe Parts being taken, with that Space, and upon the Centre aforeſaid deſcribe a Circle, the greater Arch will be to the leſſer, as 8 to 7. Wherefore 2 Arches of the greater Biſected, to 2 Arches of the leſſer Biſected, will be as 8 to 7. Likewiſe their Chords will be as 8 to 7. Therefore 2 Chords of the greater Arch being applied to the leſſer Arch, will determine the Exceſs of the 7th Part of the leaſt Perimeter, above the 8th Part of the ſame Perimeter. For it is manifeſt, that the Arch D *c* is the 7th Part of the Arch C *c*. For whether the Arch of the Perimeter be divided in 7 Parts, or more or leſs, the Demonſtration will always be the ſame.

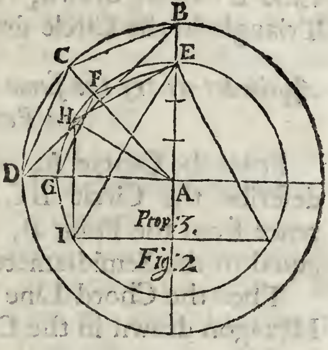
Consect. II.

An Arch being deſcribed with the Radius BC, is equal to the 8th Part of a Perimeter of a Circle whoſe Radius is AB: And to the 7th Part of the Perimeter of a Circle, whoſe Radius is AC: And to a 6th

a 6th Part of the Perimeter of a Circle, whoſe Radius is $\frac{6}{8}$ Parts of the right Line A B, &c.

Let us experience this our Method in known Polygons, and ſee whether or no by this, the Side of an Equilateral Triangle may be found from a Tetragon (*viz.* a Square).

From the Centre A, deſcribe the Circle BCD, whoſe Quadrant is B D. Then draw the Chord Line B D, which is the Side of a Square inſcribed within that Circle.



From the ſame Centre A, with a Semidiameter A E (which let be to A B, as 3 to 4) deſcribe the Circle E F G; then the Circle by B, will be to the Circle by E, as 4 to 3.

Let the Quadrant BD be cut in two in the Middle in C, and let the equal Chord Lines B C, C D, be drawn. Then draw A C, cutting E G in F, the Arch E F will be the 8th Part of the Perimeter by E, and equal to the Arch F G.

Therefore both the Arch and the Chords BC, CD will be to the Arch, and to the Chords E F, F G, as 4 to 3.

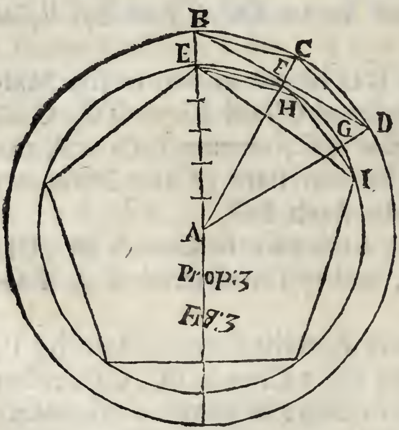
Then from the Point E in the Circle drawn by E, apply EH, HI, being the 2 Chords BC, CD, either of theſe being equal to either of thoſe. Wherefore the 2 Chords E H, H I, are to the 2 Chords E F, F G, as 4 to 3; that is, as the Arch BD to the Arch E G. Wherefore as the Arch E I to the Arch E G, ſo

ſo is 4 to 3. Therefore the Arch G I is the 3d Part of the Arch E G, that is, a 12th Part of the whole Perimeter drawn by E; and the Arch E I, a 3d Part of the ſame Perimeter. The Demonſtration is the ſame which concerns the Heptagon, which you have before in *Pag.* 50. Therefore the Chord Line E I being drawn, is the Side of an Equilateral Triangle in the Circle drawn by E.

Again let us try the ſame Method from a Hexagon to a Pentagon.

From the Centre A, with the Semidiameter A B, deſcribe the Circle B C D; and in that Circumference from the Point B, apply the Chord Line B D equal to the Semidiameter A B.

Then the Chord Line B D, will be the Side of a Hexagon drawn in the Circle by B.



In the Semidiameter A B, take A E, which muſt be to A B, as 5 to 6. And from the ſame Centre A, with the Radius A E deſcribe the Circle E F G. Then the Perimeter by B to the Perimeter by E, will be as 6 to 5. Let B D be cut by a right Line A C (cutting the Circle drawn by E, in F) in the Middle in C; and let the Chords B C, C D be drawn, equal to which apply the Chords

Chords E H, H I, in the Circle drawn by E. Then are the two Chords E H, H I, to the two Chords E F, F G, as the Arch B D to the Arch E G, that is as 6 to 5; and the Arch E I, to the Arch E G, as 6 to 5; and the Arch G I is a 5th Part of the Arch E G, that is, the 30th Part of the Perimeter; and the whole Arch E I, 6 of thoſe 30 Parts (that is, a 5th Part) of the whole Perimeter drawn by E. The Demonſtration is the ſame as in the Heptagon. Therefore the Chord Line E I, is the Side of a Pentagon drawn in the Circle by E.

Cor. 1. Therefore the Side of a Pentagon may be found, without the Work of cutting the Semidiameter in extream and mean Proportion.

Cor. 2. As from the outward Circle to the inward, the Demonſtration hath proceeded hitherto; ſo likewiſe it may proceed from the inward Circle to the outward. As from a Triangle given, may be found a Quadrat (or Square) and a Pentagon from a Square, and a Hexagon from a Pentagon, and ſo of the reſt.

For to the Side of a Pentagon given E I, the 2 Chords E H, H I are given. Wherefore if to the Semidiameter A E, be added a 5th Part of the ſame Semidiameter (to wit) E B, and the Arch B D be deſcribed; the 2 Chords E H, H I, will be equal to thoſe 2 Chords B C, C D, and equal each to other, and the Chord B D, the Side of a Hexagon inſcribed in the Circle by B.

PROP.

PROP. IV.

Of the Proportion of crooked Lines, to crooked Lines in the Circumferences of Circles.

1. As a Circle is deſcribed from a Semidiameter with one Foot of a Pair of Compaſſes being fixed, and the other Foot carried about; ſo alſo it may be underſtood to be deſcribed from a right Line given, being bent uniformly, that is, ſo as the Angles be always equal; from which certain Flexion or Bending, if the Angles be conceived to be infinite in Number, is deſcribed a Circle. For a Circle in its Nature, differs nothing from a Polygon of an infinite Number of Sides.

And bending is a departing from Straitneſs according to ſome Angle, which is Crookedneſs.

2. And the Crookedneſs of ſome to other ſome, is greater or leſſer; and therefore Crookedneſs is Quantity, and belongs to the Subject of Geometricians, and chiefly to thoſe who write concerning the Magnitude of a Circle, and the Inſcription of Polygons in a Circle, altho' concerning this Thing, nothing hath been delivered to us from the Antients.

3. What Proportion an Angle in a Circumference, to an Angle, in a Circumference in the ſame Circle hath, the ſame Proportion hath the Crookedneſs of the greater Arch, to the Crookedneſs of the leſſer Arch. For ſince Crookedneſs is no other Thing, than a Bowing (by an Angle in a Circumference) from Straitneſs; it muſt be that the greater Angle makes the greater Crookedneſs. From whence it follows, that the Crookedneſs of Arches in the ſame Circle, be to each other like their Angles.

4. In

4. In divers Circles the Crookedneſs of the greater Perimeter, is leſs than the Crookedneſs of the leſſer Perimeter, in the Proportion of the Radius to the Radius, or of the Diameter to the Diameter reciprocal. For in great Circles, as in the great Circle of the Earth, no Crookedneſs can be diſcerned in a long Space, but in a Ring it is every where diſcern'd; therefore in indifferent Circles, the leſſer Circles have the greater Crookedneſs, for this very Cauſe, becauſe the Diameter is leſſer; which is manifeſt by the Light of Nature.

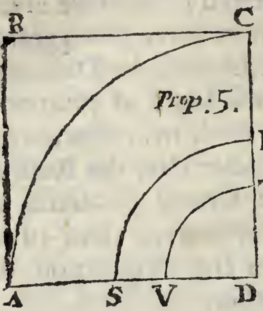
5. (If the Proportion of an Arch in a Circumference be the ſame to the Perimeter that the Radius is to the Radius) Any ſame Chord ſubtends a greater Portion of his own Perimeter, than of a greater Perimeter, according to the Proportion of the greater Perimeter to the leſſer.

For the Cauſe wherefore the ſame right Line, ſubtends a greater Part of the leſs Perimeter than of the greater, is the only and eſſential greater Crookedneſs of the ſame Line, being bowed in the leſſer Perimeter, than when it is bowed in a greater Perimeter: Therefore if the Crookedneſs of a leſſer Arch to the Crookedneſs of a greater Arch in a Semicircle be as 8 to 7, the Chord, which ſubtends the eighth Part of the greater Arch, will ſubtend the ſeventh Part of the Semiperimeter of the leſſer: I ſay in a Semicircle, becauſe the Crookedneſs of a Perimeter beyond a Semicircle proceeds a contrary Way, and becauſe the Subtenſes of the Arches, which together with themſelves in a Semicircle increaſe; but beyond a Semicircle they all decreaſe.

6. Alſo therefore an Angle in the Circumference of a leſſer Perimeter (becauſe the Subtenſes are equal) will be to an Angle in a greater Perimeter, as the greater Perimeter to the leſſer.

PROP. V.

A mean Proportional between the Semidiameter of a Circle, and $\frac{2}{5}$ Parts of the same is equal to $\frac{2}{5}$ Parts of the Fourth Part of the Circle.



DEscribe a Quadrant of a Circle D A C, and compleat the Quadrat A B C D; in the Side D C take D T, being two fifth Parts of the Side D C, and between D C and D T, let there be taken a mean Proportional DR, so that DC, DR, DT, may be continual Proportionals; also describe the Quadrantal Arches R S, T V; then the Arch T V is two fifth Parts of the Arch C A.

I say the Arch T V, and the right Line D R are equal.

Let it be supposed that there is a right Line given equal to the Arch A C, and from it describe a Quadrantal Arch (*viz.* with the right Line given being Radius, describe the fourth Part of a Periphery) the Line D C, C A, and the Quadrantal Arch above C A are continual Proportionals.

Let there be writ apart	D C, C A, the Arch above C A $\div$
And under these	D R, R S, the Arch above R S $\div$
Again under these	D T, T V, the Arch above T V $\div$

Then the Antecedents to the Consequents will be thro' all the Orders or Ranks forward, as the Semidiameter

midiamerer to the Arch of a Quadrant, and backward, as an Arch of a Quadrant to the Semidiameter.

Therefore as D C to R S, ſo is R S to the Arch above T V; therefore D C, R S, and the Arch above T V, will be continual Proportionals. And becauſe D C, C A, and the Arch above C A are likewiſe continual Proportionals, and have the firſt Antecedent D C common; the Proportion of the Arch above C A to the Arch above T V, will be (by the 28 of the 14 of *Euclid*) in Duplicate Proportion of C A to R S; and for the ſame Cauſe the Arch above R S, is a mean Proportional between the Arch above C A, and the Arch above T V. Now if D C be greater than R S, likewiſe R S will be greater than the Arch above T V; and the Arch C A greater than the Arch above R S. Wherefore, when as D C, C A, and the Arch above C A are continual Proportionals, the Arch above T V, and the Arch above R S, and the Arch above C A cannot be continual Proportionals, becauſe it is demonſtrated to the contrary. Therefore D C is not greater than R S.

Again let R S be ſuppoſed to be greater than D C; then the Arch above R S will be a mean Proportional between the Arch above T V, and the greater Arch above C A. Therefore the Inconvenience will return.

Wherefore the Semidiameter D C is equal to the Arch R S; and by Conſequence the Arch F V, that is two Fifths of the Arch C A, and the right Line D R are equal, that is, a mean Proportional between the Semidiameter and two fifth Parts of the ſame, is equal to two fifth Parts of the fourth Part of the Circumference of the Circle: Which was to be demonſtrated.

PROP. VI.

LET there be deſcribed a Quadrat A B C D, and let it be cut in the Middle both Ways by E F, G H; alſo let it be cut by the Diagonals A C, B D (concurring in the Centre I) four Ways.

Moreover, between D C and two fifth Parts thereof, let there be taken a mean Proportional D R, and join A R, cutting E F, in *a*, let it be produced until it meet with the Side B C being produced in *b*.

I ſay that the right Line B *b* is Quintuple to (*viz.* five times as long as) the right Line E *a*, or the fifth Part of the Arch A C (or it may be thus render'd, the right Line B *b* is equal to the Quadrantal Arch A C.)

Becauſe the Triangles A D R, A B *b* are alike, and the Arch of a Quadrant deſcribed from D R is equal to the Side D C, or A B; likewiſe the Arch deſcribed from A B will be equal to the right Line B *b*. Therefore whereas D R is two fifth Parts of the Arch A C, and by conſequence B *b* five of thoſe Fifths; B *b* will be five times as long as the right Line E *a*, which is the Half of the right Line D R, and the fifth Part of the Arch A C, or the right Line B *b*. Which was to be demonſtrated.

Therefore if in the Side B C, be taken a part *B i*, equal to E *a*, that part B *i* being five times repeated, will end in *b*.

A compendious Expoſition.

If 2 right Lines whatſoever have a mean Proportional between them, which is equal to the Side A B, it will be as the Arch A C to one of them; ſo reciprocally

procally the other of them will be to two fifth Parts of the ſame Arch A C; as in Example, becauſe the Side A B is a mean Proportional between the Arch A C, and two Fifths of the ſame; and the ſame mean Proportional between the Diagonal Line A C, and the Half of it D I. The right Angle under the Arch A C, and two fifth Parts of the ſame, will be equal to the right Angle under B D, and the Half of it D I. Therefore as the Arch A C is to his Diagonal A C, or B D, ſo will the Half Diagonal D I be to D R. And as the Half of the Arch A C, to the Half of the Diagonal, ſo is the Half Diagonal to D R.

CONSECT.

From hence appears the Magnitude of the eighth Part of the whole Perimeter. For if to the right Line D R, be added R *z*, being a fourth Part of D R, the whole Line D *z*, will be five of thoſe ten Parts, that is half the Arch A C. For when as D R is two Fifths, that is, four Tenths, D *z* will be five Tenths.

PROP. VII.

THE ſame Lines continuing, draw the right Line D F. I ſay D F is a mean Proportional between the whole Arch A C, and his Half.

Draw the right Line R *r* parallel to the Side B C, cutting the Diagonal D B in r, and D F in *d*. Then becauſe D F cuts the Side B C in the Middle in F, the right Line D *d* will cut the right Line R *r* in the middle in *d*. Let L N be drawn parallel to the ſame Side B C, cutting D F in *e*, and the Side D C in N. Then D N and N L will be equal, and either of

them equal to the Semidiagonal D I, and N L, will be divided in the Middle in *e*.

Therefore becauſe by the compendious Expoſition of the precedent Prop. D *z*, D N, D R, are continual Proportionals: If with the Radius D *z* be deſcribed an Arch *z f*, it will cut the right Line N L in *e*; likewiſe if with the Radius D N be deſcribed an Arch of a Circle N I, it will cut the right Line R *r* in *d*.

Then deſcribe the Quadrantal Arch N I O, which (as it is ſhewn) will paſs through *d*. Then becauſe D R is the Radius of a Circle, whoſe fourth Part of the Perimeter is equal to the Side D C; the right Line D *d* or D N will be the Radius of a Circle, whoſe fourth Part of the Perimeter is equal to the right Line D F. But the right Line D N is the Radius of a Circle, whoſe fourth Part of the Perimeter is the Arch it ſelf NIO.

Therefore the right Line D F, and the Arch NIO are equal: And the Arch NIO is a mean Proportional between the Arch A C and his Half, therefore alſo the right Line D F is a mean Proportional, &c. which was to be demonſtrated.

Cor. If from a Point *z* be drawn a right Line *z c* parallel to the Side B C, cutting the Diagonal D B in *c*; D *c* will be equal to D F. For D C will be a mean Proportional between D *z* and the double of it. Likewiſe it is made appear, that D *z* is equal to half the Arch A C.

Consect. I.

The Arch A C, that is the right Line B *b*, is a mean Proportional between the Side D C, and the Quintuple of (*viz.* a Line five times as long as) the Half

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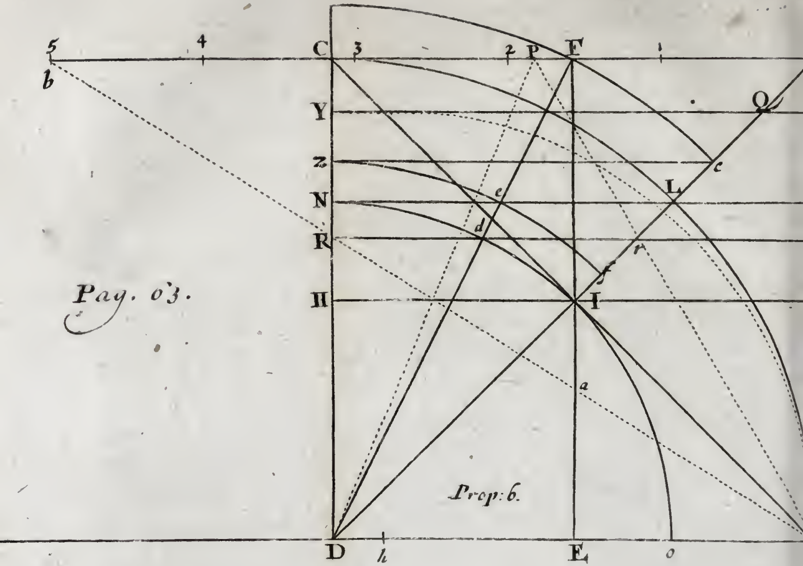


Half Side B F. For ſeeing that two Fifths of the Side D C, the right Line D R, and five of thoſe Fifths of the Side D C are continual Proportionals. If the Proportion be continued, D R, D C, and the mean Proportional between D C, and the Quintuple of the Half Side B F, will be continual Proportionals: Therefore a Quadrat being made or drawn from the right Line B *b*, is equal to ten Quadrates drawn from the Half Side B F. From whence alſo it follows, that a Quadrantal Arch being deſcribed from the Arch A C, being extended in a right Line, is Quintuple to the Half Side B F.

Consect. II.

The ſame Lines remaining, to the Side A D let there be added in a direct Line D *g*, equal to the right Line D *z*, that is equal to the Arch C L; moreover cut the whole Line A *g* in the Middle in *h*; then from the Centre *h*, with the Radius *h A*, or *h g*, deſcribe the Arch of a Circle A Y, cutting the Side D C in Y: Then D Y will be a mean Proportional between the Side D C and D *z*; and a right Line Y Q being drawn parallel to the Side B C, cutting the Diagonal D B in Q, Y Q will be the Side of a Quadrant (from *Archimedes*'s Demonſtration) equal to the Sector D C L, or an eighth Part of the whole Circle deſcribed from the Semidiameter D A.

Consect. III.

From hence it follows, that the Arch A C *is equal to the Compound of the Side* B C *and a Tangent of* 30 *Deg.*

For becauſe D C is a mean Proportional between the Arch A C and D R; if to D R and D C there be

taken a third Porportional, it will be equal to the Side B C, together with the Tangent of 30 Deg.

But the Diagram muſt be renewed (or drawn anew.) Therefore let A B C D be a Quadrate, and let it be divided into four equal Parts by the right Lines E F, G H; alſo let it be divided into four equal Parts by the Diagonal Lines A C, B D, concurring in the Centre I. Alſo draw the Quadrantal Arches A C, B D cutting E F and G H, in K and X.

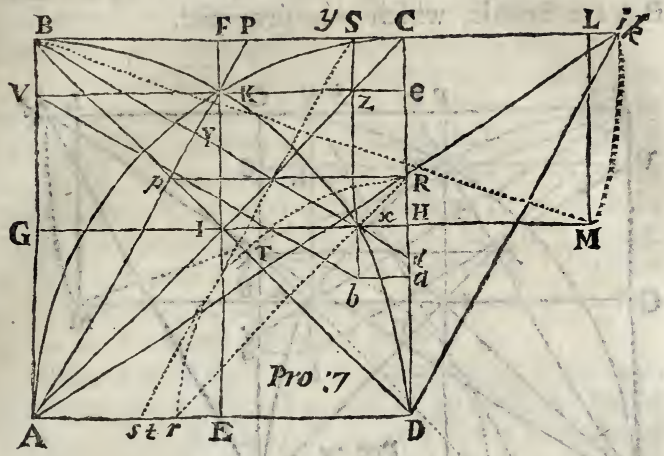
Produce the Lines B C, G H, and let there be taken the right Lines C L, H M, either of them being equal to half the Side, and let B M be joined. Then B M being taken as a Semidiameter, with it, from the Centre B, deſcribe an Arch of a Circle, cutting B C, being produced in *i*. Then becauſe B L being ſquared, contains as much as nine Quadrates (or Squares) made from the Half Side, and L M H C one of them Quadrates: Alſo the right Line B M, that is, B *i* being ſquared (to wit, a Quadrate being drawn with four Lines, each of them being the Length of B *i*) that Quadrate contains as much as ten Quadrates made from the Half Side C H; and therefore B *i* is equal to the Arch A C, or B D.

Draw A K, and produce it to the Side B C in P; A P will be a Secant of 30 Deg. and B P a Tangent of 30 Deg. and A P the double of B P. To the Side B C add C *k* equal to B P, and the whole Line B K will be compoſed of the Side B C, and a Tangent of 30 Deg. B P or C *i*.

It remains therefore to be demonſtrated, that *i* and *k* meet in the ſame Point.

Let A *i* be joined, cutting D C in R; and with the Radius D R, deſcribe a Quadrantal Arch R *r*, cutting the Diagonal Line B D in T.

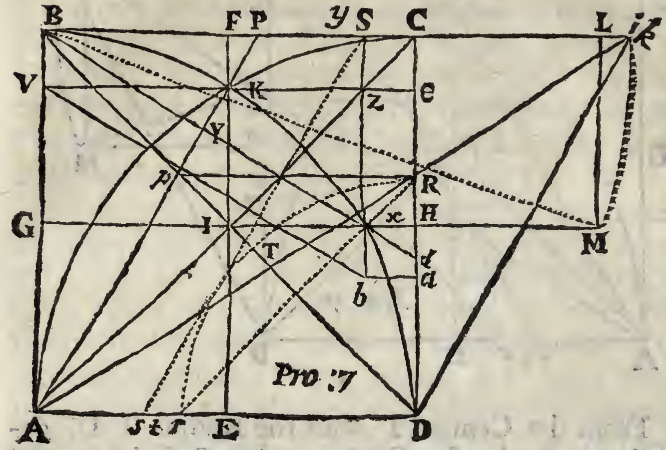
From



From the Centre T with the Radius T D, describe an Arch of a Circle cutting B C in S, and produce S T to the Side A D in S; then are D R, D T, T S Equals.

Then becauſe A B is a mean Proportional both between A P and E K, and between B *i* and D R, likewiſe between the Diagonal B D and its Half C I; it will be as B *i* or the Arch A C) to B D, ſo C I to D R or S T. And likewiſe as B D to A P, ſo reciprocally E K to D R or S. T. Wherefore if there be taken in the Side BC, a certain right Line equal to E K, ſuppoſe the right Line B *y*, and from thence to the Side A D be drawn *y t* parallel to A P, it will be as B *i* to *y t*, ſo B *y* to S T. Wherefore the right Line *y t* will paſs by T, and *y* T will be equal to S T, which is abſurd. Then B S is equal to E K, and S *s* equal to A P. Therefore becauſe A P is equal to D K, D K will be parallel to S *s*. Then as

as B k to D k, ſo is B S to S T: Therefore B i and B k are Equals, which was propoſed.



CONSECT IV.

From hence it follows, that the right Line H R (which is the Difference whereby two Fifths of the Arch A C exceeds the Half Side D H) is equal to F K the verſed Line of 30 Degrees.

For if in the Side A B, there be taken A V equal to E K, and with the Radius A V be deſcribed an Arch of a Circle, cutting A P in Y, the Arch V Y will be equal to the Half Side B F or B Y, and A Y equal to A V; and therefore B Y will paſs to x, and the ſame B Y will be a Tangent of 30 Degrees in the Circle, whoſe Semidiameter is A V or E K. Then produce V K to the Diagonal A C in z, V z will be equal to E K, wherefore a Perpendicular being dropt down from the Point S, it will paſs through z and

and x; and B x being produced to the Side D C in d, C d will be equal to C i: But S x is equal to half the Side, and therefore z x equal to K z.

In the Side C D, let there be taken the right Line C a equal to D R, and let a b be drawn parallel to the Side B C, cutting z x produced in b, and let V b be joined. Then becauſe as B i is (that is B C more the Tangent C i) to the Secant B d, ſo is B S to D R, that is to S b; and as B C to the Tangent C d, ſo V z to S x; if in S x produced, be taken z b equal to S x, z b will be a Tangent of 30 Degrees in the Arch deſcribed from V z.

Then join V b, it will be equal to the Side B C, and becauſe z b is a Tangent of 30 Degrees in the Circle, whoſe Radius is V z, and C d a Tangent of 30 Degrees in the Circle, whoſe Radius is A B; B d and V b will be Parallels; and join V b equal to the Side B C, and B C more C d will be equal to the right Line B i.

Then becauſe it is as B C more C d (that is B i) to B d, ſo V z to D R; and as the ſame B i to V b, ſo V b to D R, and S b will be equal to D R: For theſe two Analogies can no way be conſtituted in any other Point of the right Line S x.

Then the Equals D H being taken from D R, and z b from S b, there remains H R, S z, both Equals; but S z is equal to F K: Wherefore H R, F K are Equals.

Cor. Join R r, it will paſs by x.

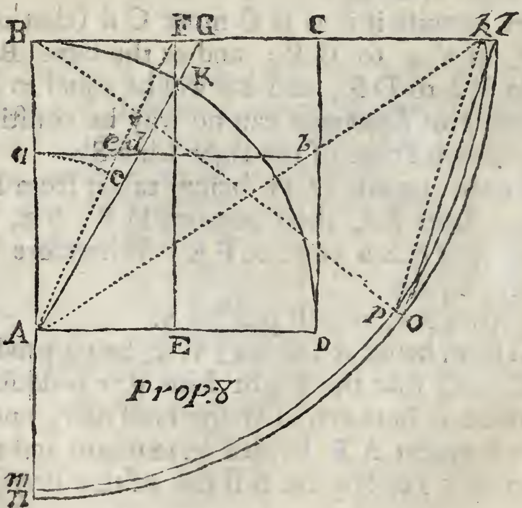
Alſo from hence it follows (V z, being produced to D C in e) that the Right Line R e is double to the Difference between G B the Half Side, and the greater Segment A B divided in extream and mean Proportion: For (by the firſt *Cor.* of this *Prop.*) the

Fourth

Fourth Part of the Side A B, or D C, is equal to the right Line F K, together with the Difference between the Half Side, and the greater Segment of the Side: Wherefore half the Side, that is, *a e*, is equal to the double of F K, and to the double of the Difference between the greater Segment of the Side and the Half Side: But *a* R is double to F K; wherefore R *e* is double to the Difference between the greater Segment of the Side and the Half Side.

PROP. VIII.

AGain describe a Quadrate A B C D, also describe a Quadrantal Arch B D; and to the Side B C, let there be added C *k* equal to a Tangent of 30 Degrees, and let the right Line A *k* be drawn; moreover in the Side A B, let there be taken A *a* equal to the



Prop. 8

the right Line A c being made equal to the greater Segment of the Side A B; (divided in extream and mean Proportion) then draw a b parallel to the Side B C, cutting A k in b.

I ſay, the right Line a b is the Subtenſe of two fifth Parts of the Quadrantal Arch k m, deſcribed with the Semidiameter B k, and thoſe two Fifths equal to the Side A B.

For becauſe A a is the greater Segment of the Side B C, being divided in extream and mean Proportion, it is alſo the Side of a Decagon in a Circle, whoſe Semidiameter is A B (by the 4 *Prop.* of the 14 *Elem.* of *Euclid*) and ſubtends the tenth Part of the whole Perimeter, that is, a fifth Part of the Half Perimeter, that is, two fifth Parts of the Arch B D.

Therefore ſince it hath been ſhewn, that the right Line B k is equal to the Arch B D, it will be, as A B to A a, ſo B k, (that is, the Arch B D) to a b.

Wherefore a b is the greater Segment of the right Line B k divided in extream and mean Proportion.

Apply to the Arch k m the right Line k p equal to a b; then the Arch k p will be two Fifths of the Quadrantal Arch k m, which is the firſt. But the Arch k m (by the 4 *Corol.* of *Prop.* 7.) is equal to five Half Sides of a Quadrate from A B; wherefore the Arch k p is equal to the Side A B, which remained to be demonſtrated.

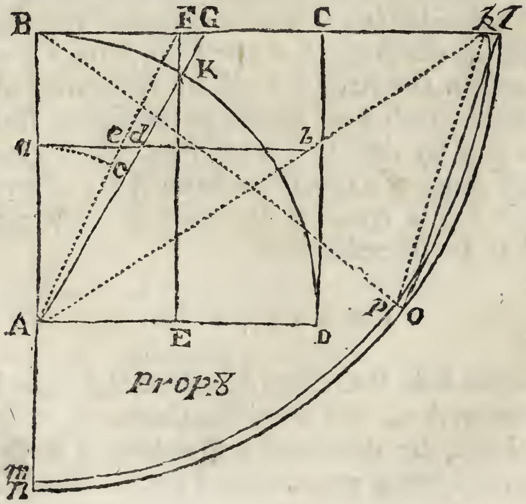
Consect. I.

If to the Side B C there be added the right Line C l equal to A a, and from the Centre B, with the Interval B l, be deſcribed a Quadrantal Arch l n, alſo join B p being produced to l n in o: The Chord Line

Line *l o* being drawn, it will be equal to the Arch *k p*, or the Side A B. For (by the 5 *Prop.* of the 13 *Elem.*) the Side B C will be the greater Segment of the whole B *l*: Wherefore the Side B C ſubtends two Fifths of the whole Arch *l n*. Therefore ſince the Angle *k* B *p* is two Fifths of the Angle *k* B *m* likewiſe *l o* will be two Fifths of the Arch *l n*: Wherefore the right Line *l o*, which ſubtends the Arch *l o*, is equal to the Side BC, and the ſame equal to the Arch *k p*.

Consect. II.

From hence it is manifeſt, that both the Quadrate it ſelf ABCD, and all the right Lines from the Centre A to the Side B C (if the Work be produced) are cut from *a b* (if the Work be produced) in



in extream and mean Proportion: For the Quadrate ABCD is divided in the ſame Proportion whereby the Side A B is in *a*; likewiſe from the ſame *a b* are divided all the right Lines drawn from A to B C (when the Work is produced) in the ſame Proportion whereby A B was divided in *a*: Wherefore (by the 2 Prop. of the 14 Elem.) they are divided in extream and mean Proportion.

Cor. From hence a manifeſt and brief Method appears of finding proportional Segments of right Lines of what Kind ſoever. As for Example; If the greater Segment of a Secant of 30 Degrees be ſought, draw the Line A K, and let it be produced to the Side B C in G, which is the Secant of 30 Degrees; it will cut the right Line *a b* in *d*, and A *d* is the greater Segment, and the Reſidue is the leſſer Segment: Or elſe the Section of a Tangent of 30 Degrees being ſought, which as aforeſaid is half the Secant, and the Side of a Cube inſcrib'd in a Circle, whoſe Diameter is A B.

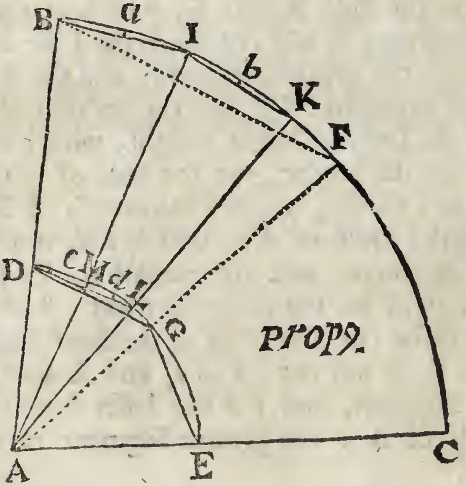
Take the Half of A *d*, that is *a d*, it will be the greater Segment, and the remaining Part of the Tangent will be the leſſer Segment. Likewiſe, if A F be to be cut according to the ſame Proportion, draw A F, it will cut *a b* in *e*, and A *e* will be the greater Segment, and *e* F the leſſer Segment; and the Half of A *e* the greater Segment of the Half of A F.

PROP.

PROP. IX.

To cut an Angle given, into any Proportion given.

LET the Angle given be B A C, and let the Proportion given be as A B to A D. Let the Arch A D E be deſcribed, and cut the Arch B C in the Middle in F. Wherefore A F being drawn, it will alſo cut D E in the Middle (ſuppoſe) in G. Then the Chords B F, D G being drawn, they will be between themſelves as the right Lines A B, A D.



Apply the two Chords D G, G E, to the Arch B F in I and K, ſo that the Chords B I, I K may be Equals to the Chords D G, G E, and either to other; alſo draw the Lines A I, A K, of which A K cuts the Arch DE in L, and AI cuts the ſame Arch in M.

I ſay

I ſay the whole Arch BC being thus divided in K, that the Arch BC (or the Angle given BAC) is to the Arch BK (or to the Angle BAK) as the right Line AB to AD.

For the Chord BK is to the Chord DL, as AB to AD. Alſo as the two Chords BI, IK, to the two Chords BM, ML; ſo is AB to AD. But the two Chords BI, IK, are by Conſtruction equal to the two Chords DG, GE. Wherefore the two Chords DG, GE, are to the two Chords DM, DL, as AB to AD. But as two Chords DG, GE, to two Chords DM, DL, ſo is the whole Arch DE, to the Arch DL; which I thus make appear.

If the Arches BI and IK, be cut in the Middle in *a* and *b*; likewiſe the Arches DM, ML, being cut in the Middle in *c* and *d*; and the Chords of the Biſegments in the Arch BK being drawn; likewiſe the Chords of the Biſegments in the Arch DL; alſo theſe will be as AB to AD, and they will always be ſo, if the Biſegments of the Biſegments proceed infinitely. Alſo the ſame is true in the Biſections of the Arches KC and LE. But the Chords of the Biſegments infinite in Number, are equal to the Arch itſelf; for if all the Chords were leſs than all their Arches, there might yet a Biſection proceed; which is contrary to the Suppoſition.

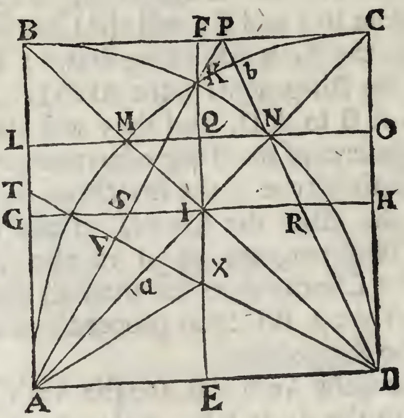
Therefore, the Arch DE is to the Arch DL, as the Arch BC to the Arch BK, as the given Line AB, to the given Line AD, and ſo alſo the given Angle BAC to BAK. Therefore the Angle given BAC is cut in K, in the Proportion given of AB to AD. Which was to be done.

PROP. X.

Of Sines, Subtenſes, and other Lines in the Quadrant of a Circle.

THE Tangent of an Arch of 22 Degrees and $\frac{1}{2}$ is equal to the Exceſs whereby the Diagonal of a Quadrate exceeds the Side of the ſame.

Let ABCD be a Quadrate, and in it a Quadrantal Arch inſcribed BD, cutting the Diagonal AC in N. Then AN is equal to the Side AB. I ſay NC is equal to the Tangent of an Arch of 22 Deg. $\frac{1}{2}$.



Deſcribe the Quadrantal Arch AC, cutting the Diagonal DB in M: Alſo draw MN, it will be parallel to the Side BC, and MN being produced to both the Sides AB, DC, in L and O. Then DO will be equal to the Sine of an Arch of 45 Degrees, or to the

the half Diagonal D I. Let the Quadrate ABCD be cut by the right Lines EF, GH, interſecting at I, four Ways. Then is DO a mean Proportional between the whole Side DC, and his half DH. Wherefore as DC to DO, ſo is DO to DH; and likewiſe the Difference CO to the Difference OH.

Likewiſe CO and NO are equal, becauſe the Angles at C and N are half right Angles. Wherefore NC is in Power double to CO; alſo CO is in Power double to OH (being double in Power ſignifies that a Quadrate whoſe Side is CO, contains as much again as a Quadrate whoſe Side is OH;) for when the Side DC is in Power double to DO, and DC, DO, OH continual Proportionals; CO is in Power double to OH; therefore alſo HO, OC, NC are continual Proportionals; becauſe NC is double in Power to CO.

Draw the Chord DN; then the Angle ODN will be an Angle of 22 Deg. $\frac{1}{2}$. Alſo the Chord DN cuts the right Line GH in R, and HR will be equal to HO.

Likewiſe becauſe RH, NO, NC are continual Proportionals in the Proportion of the Side DC to DO, the Chord DN being produced to the Side BC in P, will cut the Part CP equal to the right Line NC, being equal to the Exceſs of the Diagonal above the Side. Wherefore a Tangent of an Arch of 22 Deg. $\frac{1}{2}$, is equal to the Exceſs of the Diagonal, &c. which was to be demonſtrated.

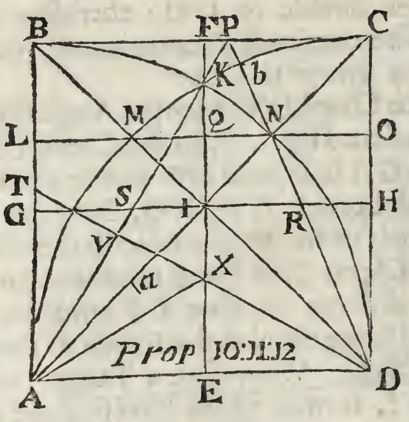
Cor. From hence it follows, that the right Line BP is equal to the double of CO. For if from the Centre C, with the Interval CD, an Arch be drawn, cutting CA in *a*, *a* N will be double to IN, and A *a* equal to NC; therefore, when as *a* C, and BC be equal, likewiſe *a* N, that is, the double of CO will be equal to BP.

 PROP.

PROP. XI.

A Tangent of an Arch of 30 Degrees, together with a Tangent of an Arch of 22 Deg. $\frac{1}{2}$, are equal to the Side of a Quadrate BC.

Divide the Arch MC in the Middle in *b*. Then either of the Arches C *b*, *b* M will be an Arch of 22 Deg. $\frac{1}{2}$, and D *b* will paſs by N; alſo MK is a third Part, that is, two Sixths of the Arch CM. Then when as MK is a third Part of MC, and M *b* an half, the Arch MK will be double to the Arch K *b*.



Then the Angle BDK is a ſixth Part of the Angle CDM, that is, a twelfth Part of the right Angle.

Produce D *b* to the Side BC in P.

Then becauſe the Angle AKD is two Thirds, that is eight Twelfths of one right Angle, and the Angle KDP one Twelfth; if AK be produced until it meet D *b* being produced, which will happen in P. For where-

wheresoever it cuts in Db being produced, it will make with it an Angle equal to seven Twelfths of a right Angle, because the Angle KAB is equal to eight Twelfths, and the Angle KDP to one Twelfth; for the remaining Angle KPD will be seven Twelfths of one right Angle; for when the Angle CPD is nine, and the Angle which the Tangent of 30 Deg. makes with his Secant is eight, the remaining Angle will be a Complement to two right Angles, that is, to the three Angles BPA, APD, CPD. Therefore when as the Angle CPD is nine, and the Angle DAK eight Twelfths, the other Angle APD will be seven, and all the three Angles together, will be twenty-four Twelfths of one right Angle, that is equal to two right Angles, that is to the three Angles of the Triangle APD. Wherefore the Tangent of 30 Deg. *&c.* which was to be demonstrated.

ANIMADVERS.

This Prop. hath long since been confuted by Tables of Sines, Secants and Tangents, calculated from several Geometricians by Dr. Wallis.

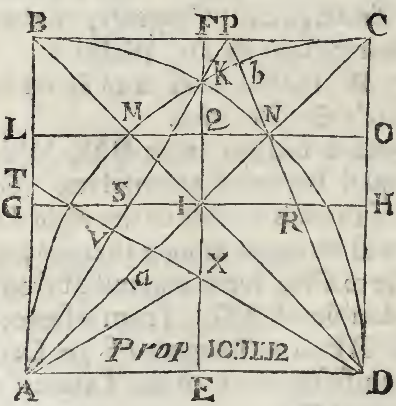
Whether Mr. Hobbs *or he is to be credited, is left to the Reader's Consideration.*

From hence it follows, that BM, MN, NC, CP, KS, are equal between themselves. For whereas every one of them is double to the right Line HO or IQ, they will be equal among themselves. Besides BP is double to GS; for it is manifest from this, that AB is the double of AG. From whence it appears again, that BP is a Tangent of 30 Deg. for AS, which is manifestly equal to the Tangent of 30 Deg. is the double of GS.

PROP. XII.

A right Line which cuts the Base AK *of an Equilateral Triangle, from any Vertical Point in the Middle, is Sesquialter of the Tangent of an Arch of* 30 *Degrees.*

LET there be taken in AB, a Tangent of an Arch of 30 Deg. AT. Join DT cutting AK in V, and the right Line EK in X, and draw AX. Then the Triangle ATX will be Equilateral, and both the Angles AVD, and KVD are right Angles; likewise the Sides AX, XD will be equal. And TX is the double of VX. Therefore DV is the Triple of VX, that is, Sesquialter (*viz.* once and an half) of DX, that is, a Tangent of 30 Deg. which was to be demonstrated. The same Proposition is demonstrated by



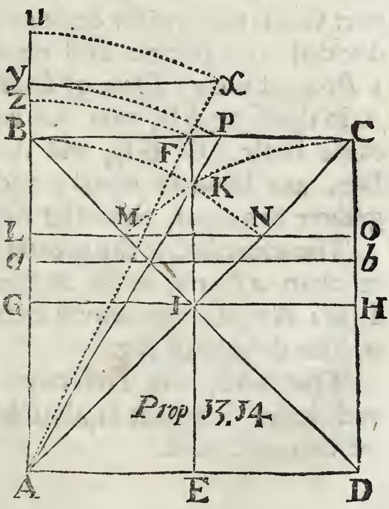
by *Euclid*, in his 18 *Prop.* of the 14 *Elem.* but becauſe it is too long to rehearſe in this Place, let it be read by him that doubts of the Truth of it.

Corol. Therefore the right Line DV or EK, is triple the Difference between the Side DC and the half Diagonal DI or DO; for CO is manifeſted to be equal to half the Tangent AT.

PROP. XIII.

The Difference between the greater and leſſer Segment of a right Line, divided in extreme and mean Proportion, is double the Difference between the ſame right Line, and a right Line whoſe Power is to it, as 5 to 4.

LET the given right Line be AB, to which let there be put at right Angles BF, half of the given Line AB; then draw AF, the Power whereof is to the Power of the given Line, as 5 to 4, and to the Power of BF, as 5 to 1.



NOTE, That the Word *Power* will be often

used; therefore, by the *Power of a Line* you are to understand a Quadrate or Square made from the Line mentioned, each Side whereof is equal to the mentioned Line; as if it had been said, a Qaudrate or Square drawn from AF (that is, a Quadrate, each of its Sides being equal to AF) is to the Quadrate drawn from the given Line AB, as 5 to 4, and to the Quadrate drawn from BF, as 5 to 1.

With the Interval AF, describe an Arch of a Circle, cutting AB being produced in *z*. Then the Square from the right Line A *z*, is to the Square made from the given Line AB, as 5 to 4, and to the Square from BF, as 5 to 1.

I say, the Difference between the greater and the lesser Segment of the given Line AB, divided in extreme and mean Proportion, is the double of B *z*.

Divide AB in the Middle in G; then AG being taken away from the whole Line A *z*, the remaining part G *z* is the greater Segment of the given Line AB divided in extreme and mean Proportion (by the 1 *Prop.* of the 13 *Elem.* of *Euclid.*) From the Point A in the Line AB, take A *a* equal to G *z*; then because both AG, GB, and A *a*, G *z* are equal, G *a*, B *z*, are likewise equal; and because A *a* is the greater Segment, the lesser Segment will be *a* B.

Therefore is G *z* the greater Segment, being greater than *a* B the lesser Segment. The two right Lines B *z*, G *a* are equal between themselves, that is, the double of B *z*.

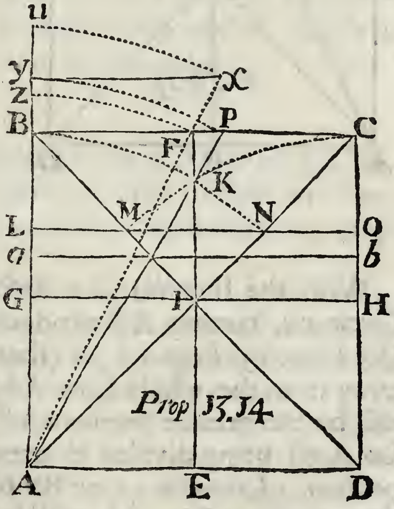
Therefore, the Difference between the greater and lesser Segment is double, &c. Which was to be demonstrated.

PROP.

PROP. XIV.

If the Secant of an Arch of 30 Degrees be cut in extreme and mean Proportion, the greater Segment will be equal to the Semidiagonal of a Quadrate made from the Semidiameter.

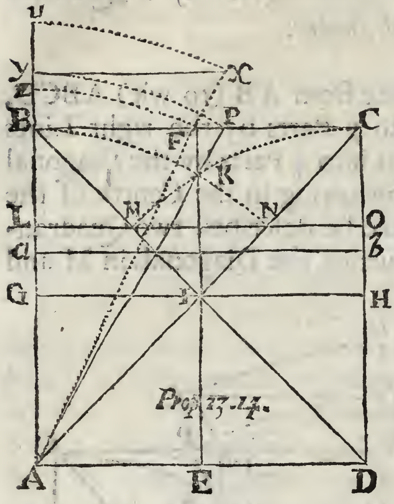
DEſcribe a Quadrate from AB (to wit) ABCD, and divide it into 4 Parts by the right Lines EF, GH, alſo divide it into 4 Parts by the Diagonal Lines AC, BD, all concurring in the Centre of the Quadrate at I; let there be deſcribed two Quadrantal Arches AC, BD, cutting the Diagonals in M and N, and the right Line EF in K draw AK, and produce it to the Side BC in P. Then AP is the Secant of the Arch BK, which is an Arch of 30 Degrees; likewiſe BP is a Tangent of 30 Deg. and the half of the Secant AP.



By the Points M and N, draw the right Line

LO equal and parallel to the Side BC. Wherefore AL, or DO, is equal to the Semidiagonal AI.

I ſay, AL is the greater Segment of the Secant AP divided in extreme and mean Proportion.



With the Interval AP deſcribe an Arch of a Circle P*y*, cutting AB produced in *y*.

From the Point *y* draw *yx* parallel to the Side BC, and equal to half the Secant AP, that is, equal to the Tangent BP. Then A*x* being drawn, is in Power Quintuple to (or five times as much as) the right Line *yx*.

With the Interval A*x* deſcribe an Arch of a Circle *xu*, cutting AB produced in *u*. Wherefore (by *Elem.* 13. *Prop.* 1.) *yx* (that is, BP, being taken away from the whole Line A*u*, the remaining Part will be the greater Segment of the right Line AP (or A*z*) being divided in extreme and mean Proportion. Likewiſe *yx* or BP being taken from A*u*, the remaining Part AL will be equal to the Semidiagonal AI.

For

For it is manifeſt by *Prop.* 10. that the right Line BP is double to the right Line CO or BL. Therefore vL is equal to yx or BP; and the remaining Part AL the greater Segment of the Secant AP, or the right Line Ay divided in extreme and mean Proportion. Which was to be demonſtrated.

Corol. From hence it follows, that the half Diagonal AI is the greater Segment of half the Secant AP, that is, of a Tangent of 30 Degrees, that is, of a Side of a Cube inſcribed in a Circle, whoſe Diameter is AB.

PROP. XV.

A Digreſſion concerning the Diſcord between the Computation of Lines, of Superficies, and of Numbers in the Demonſtrations of Geometricians.

BY the 5th *Definition* of the 5 *Elem.* of *Euclid*, Magnitudes are ſaid to have Reaſon or Proportion to one another, which being multiplied, may exceed one another.

From which Definition it is manifeſt, that Lines, Superficies, and Solids, can have no Proportion among themſelves; for being multiplied together in themſelves, they can never mutually exceed.

If, notwithſtanding, for a Line there be uſed a ſmall right Angle, there may truly ſometimes a Proportion be found between a Superficies and that right Angle, tho' it be very ſmall; to wit, when two Quadrates are to each other as a Quadrate Number to a Quadrate

drate Number. And one is the Meaſure of the other, becauſe they may be compared.

But the Quadrates which are between themſelves as Quadrate Numbers, are much fewer than thoſe which are not as Quadrate Numbers, altho' both are innumerable.

Therefore the Doctrine concerning the Quantity of Lines is a Science ſubſiſtent by itſelf, and diſtinct from the Science of Superficies, and this diſtinct from the Science of Solids.

Moreover, becauſe all Menſuration begins from a Point, and a Point cannot be conſidered as Figurative, how can a Point in a Quadrate Angle any otherwiſe be conſidered, than as a *Quantum* common to the Quadrate and Side of it? For to conſider it as Figurative, and not Figurative, is abſurd.

Beſides, how can a Point, which is in the Centre of a Circle, be accounted for nothing, when as it is diviſible? For in how many Parts ſoever any Sector is divided, in ſo many Parts likewiſe is the Centre divided.

Perhaps, ſome may ſay, without the known Quantity of Figures, very few of Theorems hereafter concerning the Proportions of Lines are demonſtrable, beſides Mechanick Menſuration. But they err, Firſt, Becauſe the Proportions of Lines between themſelves, and the Conſtruction of Figures, and all their Affections of Motion, are deliver'd by *Euclid*, without the known Quantity of a Quadrate, or of any other Figure, or the Proportion of Figure to Figure. Neither hath any one attempted to demonſtrate the Length of a Line by the Magnitude of a Quadrate, before *Archimedes*; neither after him (that I know) beſides *Eutocius*, before *Copernicus*; truly neither hath he demonſtrated accurately.

Neither

Neither do I ſay theſe things, becauſe I would admit Mechanick Operations for lawful Demonſtrations. But in every Geometrical Queſtion, I think it much more Prudence, before mechanically Meaſuring the Magnitude ſought, as much as may be done with Truth, to attempt the neareſt Way, and afterwards inquire into the Cauſe of that nearneſs, which being found out, will detect the Truth or Falſhood; than raſhly believing of uncertain Reaſoning, or the Reckoning of thoſe, or the Authority of others, who pronounce unknown things, eſpecially, not only where the Certainty of one, but of many Theorems is deſtroyed. A diligent Meaſurer declares more credible things by Meaſuring, than he that reaſons from falſe Principles; and will deſervingly ſcorn the Algebraiſts, that is, the Arithmeticians, diſputing againſt Meaſure, and ſaying, That the Side of a Quadrate, and the Root of a Number are the ſame.

Alſo no Man will think thoſe ſtudious of Truth, who ſeeing a Concluſion contrary to their own Knowledge, although ſupported with very probable Arguments, are content to reſiſt the ſole Demonſtration, and neglect the Verity of the thing (which ſometimes proceeds from the Debility of their own Wit, or from the Omiſſion of ſome Propoſition, which the Demonſtrator ſuppoſed to be well known to Geometricians, eſpecially the chiefeſt of them) for theſe Kind of Men are not ſeeking the Truth, but Victory.

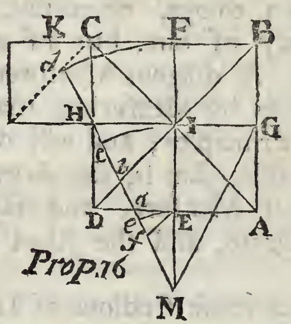
What I have ſaid concerning the Incongruity of Lines and Superficies, will appear clearly in the following Problem, being propoſed to Algebraiſts.

PROP.

PROP. XVI.

DEscribe the Quadrate ABCD, and divide it (both) with the right Lines EF, GH, and with the Diagonals AC, BD, interfecting in I four Ways.

In FC being produced, take FK equal to the half of MF, and join MK; then with the Radius MF, defcribe an Arch Fd, it will cut in d, fo that MF, Md are equal. And MK cuts the right Line Ed in a, and the Diagonal ID in b. Alfo MK will pafs by H.



Prop.16

Draw Ef parallel to ID cutting MK in f. Moreover with the Radii ME, MI, def ribe the Arches Ee, Ic, cutting MK in e and c.

It is manifeft from this Conftruction; Firft, That Ma, aH, HK, are Equals between themfelves.

Secondly, (Becaufe MI is double to ME) Mb is double to Mf.

Thirdly, (Becaufe ME is the double of Ea) Mf is the double of fa; and (becaufe MI is double to IH) Hb is double to ba; and then it will be as Ha to Hb, fo Ma to Mf, as 3 to 2, or 9 to 6.

Fourthly, It is manifeft, that as well Kd as ba is to ae, as 3 to 1, and to Hc as 3 to 2, and fo likewife fe will be to ae; to wit, as 3 to 2, or 9 to 6, and

and therefore C*d* being joined, is parallel to I D; therefore *d*H, H*b* are equal.

Fifthly, It is manifeſt, that M*f*, *f b* are equal, and *b* K, *d a* are equal.

Then M*d* is four times two, whereof M K is thrice three.

Wherefore M K is to M*d* or M F, as 9 to 8.

Then becauſe M K is Quintuple in Power to F K or M*b*; if F K be taken away from the right Line M K, the remaining Part *b* K (by *Elem.* 13. *Prop.* 1.) will be the greater Segment of M K, or *b d* of M*d* being divided in extreme and mean Proportion.

But *b*K is equal to *d a*. Wherefore *d a* is the greater Segment of M*d* divided in extreme and mean Proportion.

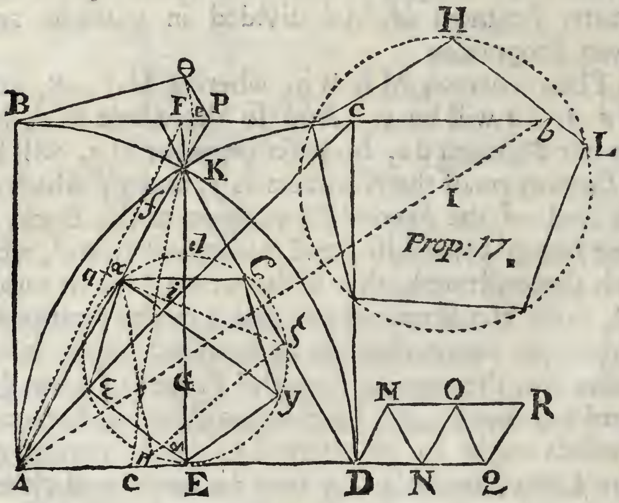
Then whereas M K is 9, whereof M*d* is 8, and M*a* 3, *da* will be 5. And ſo the whole M*d*, his greater Segment *d a*, his leſſer Segment M *a*, will be in Proportion of the Numbers 8, 5, and 3; which at the End of the Second Propoſition of this Book, I have ſhewed to be falſe; and it is againſt *Euclid*, who hath demonſtrated, that if the whole Line be rational, both the Segments of that Line be irrational. Unleſs this Demonſtration be confuted, there is no reaſon that the arguing from the Powers of Lines be heard any more. No Line drawn obliquely between Parallels ought to be referred to the Proportion of pure Lines; becauſe they may be conſidered either as Triangles, or as ſmall oblique angled Parallelograms, whoſe Longitude is not determined. For the Longitude of a Figure is nothing ſurely beſides that which is called Altitude.

PROP.

PROP. XVII.

The Side of an Icosahedron is equal to the third Part of the greatest Semicircle in its own Sphere.

DEscribe the Quadrate ABCD, and divide it in the Middle by the right Line EF parallel to the Sides AB, DC; also describe a Quadrant ABD, whose Arch will cut the right Line EF in K. Join AK



being produced to BC in P. Then is BP a Tangent of 30 Degrees, to which add Pb equal to the Side BC, the whole Line Bb will be compounded of the Side, and a Tangent of 30 Deg. Likewise draw the Diagonal

Diagonal A C, which the right Line E F will cut in the Middle in *i*.

Join D K, and A K D will be an Equilateral Triangle.

In the Side A D take A *c* a 3d Part of it, and join *c* K, and produce it to the Side B *c* in *e*; and F *e* will be a 3d Part of F P, and therefore B *e* will be a 3d Part of the whole Line B *b* (by *Confect.* 3. of the 7 of this.)

In the right Lines K A, K D, let there be taken K α, K β, either of them equal to a 4th Part of the Diagonal A C, or the Half of A *i*, and join $\alpha\beta$. Then the Triangle K $\alpha\beta$ will be Equilateral; and its Base $\alpha\beta$ will be cut by the right Line K E in the Middle, and at right Angles.

Of the right Line B P, which is a Tangent of 30 Degrees, and of 2 right Lines, whereof either is equal to $\alpha\beta$, make the Triangle $\alpha\beta\gamma$; and $\beta\gamma$, $\alpha\beta$, will be equal.

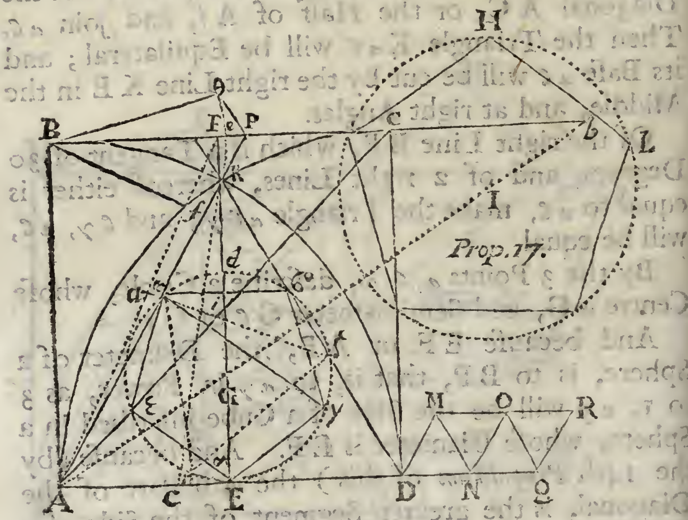
By the 3 Points α, β, γ, describe a Circle, whose Centre is G, and Semidiameter G β or G α.

And because E F or A B, the Diameter of a Sphere, is to B P, that is, to $\alpha\gamma$ in Power, as 3 to 1, $\alpha\gamma$ will be the Side of a Cube inscribed in a Sphere, whose Diameter is E F. And because (by the 14th *Proposition* of this) the 4th Part of the Diagonal, is the greater Segment of the Side of a Cube, divided in extream and mean Proportion; it will be (by the 8 *Prop.* of the 13 *Elem.*) that the right Line $\alpha\beta$ is the Side of a Pentagon in the Circle $\alpha\beta\gamma$; and that Side of a Pentagon one of the 12 Seats or Points of a Dodecahedron inscribed in the same Sphere with the Icosahedron.

Compleat the Pentagon $\alpha\beta\gamma\delta\varepsilon$.

Draw the right Line A F cutting the Arch A K C in f; and the Power of B f will be (as is well known to Geometricians) a 5th Part of the Power of the Side A B, or Quintuple in Power to the Diameter of the Sphere.

Describe aloof off with the Radius I H, being equal to B f, the Circle H L, in which the Side of an Equilateral Pentagon is H L. Then H L will be the Side of an Icosahedron in the same Sphere, by the 16 *Prop.* of the 13 *Elem.*



Then (by the 5 *Prop.* of the 14 *Elem.* of *Clavius's* Edition) the right Line H L is the Side of an Equilateral Triangle inscribed in the same Circle. Inscribe in the Circle $\alpha\,\epsilon\,\gamma\,\delta\,\varepsilon$, the Equilateral Triangle $\alpha\,\zeta\,\eta$. Then $\alpha\,\zeta$ will be the Side of an Icosahedron in the Sphere, whose Diameter is E F.

It

It is to be ſhewn, that the right Line $\alpha\zeta$ is equal to a 3d Part of the Semicircle, whoſe Semidiameter is BF, that is, to the Arch BK, which is a 3d Part of a Quadrant of a Circle deſcribed from A B as a Semidiameter.

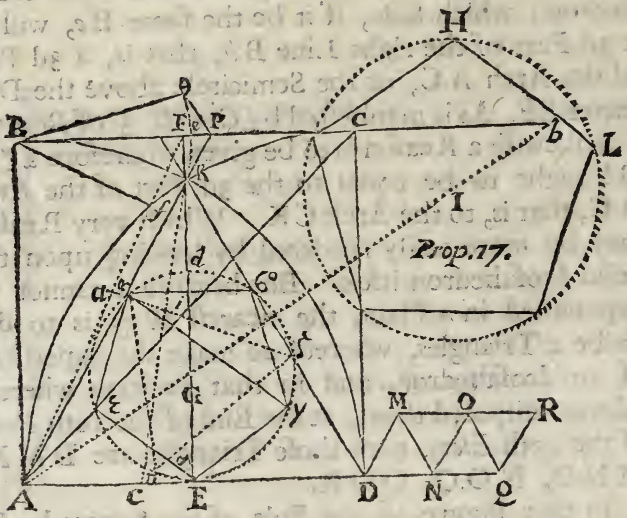
With the Semidiameter B e, deſcribe an Arch $e\theta$, in which take $e\theta$ equal to the Arch $\gamma\zeta$. Then draw Bθ, it will be equal to $\alpha\zeta$, that is, to the Side of an Icoſahedron in a Sphere whoſe Diameter is E F. Then with the Radius Bθ, deſcribe an Arch of a Circle cutting B P, it will give a Side of an Icoſahedron; which Side, if it be the ſame B e, will be a 3d Part of the right Line B b, that is, a 3d Part of the Arch A C, or the Semicircle above the Diameter EF. As is manifeſted by *Conſect.* 3. of *Prop.* 7.

Likewiſe a Reaſon is to be given wherefore $\alpha\zeta$ or Bθ ought to be equal to the 3d Part of the Arch A C, that is, to the Arch CK. Which very Reaſon may be more eaſily rendered by looking upon the Solid Icoſahedron itſelf. But becauſe it cannot be expounded in a Plain, the neareſt Way is to deſcribe 4 Triangles, whereof 20 make the Superficies of an Icoſahedron, and in that Poſition wherein *Clavius* diſpoſed them, at the End of the 16th *Prop.* of the 13th *Elem.* then thoſe Triangles are D M N, M N O, N O Q, Q O R.

In this Figure let the Pole of the Sphere be D, then the Points D, M, N, O, Q, R, will be in the Concave Superficies of the Sphere; and therefore the right Lines M N, N O, O Q, will not be in the ſame Plain with the Points D and R, which are in the Plain by the Diameter of the Sphere.

Wherefore the Side of an Icoſahedron proceeds from the Pole D to the Pole R, by 5 equal right

Lines, to wit, from D to M, from M to N, from N to O, from O to Q, from Q to R. And first in the Motion from D to M, it is moved forward towards R. Again, from M to N, it is not moved forward towards R. Thirdly, from N to O, it is moved forward, as much as from D to M. Fourthly, from O to Q it is not moved forward towards R. Fifthly, from Q it is moved forward to the very Point R. Therefore by 5 equal right Lines, which are the Sides of an Icosahedron, is the Motion made



from Pole to Pole; but by reason of the Digression to the nearest Circles, which divide the Superficies of the Sphere in 5 Parts, the Motion is made by the 5 Sides of an Icosahedron.

But in the Circumference of a Semicircle, the Motion is made from Pole to Pole by 3 Arches, every

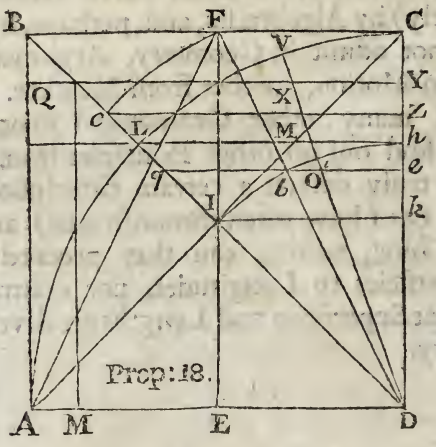
every one of them being equal to the Arch C K or B K. The Motion therefore being taken away, which is made by thoſe two Sides of an Icoſahedron which move not forward, the Ways by the 3 Sides of an Icoſahedron, and by 3 Arches of a Semicircle, every one of which being equal to the Arch C K, or at leaſt, by the Chords of thoſe Arches, will be equal. But B θ will be found greater than the Chord B K. Wherefore the Arches every one being equal to B K, are equal to 3 Sides of an Icoſahedron, and one to one. Which was to be demonſtrated.

This kind of Demonſtration will be condemned (I know ſurely) by Algebraiſts, and, perhaps, by others, who do not admit in Geometry, Arguments to be taken from Motion, neither from Meaſure. But to this, and many other Geometrical Propoſitions, they can find out no other Principles from whence they can truly derive a certain Concluſion. For Numbers (as I have often demonſtrated) are unapt for this Thing, neither can they proceed rightly from Superficies to Longitudes, nor contrarywiſe, becauſe that Superficies and Lengths are divers kinds of Quantity.

PROP. XVIII.

A Circle being given, to find a Quadrat equal to it.

LET the Circle given, whose Quadrat is D A C, be inscribed within a Quadrat A B C D; and D L C the 8th Part of the Circle. Cut the Quadrat A B C D with the Diagonals A C, B D; also with the right Lines E F, I *k*, all intersecting in I, and



dividing the Quadrat into 4 Parts both Ways; and draw D F cutting the Arch C L in P, and by the Point P draw Y Q, cutting the Diagonal B D in Q; then D Y, Y Q will be equal.

With the Radius D F, describe an Arch *Fc* cutting the Diagonal B D in *c*, so that D F, D *c* may be Equals.

I say,

I ſay, the Quadrate from YQ or DY is equal to an 8th Part of the Superficies of the Circle DCL.

From the Point *c* draw *c z* parallel to Y Q, cutting D C in *z*. Then is *z c* (by the 6th Prop. of this) half the Arch A C, and therefore equal to the Arch C L.

In the Arch C L, take the Arch L V equal to C P, and join D V, cutting Y P in X; and the 3 Lines C Y P, will be wholly within the Sector D C L; but the 3 Lines P Q L are all without the ſame Sector.

Likewiſe both the 3 Lines together are (as before I have ſhewn, and now will ſhew) equal to the Sector A P V. For becauſe the right Line B C is cut in the Middle at F, and the Baſes of the Triangles D C B, D Y Q are parallel; alſo the Baſis Y Q is cut in the Middle in P, and the Triangles D Y P, D P Q are equal.

Now D P L more P Q L more C P Y, are equal to D V L, or D C P (becauſe D P L more P Q L is equal to D Y P). For D C V more D V P is equal to D C P, or D V L.

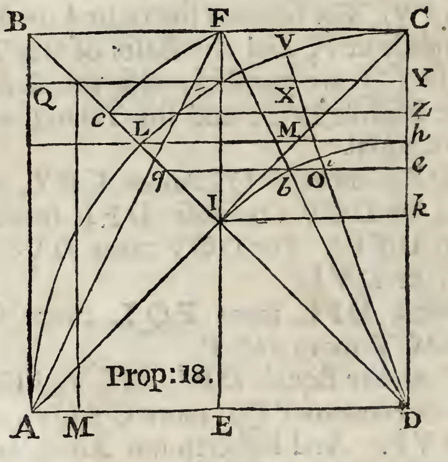
Wherefore D P L more P Q L more C Y P is equal to D C V more D V P.

Then both the Equals D P L, D C V, being taken away, there remains P Q L more C Y Q equal to the Sector D V P. And hitherto our Adverſaries conſent or agree. This alſo they muſt condeſcend to (for it is manifeſt) that if C Y P, P Q L, are equal between themſelves, the Triangle D Y Q, and the Sector D C L are likewiſe equal.

And I had thought it demonſtrated before from this, becauſe the Triangle whoſe Vertex is D, and Baſis parallel to the Side B C, no Triangle can be

made equal to the Sector DCL, whose Base passeth not by P; but the Force of the Demonstration they have not clearly understood. Go to then, we will produce, if it be possible, a more clear Demonstration.

With the Radius DI, which is a mean Proportional between the Side DC, and the half Side D *k*, describe an Arch of a Circle cutting DF in *b*, and DV in *i*, and DC in *h*; and by the Point *b* draw *e q*, cutting DC in *e*, and the Diagonal BD in *q*, and DV in *o*. Then the Sector DCP will be dou-



ble of the Sector D *h b*, as also of the 4 Lines CP *b h*; likewise the Triangle DXY double of the Triangle D *o e*; it is likewise double of the 4 Lines YX *o e*. Wherefore the remaining Sector DVP is double to the 3 Lines CYP, and likewise double to the 4 Lines VP *b i*. And D *b i* double to the 3 Lines *h b e*.

Then

Then when as the Sector D V P is double to the three Lines C Y P, and the ſame equal to two three Lines, C Y P, P Q L; then C Y P, P Q L will be equal between themſelves.

Which was to be demonſtrated.

Therefore a Quadrate is found (to wit, a Quadrate from Y Q) equal to the 8th Part of a Circle, to wit, to the Sector D C L; and ſo is effected the ſquaring of a Circle, nor now the firſt Time, but many Years ago ſufficiently demonſtrated by divers Methods.

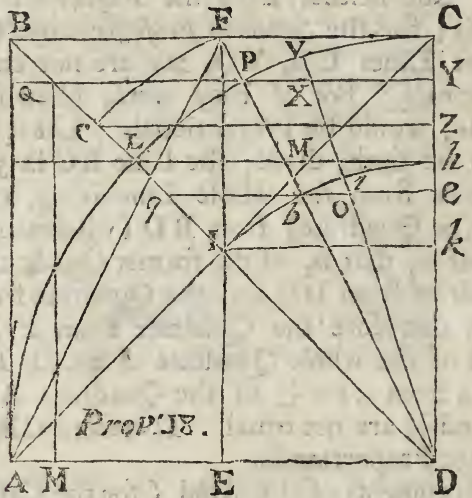
From this Demonſtration likewiſe I have deduced the duplicating of a Cube, ſhewing that the 4 right Lines C B, zc, eq, and I k, are continual Proportionals. But neither have the Algebraiſts underſtood this; for the *Savilian Profeſſor* objects, that the right Lines C B, Y Q, zc are not continual Proportionals. For if they were, likewiſe B D, D Q, D c, would be Proportionals. Let it be ſuppoſed as he ſaith, divide the Line B C in 5 Parts, the Square from the whole Line is 25, and the Square (or Quadrate) from B D (whereas it is the Double of it, that is, of the former Quadrate) 50; the Quadrate from D Q 40, the Quadrate from D c 32; and therefore the Quadrate from zc, 16 of thoſe 25 of the whole Quadrate A B C D. But the Quadrate from zc is $\frac{10}{16}$ of the Quadrate A B C D. But $\frac{16}{25}$ and $\frac{10}{16}$ are not equal. Therefore C B, Y Q, zc are not Proportionals.

But Arguments of this Kind (for the Cauſe declared at the 15th Propoſition) are the mere Spirit of Deluſion in Algebraiſts, in applying thoſe Numbers to Quantities, which have not the Proportion of Number to Number.

If

If BC be divided in 5 Parts, the Quadrate from DF will be 6 $\frac{1}{4}$, to wit, the 4th Part of 25. Wherefore D*c* will be 31 $\frac{1}{4}$, and the Quadrate from BD will be 8 Times as much as the Quadrate from BF, or 50.

But the mean Proportional between 8 and 5, will be the Side (I call it the Side, not the Root) of the Number 40. Then not only 50, 40, 32, but alſo 50, 40, 31 $\frac{1}{4}$ will be continual Proportionals. Therefore what the Cauſe of this Diſcord ſhould be (ſince it is plainly demonſtrated, that both *zc* is equal to Half the Arch AC, and that the Quadrate from YQ is equal to the Sector CL) unleſs that Lines drawn (that is diviſible according to Breadth) cannot be compared



with pure Lines, that is, without Breadth. But the right Lines CB, *zc*, *eq*, *k*I, are continual Proportionals, as I ſhall make appear moſt clearly in the following Demonſtration. Con-

CONSECT. I.

The Arch CL is leſs than four fifths of the Radius DC; for all the verſed Sines which are poſſible to be drawn in the Quadrant DAC, being taken together, are equal to the Area or Content of the Quadrant, and by conſequence to the Quadrat DYQM, or to four fifths of the Quadrate ABCD. And for becauſe thoſe verſed Sines are all bounded within the Arch, the 2 Sides BC, CD of the Quadrat ABCD, ought to be to the Halfs of the Arch AC, in Proportion double of CD to DY; but it is not ſo. For altho' you ſhould divide Lines, or other continued Quantity infinitely, notwithſtanding you will never attain to nothing; becauſe continual Quantity is always diviſible into Diviſibles. Then thoſe Sines, whereof the whole Number of them fill the Area of a Quadrant, will have every one their Breadth, and will always be finite in Number. Alſo thoſe verſed Sines are Parallels between themſelves, whereof the greateſt DC is the Radius of a Circle, whoſe Bound (or End) at C is a ſmall Arch. Alſo DC is the Side of a Quadrat ABCD, and therefore (becauſe it hath Breadth) it will be a right Angle, and therefore more than the Radius DC. And part of it will be without the Circle. The Proportion of the reſt of the verſed Sines is the ſame; and for becauſe according to Breadth, they may be divided as often as it is poſſible to divide, they will never proceed to an Indiviſible.

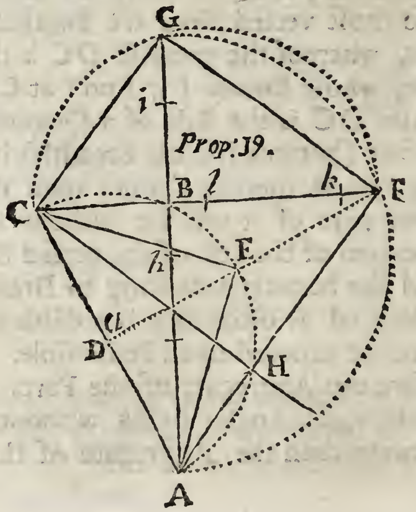
Therefore the Aggregate of the Parts compaſſed about of the right Angles being without the Quadrant, is more than the Aggregate of the Parts of the

the Quadrant itſelf AC. Alſo the Arch AC is ſome ſmall matter leſs than four fifths of the Diameter, and the Arch CL, or the right Line zc leſs than four fifths of the Side DC.

PROP. XIX.

Between a right Line given, and the Half of it; To find two mean Proportionals.

LET the given Line AB, and the Half of it BC, be diſpoſed at right Angles; join AC, and let AC be cut in the Middle in D; from the Centre D with the Radius DA, deſcribe a Semicircle, and divide it in the Middle in E. Draw AE, CE, and in CB being produced, put BF equal to CE. In AB being produced, take 2 5ths of the right Line AB, which let be Bi, and between AB and Bi find



find a mean Proportional Bk: Equal to it in AB produc'd, put BG (which, as above, is demonſtrated, is equal to 2 5ths of the Quadrantal Arch deſcribed from AB) and draw FG, GC. Moreover, divide CF in the Middle in l, from the Centre l with the Radius lF, deſcribe a Semicircle by F and G, which will paſs by C; which I ſhew thus.

The right Line AC, whoſe Quadrate is Quintuple of the Quadrate BC, is a mean Proportional between the Quadrantal Arch deſcribed from AB and its Half, as is ſhewn at Prop. 6. Therefore CE is equal to half the Quadrantal Arch deſcribed from AB, for becauſe the Angle by E is a right Angle, CE is a mean between both the right and circular Lines CE, EA, and either of them. Wherefore BF is equal to half the Quadrantal Arch deſcribed from AB. And BG is a 3d continual Proportional to BF, and the whole Line AB by Prop. 7. Then becauſe the Quadrantal Arch from AB, and the right Line AB, and 2 5th Parts of the Quadrantal Arch from AB, are continual Proportionals; it will be, as 2 5ths of the Quadrantal Arch from AB (or the right Line Bk) to the Half AB, ſo AB to the Half of the Quadrantal Arch from AB. Therefore it is as AB to BF, ſo BG to BC.

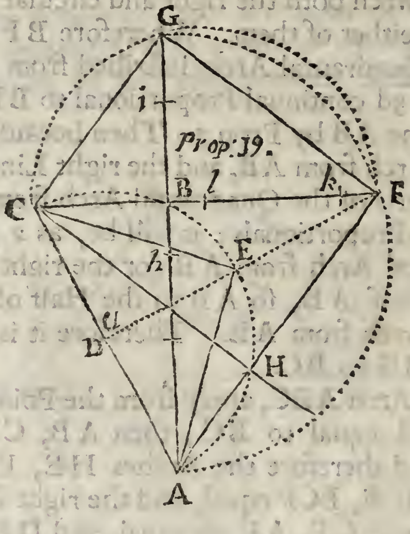
In the Arch ABC, apply from the Point A, a right Line AH equal to BC; then AB, CH will be equal, and therefore the Arches HE, EB, or the Angles HCE, ECF equal, and the right Lines HF, BF, likewiſe CF, AF are equal, and DE produc'd, divides the Angle AFC in the Middle.

Alſo the Triangles ABF, GBC are alike, wherefore the Angles BGC, BAF are equal, alſo the Angles BCG, BFA equal. And (for becauſe the Angle CHA is

is a right Angle, and CH equal to AB) the Triangle ABF, FBG are alike and equal. Therefore both BF, GC, alſo CH (or AB) and FG, are equal between themſelves.

And AG being divided in the Middle in *h*, from the Centre *h*, with the Radius *h* A, a Semicircle being deſcribed, it will paſs by F.

Then is FGCH right Angled, and the Triangles ABF, FBG alike. Therefore it is as AB to FB, ſo FB to BG, and ſo BG to BC. Therefore BF, BG are found to be two mean Proportionals between the given Line AB, and the Half of it BC. Which was to be done.



C o r.

It is manifeſt from the Precedings, that the greater of the 2 Means is an 8th Part, and the leſſer Mean

2 5th

2 5th Parts of the Quadrantal Arch described from A B. Alſo A B (or C H) is equal to F G; and B G is equal to that Part of the right Line C H, which is cut off by A B, being computed from the Point C, and the Conſtruction of the Problem demonſtrated to the Eyes of all Men.

PROP. XX.

Of the Centre of Gravity of a Quadrant of a Circle.

THE Centre of Gravity (that is, the Centre of Weightineſs or Heavineſs) of a Quadrant of a Circle is in a right Line from its Centre, dividing the Arch in the Middle, and diſtant from the Centre of the Circle, ſo much as is the mean Proportional between the Semidiameter, and 2 5th Parts of it.

Deſcribe the Quadrate A B C D, and in it a Quadrant A D C, and draw the Diagonals A C, B D, whereof B D will cut the Arch A C in the Middle in L.

Between the Semidiameter D C, and 2 5th Parts of it, find a mean Proportional D R, and with the Radius D R, deſcribe an Arch of a Quadrant R S, cutting the Diagonal B D in *z*. I ſay *z* is the Centre of Gravity of the Quadrant D A C.

Cut the Quadrate A B C D into 4 Parts by the right Lines E F, G H, cutting themſelves mutually and at right Angles in I. Join D F, cutting the Arch A C in P; and by P draw Y Q parallel to B C, cutting the Diagonal B D in Q, and E F in V; alſo compleat the Quadrate D Y Q M, whoſe Side Q M cuts the Arch A C in N.

Then as the Quadrate from D F, to the Quadrate from

from DP or DC, so is the Quadrate from DC to the Quadrate from DY. And the Quadrate ABCD is 5, whereof the Quadrate DYQM is 4.

Join YM, and it divides DQ in the Middle; also draw the right Lines *Pk*, NO, that parallel to the Side AB, this parallel to the Side BC, they will cut each other mutually, and at right Angles, in the Middle of the right Line DQ.

Therefore because the Quadrate ABCD is to the Quadrate DYQM as 5 to 4, the Quadrate of DQ will be 8, whereof the Quadrate from YQ is 4, and the Quadrate from DC 5, and the Quadrate from the half DQ 2.

Then the Quadrate from the Half of DQ, is 2 5ths of the Quadrate ABCD; therefore the Half of DQ is a mean Proportional between the Semidiameter DC, and 2 5ths of it, and therefore equal to D*z*. Likewise *z* is both the Centre of Magnitude, and also the Centre of Gravity of the Quadrate DYQM; and the Point I the Centre of Magnitude and of Gravity of the Quadrate ABCD.

Also it is shewn at the 18 Prop. of this, that the Quadrate DYQM, and the Quadrant DAC between themselves are equal. And that the 3 Lines CYP, PQL, AMN, NQL between themselves are equal. Therefore if from the Quadrate DYQM be taken two three Lines equal to PQL, NQL, and to the same Quadrate be added two three Lines CYP, AMN at equal Distances from the Diameters of even Height, the 3 Lines CYP, AMN will be equiponderant (or weigh alike) but the Distances YP, MN, PQ, QN are equal and alike distant from the Diameters of equal Height NO, P*k*; likewise the Points Y, Q, M, D are equally distant from the Centre

Centre of Gravity of the whole z; therefore z is the Centre of Gravity of the Quadrant D A C, which was to be demonſtrated.

Consect.

The Centre of Gravity of a Semicircle, is a mean Proportional between two Fifths and one Fifth of the Arch A C. For it is ſhewn (*Prop.* 5.) that the right Line, which is a mean between the Semidiameter, and two Fifths of it, is equal to two Fifths of the Arch A C; therefore if z O be produced to p, ſo that z O, O p be equal, then draw $D\ p$, p will be the Centre of Gravity of a Quadrant equal to D A C· And becauſe the Point O divides $z\ p$ in the Middle, the Point O will be the Centre of Gravity of the double of the Quadrant D A C, that is, of a Semicirle deſcribed with the Radius D C; alſo $D\ p$ produced, cuts the Arch C l equal to the Arch L C. Then when as the Quadrant D A C reſteth in z, and the Double of the Sector C l reſteth in p, the whole Semicircle will reſt in O: But z O is a mean Proportional between D z, and its Half, that is, between two Fifths, and one Fifth of the Arch A C, becauſe the Triangle DO z, is right angled and equicrural; (*viz.* it hath two Sides equal.)

I PROP.

PROP. XXI.

The Centre of Gravity of the Circular Line and Strait Line, ALCA *is in the Diagonal* BD, *distant from the Point* B, *so much as is the Length of a Tangent of* 30 *Degrees.*

FInd a Tangent of 30 Deg. B X, equal to which, from the Point B in the Diagonal BD take B T. I say that the Point T is the Centre of Gravity of the two Lines A L C A.

For becauſe the Quadrat from A B, is to the Quadrat from B X, or B T, as 3 to 1, and the Quadrat from the Semidiagonal B I is half of the Quadrat from A B, the Quadrat from B I will be to the Quadrat from B T, as $\frac{3}{2}$ to 1, as 3 to 2.

And the three Lines A B C L A, to the two Lines A L C A, as 2 to 3: Which I thus ſhew.

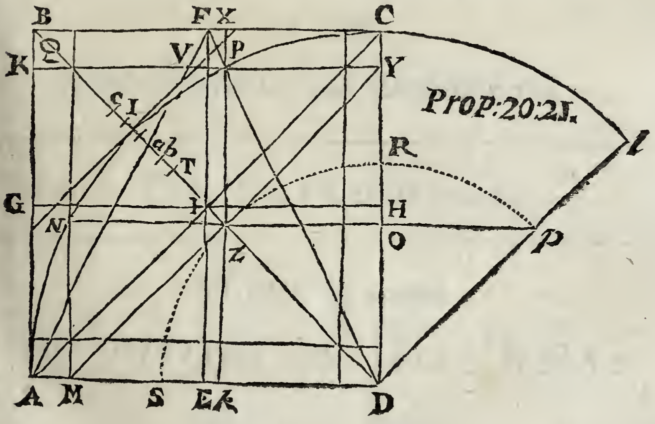
The Gnomon Y B M, is a fifth Part of the Quadrat D Y Q M, that is a fifth Part of the Quadrant D A C: Wherefore alſo the three Lines A B C L A is a fifth Part, or two tenth Parts of the Quadrat A B C D; and the Triangle A B C is the Half, or five Tenths of the Quadrat A B C D: But the Quadrant D A C is four Fifths, or eight Tenths of the Quadrat A B C D; therefore the two Lines A L C A is three Tenths of the Quadrat A B C D.

Then is the Proportion of the three Lines to the two Lines, the ſame which 2 is to 3, that is, reciprocal of the Proportion, as well of the Magnitudes ALCA, ABCLA, as alſo of the Quadrats BI, BT Join A F cutting the Diagonal B D in *a*. Then becauſe A B is double to B F, and the Angle A B I divided

divided in the Middle by the right Line B *a*, A *a* will be the double of *a* F: Therefore the Point *a* is the Centre of Gravity of the whole Triangle A B C.

Cut *a* T in the Middle in *b*, and take *a c* the Triple of T *b*, and let *a* be the Centre of the Ballance. It will be therefore as the three Lines A B C L A, to the two Lines A L C A, that is, as 2 to 3, so reciprocally *c a*, to *a* T, to wit, as 3 to 2.

Therefore is T the Centre of Gravity of the two Lines A L C A; and the Point *c*, the Centre of Gravity of the three Lines A B C L A: Which was to be demonstrated.



The End of the SECOND BOOK.

SOME

PRINCIPLES

AND

PROBLEMS

IN

GEOMETRY,

Thought formerly Desperate;

NOW

Briefly Explained and Demonstrated.

The THIRD BOOK.

Written in *Latin*

By *THOMAS HOBBES*:

And made *English*

By *VEN. MANDEY.*

LONDON:

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SOME

Principles *and* Problems

IN

GEOMETRY, *&c.*

CHAP. I.

Of the Subject, Principles, and Method of the Mathematicks.

HE that would apply himſelf to the Study of the Mathematicks, muſt, in the firſt place, underſtand what the Matter is concerning which Mathematicians treat, and what it is that is enquired into about that Matter.

We are therefore to know, that the Matter which they treat of, is every thing that hath Magnitude, that is to ſay, all about which the Queſtion may be asked, *How great is it?* So that the ſubject Matter of Mathematical Sciences, are the *Lengths*, *Superficies* and *Thickneſs*, or Profundity of Bodies: For every Body hath three, and no more Dimenſions, and theſe differing in Kind. From the Motion of a Body, a thing long is produced, which is called a *Line*; then from the Motion of the Line ſprings a Superficie; and from the Motion of the Superficie, *ſomewhat thick or groſs*, which

which is alſo called a *Solid and Mathematical Body* There is no farther Progreſs; for which way ſoever a ſolid Body is moved, a ſolid Magnitude will be deſcribed. They therefore, who among the Kinds of Magnitudes rank Surdoſolids, Quadratoquadrats, *&c.* do idly; for theſe are not Figures, but only Numbers, that is, not Magnitude, but Magnitudes.

Motion, Time, Force, Weight, belong likewiſe to the Mathematicks; for one of theſe may be ſaid to be bigger or leſs than another: Nay in ſo far the Mathematicks treat of Qualities, as one may be called more or leſs ſuch than another. For of whatſoever may be adverbially ſaid, *more or leſs*, that hath a Foundation in ſome Magnitude. The Compariſon of Motions, Magnitudes, Times, *&c.* amongſt themſelves, pertains alſo to the Mathematicks; for by reaſon of the various *Inequalities* of Things, one *Inequality* is more or leſs than another. *Inequality* therefore hath its own Magnitude, and that is it which is uſually called *Proportion* or Reaſon. Number belongs to the Mathematicks alſo; for to what Science a Body, or Motion, or Time, or a Line belongs in the Singular; all theſe belong to the ſame in the Plural. For when One is nothing elſe but a *thing number'd*, Number can be nothing, other than *Things number'd.*

Now to ask how big a thing infinite is, would be an abſurd Queſtion, and not at all to be anſwer'd. For neither can a finite Magnitude be divided into Parts infinite in Number, nor can an infinite Number be given: when Mathematicians frequently ſay, *Let a Line be drawn out in infinitum*, they are not ſo to be underſtood, as if they thought that would

be done, but that they leave it to be produced at the Reader's pleaſure.

The thing that is enquired of thoſe Subjects of the Mathematicks, is in a manner nothing elſe, than how great the propoſed Magnitude is, as it is compared with another Magnitude of the ſame Kind; or that I may ſay it in a Word, the Proportion is enquired. It is therefore neceſſary to one that ſtudies Mathematicks, exactly to underſtand theſe, and all other Terms which Mathematicians uſe; that is to ſay, to know the true and clear Definitions of the ſame.

Quantity (which the Antients no where defined, and who, for want of a Definition thereof, raſhly diſputing about the Nature of an Infinite, have erred) I thus define: Quantity is a Magnitude every way determined either by Expoſition, ſo that to one asking, How great is it? It may be anſwered, As great as you ſee; or by Compariſon, ſo that it may be anſwered, It is one or more Feet great, or any other ſuch way. But above all things, he ought to have an accurate Definition of Proportion, or *Mathematical Reaſon*; and beſides, to know how to Add, Subtract, Multiply, and Divide Proportions, and the Varieties of Compariſons; of which, though moſt of them be taught by *Euclid*, and his Interpreters, yet I ſhall reduce almoſt all of them to the next Chapter.

The Matter being known, and what we enquire into concerning it, the next thing is to take the Principles: The Principles of what? the Principles of the Science, or of knowing that we ſeek for. Now nothing but Truths can be known. The Principles then, of the Mathematicks, are the *firſt Truths*, which we are not taught, but know them by the Light of Nature, ſo ſoon as we hear them ſpoken. They are therefore called

called the firſt Propoſitions, becauſe of them the firſt of all Syllogiſms conſiſt. The Subject then of a Science, or any part of it, is not to be called a Principle; as a Point of Geometry, or an Unity of Arithmetick (which as a certain Maſter taught publickly) tho' the firſt Book of *Euclid*'s *Elements* begins with a Point, and the ſeventh Book with an Unity.

The Principles of Mathematicks, or firſt Propoſitions, are Definitions; but that theſe may be uſeful, they muſt not only be true, but likewiſe accurate, leſt that through ſo great Variety of Significations, we either proceed with Uncertainty, or by continual Diſtinctions being beat out of the Path, loſe the Truth by jangling. For as if a Hare ſhould ſtart amidſt a Company of Maſtiffs, the Dogs let looſe would fall upon one another, whilſt the Hare is running for it; ſo likewiſe in a Theatre of thoſe who diſpute for Victory, Truth flies away undiſcerned. And for this Cauſe (I mean, to remove Diſtinctions) wiſe Men have introduc'd Definitions.

But to define well, that is, ſo to circumſcribe with Words the Thing in Hand, and that clearly, and with as much Bevity as can be, that no Ambiguity be left, is a very difficult Task, and not ſo much a Work of Art, as of the natural Intellect. However we are eaſed of moſt part of this Difficulty by the Induſtry of the antient Geometricians; ſave that *Euclid*'s Definition of a Line needs ſome Correction, or at leaſt a ſound Interpretation. It is true indeed, that in comparing of Longitudes, there is no reſpect at all to be had to Latitude; but yet in the Conſtruction of Figures, Latitude is neceſſary; for without Latitude, it is impoſſible to deſcribe a Figure; nor can it be known in what part of a broad

Line

Line that inviſible Line is to be underſtood. The reſt, tho' all of them be not moſt exact, yet becauſe they are true Propoſitions, they ſpoil not his Demonſtrations.

Now Definitions are of two Kinds, of which the one barely explains the Nature of the Thing, and the other the Cauſe or Manner of producing it. But theſe Definitions are moſt uſeful to the advancing of Knowledge, which contain the Cauſes of, and Manner of producing the Thing defined. For that Saying of *Ariſtotle's* is true, *To know is to know by the Cauſe*: The reſt, which only declare the Eſſence of the Thing defined, are generally leſs fruitful; for nothing follows from them, that was not before contain'd in them; nor matters it, whether their Properties be called Definitions, or the Definitions, Properties.

Petitions are uſually reckoned among Principles alſo; but theſe are not the Principles of knowing, but of conſtructing, that is, of the Deſcription of Figures, wherein nothing is required, but that they be poſſible.

All Propoſitions likewiſe, whoſe Truth is obvious to the Light of Nature, are to be held for Principles. Now there no Doubt to be made of the Truth of a lawful Definition, becauſe they have their Truth from the Conſent and Will of Men, who, at their Pleaſure, give Names to Things explained. But there are ſome Propoſitions, which tho' they depend on Definitions, and may by them be demonſtrated; yet are ſo perſpicuous, that even without a Demonſtration they can force an Aſſent. Theſe are called *Axioms*; but care is to be had, that they be not held for true upon the bare Authority of Maſters. There

is

is great Difference between the Authority even of the beſt Maſter, and the Light of Nature: For there is nothing ſo abſurd, but that ſome Maſter (in Defence of ſome Error) hath ſome time written Things as abſurd.

The Manner likewiſe of adding, ſubtracting, multiplying, and dividing of Coſſick Numbers (that is, Things number'd) is to be foreknown; which is eaſy, and taught by *Vieta* in his *Iſagoge*, but moſt clearly and briefly by *Oughtred*, in the *Key of Arithmetick*; from whence I have transferred as much as I thought fit into the third Chapter. The Nature and Manner of finding out Square and Cubick Numbers, with their Roots, is alſo to be learnt. This is handled in the Fourth Chapter.

The Nature of an Angle is likewiſe to be known; and therefore I have explained the Properties thereof in the Fifth Chapter.

To conclude, if he uſe any thing that is well demonſtrated by any Author for a Principle, he will do well; for every thing known ſtands inſtead of a Principle, for a farther Progreſs in Knowledge.

Some perhaps may ſay, that I place all Geometry in *præcognita*, or *Things to be foreknown*; but they are miſtaken: For he is a Geometrician (as I ſaid before) who knows how to determine a Quantity by Compariſon, amongſt Quantities of the ſame Kind. But thoſe Things which I would have foreknown, are only Properties that attend Geometry, and Steps whereby we are initiated in the Myſteries of ſublimer Geometry, and whereof we ſtand in need not only for Figures and Mechanick Works, but alſo for the Knowledge of all natural Cauſes, as being all contained in Motion.

The

The Principles and Theorems, which are well demonſtrated in the Books of the ancient Geometricians, being well underſtood (for I write to thoſe who deſire to improve the Science, the Thanks for the Work being due to the firſt Inventors; and no Man can reject what is already done, but who is very learned in his own Conceit) the next Thing is, (not to demonſtrate, but) to ſeek out the Cauſes of Proportion, that one Magnitude hath to another: For no Man can demonſtrate what he knoweth not, nor know but what he hath found out, nor (unleſs very ſeldom) find out what he hath not ſought after. There is no certain Art of Invention by Sagacity, trying, ſuppoſing, deducing Conſequences from Things ſuppoſed: Men often attain either to the Cauſes of the Matter ſought after, or to ſomething impoſſible. And by this Sagacity (not by Art) all the ancient Geometricians performed what they did, as manifeſtly appears, in that moſt of their Theorems begin with the Note of Suppoſition [*If*]. But (one may ſay) how ſhall one chance to take that ſuppoſed, which he ſtands moſt in need of? There is no certain Method whereby the moſt convenient Suppoſitions may be choſen; and therefore we muſt try and experiment by meaſuring with a Compaſs, and other Inſtruments, how near we can come mechanically to the Truth of the Thing ſought. For Inſtance, in ſeeking the Length of the Circumference of a Circle (all Men knowing, by the Light of Nature, that any Arch is bigger than its Chord) it is an eaſy matter to come very near the true Length. The remaining Work is no more but to excogitate the poſſible Cauſes of ſo great a Propinquity, wherein if they ſucceed, they have the Principle of a Demonſtration. For indeed, in all Doubts,

Nature

Nature it ſelf ſets us upon various Suppoſitions. But moſt Geometricians enquire nothing now adays, but if it be *Euclid's*; tho' it be well known that the moſt noble Theorems were invented, and the moſt beautiful Fabricks of the whole World built long before the Time of *Euclid*. Now all our modern Geometricians, almoſt, imagine to find not only the Proportions of Quantities, but alſo of Motions, that is, the Cauſes of all natural Effects, in the ſole Extraction of the Numeral Roots, and that a falſe one too; and the Reaſon is, becauſe they are ſo taught.

The laſt thing in Geometry is, the efficient Cauſes of the thing ſought being found, to demonſtrate from them as it were backwards: I ſay, from the efficient Cauſes, becauſe the Cauſes of Numerical Properties are, for moſt part, nothing but arbitrary Impoſitions of Numeral Names; and Arithmetical Operations are the Demonſtrations of the Works themſelves.

CHAP. II.

Of Reaſon or Proportion.

1. REaſon is the *Relation* of a Magnitude to a Magnitude of the ſame kind, according to Quantity; as two Magnitudes, 2 and 5 being propoſed; the Reaſon of the firſt to the ſecond is the Relation of the Quantity of 2 to 5, that is, the Quantity of the Magnitude 2, as it is compared with the Magnitude 5: For the Inequality of two Magnitudes may be more or leſs than the Inequality of two other things unequal.

2. Therefore, for ſhewing Proportion or Reaſon, it is neceſſary that two abſolute Magnitudes be expoſed,

posed, of which the former is usually called the Antecedent, and the second the Consequent.

So that neither both Quantities together, as 2 and 5, nor one alone, as 2 or 5; neither $\frac{2}{5}$ (therefore our Algebraists do err, who say that Fraction and Reason are the same thing) neither the Difference betwixt 2 and 5, but the Quantity of Inequality that is betwixt two Magnitudes, is that which is called Reason or Proportion: For though both 3 and 2, and 9 and 8, differ only by a Unity, yet their Inequality is not the same. Yet if there be three Magnitudes, the least, the mean, and the greatest; the Reason or Proportion of the least to the mean, is greater than the Reason of the same to the greatest. But on the contrary, the Reason or Proportion of the greatest to the mean, is less than the Reason or Proportion of the same to the least. We might say that *Teucer* was bigger, compared to *Ulysses*, than to *Ajax*; and *Ajax* was less, compared with *Ulysses*, than with *Teucer*; the Reason of this is manifest; for by how much a lesser Magnitude comes nearer a greater Magnitude, by so much it hath a greater Reason or Proportion to it; and by how much more a greater Magnitude exceeds a less, by so much it hath also to it a greater Reason or Proportion.

3. Now the Proportions or Reasons of less to greater, and of greater to less, are different Kinds of Quantity; whereof the one is the Proportion of Defect, the other of Excess; of which the former, by how much the more it is multiplied, by so much the Defect becomes always the less; therefore it can never exceed or equal the Proportion of Excess.

4. Reasons or Proportions are the same, or alike, or equal, in Numbers, when they are of the same Denomi-

Denomination, or may be reduced to the ſame Denomination; as if both the firſt of the ſecond, and third of the fourth be $\frac{2}{3}$ or $\frac{4}{7}$: But where the Proportions are not expreſſible by Numbers, both the firſt of the ſecond, and third of the fourth, are correſpondent Sides of like Triangles, and are called Proportional, as having Portion for Portion, by the *Greeks* Ἀνάλογον, as the ſame thing ſaid again.

5. Continual Proportionals are ſuch, as, being proportional, have a middle Quantity common, as 1, 2, 4.

6. The Root of a Square Number is that Number, which, being multiplied by it ſelf, produceth ſome Number: Now the Number produced is called a Square or Quadrat, and the Root is always an aliquot Part of its own Square; as, if the Root be 8, the Square will be 64, and the Root the eighth Part of it.

7. It is then manifeſt, that the Root of a Square Number is not the Side of a Square Figure, though ſome Algebraiſts deny this, being now aſhamed to confeſs the Truth.

8. If there be three Magnitudes continually proportional, and from the ſame Antecedent three other continual Proportionals; the Proportional of the laſt to the laſt, will be the double of the Proportion of the ſecond to the ſecond; as is manifeſt in theſe Numbers, 2, 4, 8, and 2, 6, 18; the Proportion of 18 to 8, is the double of the Proportion of 6 to 4. For as 18 is to 8, ſo is 9 to 4; but 9 to 4 hath a double Proportion of 6 to 4. *Elem.* 14. *Prop.* 28.

9. Two Proportions are compounded or added together, when both Antecedents and Conſequents being multiplied among themſelves, a Proportion is made. For Example, if the Proportion of 2 to 3 be to be added to the Proportion of 4 to 5; let the Antecedents

tecedents 2 and 4 be multiplied one by another, and they make 8; then let the Consequents 3 and 5 be multiplied one by another, which make 15. This being done, the Proportion compounded of the Proportions of 2 to 3, and 4 to 5, is the Proportion of 8 to 15, according to *Eucl. Elem.* 6. *Def.* 5.

10. Otherwise, if it be thus; as 4 is to 5, so 3 to another which shall be 3 $\frac{3}{4}$, the Proportion of 2 to 3 $\frac{3}{4}$ (that is, both being multiplied by 4) will be of 8 to 15, that which is compounded of the Proportions of 2 to 3, and 4 to 5.

11. The Subtraction of a Proportion from a Proportion, is done by dividing the Quantities of the whole Proportion to be subtracted by the Antecedent, and the Consequent of the whole to be subtracted by both. For Example, Let the whole Proportion be of 8 to 15, the subtracted of 4 to 5; I divide 8 and 15 by 4, the Quotients are 2 and 3 $\frac{3}{4}$. Again, I divide the Consequent of the whole Proportion 15 to be subtracted by both the Quantities, to wit, 4 and 5, the Quotients will be 3 $\frac{3}{4}$ and 3, which are in the Proportion of 12 to 15, placing them in order, 8, 12, 15. If from the Proportion of 8 to 15, the Proportion of 12 to 15, that is, of 4 to 5, be taken; it is manifest that there remains the Proportion of 8 to 12, that is, of 2 to 3.

12. Otherwise, if it be thus; As the Consequent of the Subtrahend, which is 5, is to its Antecedent 4; so the Consequent of the whole, which is 15, is to the other 12; placing in order 8, 12, 15, it is plain that the Proportion of 12 to 15, that is, of 4 to 5, being subtracted from the whole Proportion of 8 to 15, there remains the Proportion of 8 to 12, that is, of 2 to 3.

13. Pro-

13. Proportion is multiplied by ſaying, As the Antecedent is to the Conſequent, ſo the Conſequent to a third, and ſo the third to a fourth, *&c.*

14. A Proportion is divided by Number, when between the Antecedent and Conſequent, one or more mean Proptionals are interpoſed.

15. If there be never ſo many Magnitudes of the ſame Kind, the Proportion of the firſt to the laſt, is compounded of the Proportions of the firſt to the ſecond, of the ſecond to the third, and ſo forth unto the laſt; as in any Numbers, 4, 9, 10, 3, the Proportion of 4 to 3, is compounded of the Proportions of 4 to 9, and 9 to 10, and 10 to 3, which is demonſtrated by many.

16. But there is another Kind of compared Quantity, which is alſo called Proportion, not Simply, but Arithmetical Proportion. Now four Quantities are ſaid to be in Arithmetical Proportion, when the one conſtantly exceeds the other by equal Differences. Concerning which Proportion, I ſhall only ſay, *If a Line were cut into as many equal Parts, as it is intelligible it may be divided into, the Geometrical Proportional Mean between every next two of theſe Parts, will be the ſame with the Arithmetical Mean.* If (for Example) a given Line ſhould be divided into the Parts 2 and 16, the Arithmetical Proportional Mean will be 7; ſo then 2, 9, 16 are continual Arithmetical Proportionals. Now the Geometrical Proportional Mean is leſs than 6; but if the ſame Line had been divided into 100000 equal Parts, and both a Geometrical and Arithmetical Mean taken betwixt the Whole and its Parts 99999, how ſmall a Difference would be between thoſe two Geometrical and Arithmetical Proportions? Therefore by how much leſs the Difference

betwixt the Extremes is, by ſo much leſs is always the Difference between the Geometrical and Arithmetical Mean. Wherefore if ſuch Means were every where interpoſed, all their Difference would vaniſh. This is a moſt uſeful Propoſition for finding out the Proportions that ſtrait Lines have to crooked.

CHAP. III.

Of Algebraical Operations.

1. THE Algebraiſts inſtead of Things, that is, inſtead of Magnitudes, put Letters as the Species of Things, which in this manner we are taught to Add, Subtract, Multiply and Divide. If the Species be ſolitary, as A and B, they are added by interpoſing the Sign +, which ſignifies more. A and B then being added, the Sum is A + B; but if B be to be ſubtracted from A, they interpoſe the Sign —, which ſignifies leſs, or a Mulct. Therefore A — B is that which remains of the Magnitude A, after that B is taken from it: Multiplying by a Number, is when the Number Multiplier is prefix'd, as 2 A, 3 A, *&c.* The Multiplication of A by itſelf is performed by writing A A or A q; a Line cannot be multiplied by itſelf; and this is the firſt thing wherein the Algebraiſts of this Age differ from *Diophantus* and the Ancients, who contained themſelves within the Bounds of Arithmetick. If A be to be multiplied into B, they write AB, if it be written $\frac{A}{B}$, A is underſtood to be

divided

divided by B, which ſignifies the Quotient, as if A be 10, and B 2; that Mark $\frac{A}{B}$ is 5: This does indeed, very well in Numbers; but if A A be a ſquare Figure, and B the Diviſor a Line; then that Quotient $\frac{AA}{B}$ ſhews how often the Line B is in the ſquare Figure A A, which is againſt the Nature of true Algebra, and likewiſe abſurd; for Planes or Flats are not compoſed of aggregated Lines.

But if the Species be affected with like Marks or Signs, as + A + A, they being added together make 2 A. For this is a true Rule of *Oughtred*, and the Algebraiſts; in Addition the Species are to be added, and a common Mark to be affixed. If therefore — A be added to — A, the Sum will be — 2 A; but if the Marks be different, then this is the Rule; let Species be ſubtracted from Species, and the contrary Mark prefixed to what remains; ſo that + A and — A added together make 0, that is nothing.

3. But if two affected Species be to be added to two affected Species, this is the Rule: If they be like Signs, let the Species be added, reſerving the Signs. So A + B, and A + B added together make 2 A + 2 B. But if the Marks be different, as in theſe + A — B, and — A + B, the Rule is; let the like affected be reduced under their proper Signs, and the Sum will be + A + B — A — B, that is nothing.

4. In Subtraction, if the Species be ſolitary, and each have the Sign +, as + A + B, and + B be to be ſubtracted from + A, the Remains is the Species from which the Subtraction is made, with the Sign +, together with the Species ſubtracted having the con-

trary Sign. For B taken from A leaves A — B; but if both the Signs be —, the Difference of the Species with the Sign + is to be taken for the remaining Part, as if —5 be to be ſubtracted from —7, the Remains will be + 2. For it is manifeſt, that — 5 is more than — 7 by two Unities. And that it is ſo, is eaſy to be underſtood by any that will take the Pains to conſider it. For what is it elſe, to take the Defect from a given Magnitude, but to add to it the deficient Magnitude which it wanted? If therefore — A be to be taken from A, it is manifeſt that there remains 2 A; for by the very taking away of the Defect, the Acceſſion of that which was wanting is added, in the ſame manner as he that remits a Debt, beſtows upon his Debtor as much as he hath remitted. If ſeveral Species affected with Signs be to be multiplied by one or more Species affected, this is the Rule: If the Signs be alike, let the Sign + be prefixed to the multiplied Species; but if different, the Sign —; as in this Example,

$$\begin{array}{r} A+B \\ A+C \\ \hline +CA+BC \\ AA+AB \qquad \\ \hline AA+CA+AB+BC \\ \hline \end{array} \qquad \begin{array}{r} A-B \\ A-C \\ \hline -CA+BC \\ +AA-AB \qquad \\ \hline +AA-CA-AB+BC \\ \hline \end{array}$$

Which is true indeed in Numbers; but if they be Lines, falſe. For nothing can be multiplied but by a Number.

As many Species as you pleaſe, and howſoever affected, are underſtood to be divided, when they place the

the Divisor under the Dividend, as $\frac{A+B}{B-C}$ is the Quotient of $A+B$ divided by $B-C$, that likewise is true in Numbers; but if $A+B$ be a Superficie, and $B-C$ a Line, it is absurd, as we observed before. What remains is no more but Labour in seeking out a commodious Supposition, from whence they may begin, which is not Art, but groping and chance. Now when they have found out any Proportion by changing, converting, multiplying, dividing, compounding, and elevating, and depressing it by a Scale of continual Proportionals, they toss the same, until that at length unawares, and by hap-hazard, they stumble upon some Equation that may satisfy an Arithmetical Problem: But it is impossible by this Method to answer a Geometrical Question. Algebra is, indeed, one of the Rules of Arithmetick, to the attaining the Theory whereof, two or three Days at most are required, though to the Promptitude of Working, perhaps the Practice of three Months is necessary.

CHAP. IV.

Of Square Figures, and Square Numbers.

1. A Square Figure, and a square Number, when they have so many square and equal Parts, are one and the same thing; for a whole Figure, and the Number of its Parts are the same.

2. If ſquare Figures be deſcribed from *creſcent Radii* (or Radius's increaſing) as Numbers in natural Order from an Unity, they differ among themſelves as odd Numbers of the ſame Order. For the leaſt Figure is one, to this the next odd Number (which is 3.) being added, makes the next ſquare Figure (4.) To this again, the next odd Number (5) being added, makes the third Square (9) and ſo on. Therefore the Exceſſes of the ſaid Squares increaſe as odd Numbers from an Unity, to wit, 1, 3, 5, 7, *&c.*

3. If there be three ſquare Numbers next to one another in order, and a Unity be taken from the middle, the Number that is left is the mean Proportional betwixt the Extremes. For Example, 100, 81, 64 are ſquare Numbers next to one another in order. Let the *Unity* be taken from 81, there remains 80, which is a mean Proportional betwixt 100 and 64. In like manner 49, 36, 25, are ſquare Numbers next to one another in order. Take an Unity from the middle 36, there remains 35 the mean Proportional betwixt 49 and 25. So in theſe 9, 4, 1; if from the middle 4 one be taken, there remains 3 the mean betwixt the Squares 9 and 1. Laſtly, 400, 361, 324, are Squares next to one another in order: If from the middle 361 an Unity be taken, there remains 360, the Mean betwixt the Extremes 400 and 324, and ſo in all.

4. Between two next ſquare Figures, if to the leaſt Square its Root be added, the whole will be the mean Proportional. For if to the Square 1, its Root (which is 1) be added, the whole 2 is the mean Proportional betwixt 1 and 4. In like manner, if to the Square 4 its Root be added, the whole 6 is the mean Propor-

tional

tional betwixt 4 and 9; and ſo in all, as is manifeſt by Induction.

5. If from a Square its Root be taken, there will remain the Mean betwixt it and the next leſſer Square. Let 100 be a ſquare Figure, if its Root 10 be taken from it, there will remain ninety, the Proportional Mean betwixt 100 and 81, the next leſſer Square. In like manner if from the Square 9, its Root 3 be taken, the remaining 6 will be the Mean betwixt 9 and the next inferior Square 4, and ſo always.

6. But if the Squares be not next to one another, but that one interpoſes, the leſs Square with two Roots, is the mean Proportional betwixt thoſe two Squares. For if to the Square 1 its two Roots, that is 2 be added, there wlll be 3, the Proportional Mean betwixt 1 and 9. Likewiſe if to the Square 4 its two Roots, that is 4, be added, there will be 8, the Mean betwixt 4 and 16, and ſo perpetually. The ſame Mean will be had, if from the greater Square, its two Roots be ſubtracted: For if from 16, its two Roots be taken, there will remain 8, the Mean betwixt 16 and 4.

If laſtly, Two Squares have other two between them, and to the leſs Square, the Triple of its Root be added, you will have the Mean betwixt the two extreme Squares. For if 3 be added to the Square 1, there will be 4, the Mean betwixt 1 and 16; or if to the Square 4, the Triple of its Root, to wit, 6, be added, there will be 10, the Proportional Mean betwixt 4 and 25; or if from 25 the Triple of its Root, to wit, 15, be taken, there will remain 10, the Mean betwixt 25 and

4, and ſo always. From which follows a general Rule, that the mean Proportional betwixt two Squares, is ſo many Roots of the leſſer added to the leſſer; or what remains after ſo many Roots are taken from the greater, as the greater Square is diſtant Places in the natural Order of Numbers from the leſſer.

7. Hence it is manifeſt, that the Root of a Square Number is not (as Algebraiſts imagine) the Side of a ſquare Figure, nor at all a Line, becauſe a Line adds nothing to a Flat, or Plane.

8. If two Squares are next to one another, both taken together, are greater than the double Mean, by a Unity, or one ſquare Particle. For 1 and 4 are next to one another, both together make 5, the double of the Mean is 4, the Difference is 1. In like manner 100 and 121 are next, both together are 221, the Mean is 110, the double of the Mean 220, the Difference 1, and ſo in all.

9. If Squares admit of one between them, the Extremes taken together, are greater than the double Mean by four; for 1 and 9 admit of one betwixt them, both together make 10, the double Mean is 6, the Difference 4. Likewiſe 100 and 144 receive one between them; the Extremes put together are 244, the Mean is 120, the double Mean 240, the Difference 4, and ſo every where.

10. If between two Squares, other two interpoſe, the Extremes together are greater than the double Mean by the Number Nine. For 1 and 16 have two betwixt them, added together they make

17,

17, the double Mean is 8, the Difference 9. Likewiſe 100 and 49 have two betwixt them, put together they make 149, the Mean is 70, the double of that 140, the Difference 9. In like manner, three being interpoſed, the Extremes exceed the double Mean by ſixteen, and ſo forth by ſquare Numbers throughout the whole Series of Squares.

11. Betwixt two Cubick Numbers, two mean Proportional Numbers intervene: Now they are found out after this manner; if the Cubes be next to one another, let the leſs Cube be divided by its Root, then let the Quotient be multiplied by a Number that exceeds the Root by an Unity, the Product will be the leſs of the Means. For Example, Take the next Cubes 1 and 8; let the leſſer Cube 1 be divided by its Root 1, the Quotient will be 1, then multiply 1, by one more 1, that is 2. This Number 2 is the leſs of the Means betwixt 1 and 8; by the ſame Method the greater Mean is found out, 2 being divided by 1, the Quotient is 2; then 2 being multiplied by one more 1, makes the greater Mean 4. Laſtly, 4 being divided by 1, the Quotient is 4, and 4 being multiplied by one more 1, it makes 8; or let the leſs Cube be 64, its Root 4, the next greater Cube 125; 64 being divided by 4, the Quotient is 16; 16 being multiplied by 4 more 1, makes 80, the firſt of the Means. Again, 80 being divided by 4, the Quotient is 20; which multiplied by 4 more 1, makes 100 the greater Mean. In the laſt Place, if 100 be divided by 4, the Quotient will be 25, which being multiplied by 4 more 1, makes 125.

12. If

12. If two Cubick Numbers have one Cubick Number betwixt them, let the less Cube be divided by its Root, and let the Quotient be multiplied by the Root more 2, the Product will be the lesser Mean; let the Cubes be 1, 27: 1 being divided by 1, the Quotient is 1, which multiplied by 1 more 2, makes 3 the lesser Mean. Again, 3 divided by 1 is 3, then 3 multipled by 1 more 2, makes 9 the greater Mean. Lastly, 9 divided by 1 is 9, which being multiplied by 1 more 2, is 27.

13. In like manner, if the Cubes be 8 and 64, 8 being divided by its Root 2, the Quotient is 4, and 4 multiplied by 2 more 2, makes 16, the first Mean; and 16 being divided by 2, makes 8, which multiplied by 2 more 2, makes 32 the second Mean. To conclude, 32 being divided by 2, the Quotient is 16, which multiplied by 2 more 2 makes 64. In like manner, if the Cubes be 27 and 125; 27 being divided by 3, the Quotient is 9, which multiplied by 3 more 2 makes 45 the first Mean. Again, 45 being divided by 3, the Quotient is 15, which multiplied by 3 more 2 is 75, and 75 divided by 3, the Quotient is 25; and this multiplied by 3 more 2 makes 125, and so always in Cubes that intermit one.

If two Cubick Numbers intermit two Cubick Numbers, the less being divided by its Root, the Quotient multiplied by the Root more 3, will give the first Mean. Let 27 and 216 be two Cubes intermitting two Cubes (to wit 64 and 125) 27 being divided by its Root, the Quotient is 9, which being multiplied by the Root 3 more 3, makes 54 the first

first Mean; 54 divided by 3 makes 18, which being multiplied by 3 more 3 is 108. Lastly, 108 being divided by 3, the Quotient will be 36, which multiplied by 3 more 3, makes 216.

If two Cubick Numbers intermit three Cubick Numbers, and the less Cube be divided by its Root, and the Quotient multiplied by that Root more 4, the Product will be the first of the Means. And so through all the Cubes of Numbers that are in a natural Order, adding 5, 6, 7, &c. you may find the first Means betwixt any two Cubes whatsoever.

C H A P.

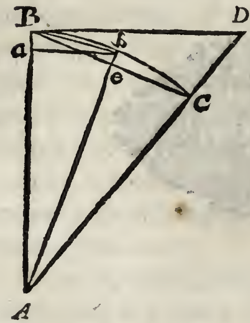
CHAP. V.

Of ANGLES.

1. A Strait or right Line Angle, is the Digreſſion of two ſtrait (or right Lines) from the ſame Point in the Circumference.

2. The Quantity of a ſtrait Line Angle, is the Quantity of the Proportion that the intercepted Circumference hath to the Perimeter of the Circle.

3. The Quantity then of an Angle is not a Line, neither a Superficie, nor yet a ſolid Body, but the Quantity of the Proportion which the Revolution of the *Radius* hath to another Revolution of the ſame.



4. The Chord of an Angle, Circumference or Arch, is a ſtrait Line that joins together any two Points of the Circumference; as if there be ſtrait Lines from the ſame Point AB, A*b*, AC, and the Angle ABD a right Angle; but the Arch B*b*C. Then the ſtrait Line BC is the Chord or ſubtenſe of the Arch BC, and the ſtrait Line B*b*, the Chord of the Arch B*b*.

5. The right Sine of the Circumference is a ſtrait Line drawn from one Term of the Circumference, to the Radius which paſſeth thro' the other Term perpendicularly; as B*e* or *ab*, is the right Sine of the Arch B*b*.

6. The

6. The Secant of an Arch or Circumference, is a ſtrait Line drawn from the Centre by one Term of the Arch to the ſtrait Line which is drawn perpendicularly in the other Term to the Radius; as A D is the Secant of the Arch B C.

7. The Tangent of an Arch, is that Part which is intercepted of the right Line which is drawn perpendicularly from the Beginning of the Arch; as B D is the Tangent of the Arch BC, or of the Angle ABC.

8. Hence it is manifeſt, that the right Sine of any Arch, is the half of the Chord of the double Arch, or that the Chord of the Arch, is equal to the double right Sine of the half of the Arch, as if the Arch B *b* be the half of the Arch B C, *a b* doubled will be equal to the Chord B C.

9. A leſſer Arch hath a leſs Proportion to its Chord, than a greater Arch hath to its Chord. For the Proportion of the Arch BC to its Chord B C, is the ſame with that of B *b*, the half of the Arch B C to the half Chord B *e*. But the Chord of the half Arch BC is the ſtrait Line B *b*, which is greater than B *e*. Therefore (by Chap. 1. Num. 2.) the Proportion of the leſſer Arch B *b* to its Chord B *b*, is leſs than that of the greater Arch B C to its Chord B C.

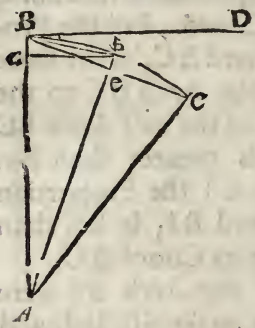
10. Hence it follows, that if the Arch B *b* were biſected, and the Biſegment of it again biſected, and ſo on as often as can be done; we would at length come to a Segment of an Arch, the Exceſs of which upon its own Chord ſhould be leſs than any given Longitude, and by conſequence, leſs than the Breadth of any Line having never ſo little Latitude.

11. In like manner, if a ſtrait Line were biſected, and the Biſegments thereof again biſected as often as might be, we would come at length to a Segment

leſs

leſs than any aſſignable Quantity, and therefore leſs than the Breadth of a Line having never ſo little Latitude.

12. Now that univerſal Saying, *Every Arch is greater than its Chord*, would hold univerſally true, if there could be given an Arch of a Circle, and a ſtrait Line, that had no Breadth; but neither is there a Line without Latitude, nor if there were, or could be but ſuppoſed, could it enter into a Demonſtration, that handles the Proportion of Superficies and Solids; becauſe they are different kinds of Quantity, as appears by the fifth Definition, El. 5. of *Eucl.* Neither (as I conceive) would Dr. *Wallis* have thought otherwiſe but that that Definition is not found among the Principles of Algebra in *Oughtred's Key of Arithmetick.*



13. In any Triangle, if the Angle be cut in two equal Parts, and the ſtrait Line that cuts the Angle, cut alſo the Baſe; the Segments of the the Baſe will in Proportion of the Sides. This is clearly demonſtrated in *Eucl.* Elem. 6. Prop. 3.

14. There is another Kind of a plain Angle, which they call the Angle of Contact, ſuch as the Angle that the ſtrait Line BD makes with the Arch BC in the Point B. Which *Euclid* (or his Interpreter *Theon*) and many that have followed him, have affirmed to be greater indeed, than any acute Angle; and the Angle which AB makes with the ſame Arch BC, bigger than any acute Angle, yet leſs than a right, which is manifeſtly abſurd. For then

then, as Dr. *Wallis* will have it, the Angle of Contact had been of the ſame Kind with the Angle that is made by the Revolution of the Radius; for every Angle that is not obtuſe, is either right, or leſs than a right Angle, by ſo much as any Angle is acute.

They were in the firſt Place miſtaken, in that they thought the Space betwixt the ſtrait Line BD, and the Arch BC to be an Angle, which (by Numb 3. of this Chapter) is falſe. They were alſo miſtaken, in that they took thoſe Angles for Quantites of the ſame Kind, which is falſe. For an Angle of Contact, howſoever multiplied, can never equal nor exceed an Angle made by the Circulation of the Radius.

15. If it were true, that the Angle made by the right Line AB, with the Arch BC, were leſs than a right Angle; it might come to paſs that an acute Angle increaſing uniformly, would at length become greater than a right, which yet could never equal it: The Ground of which Abſurdities is, that they thought no Line to have any Latitude, nor a Sine any Quantity at all.

16. That an Angle in the Centre is the double of an Angle in the Circumference (if they inſiſt upon equal Arches) is demonſtrated by *Eucl. El.* 3. *Prop.* 20. But it is to be obſerv'd, that to the Eſſence of an Angle in the Circumference, it is not required that both the Sides ſhould terminate in the Circumference; for DBC is an Angle in the Circumference, though the Point D be not in the Circumference.

CHAP. VI.

Of the Proportion of the Perimeter to the Radius of the Circle.

1. A Quadrantal Arch is equal to the Radius, together with the Tangent of 30 Degrees.

Let the Radius be AB, the half Radius BE, or AF, draw the Line then EF, the Rectangle ABEF, will be the half of the Square from the Radius AB. By the Radius AB let the Arch BG be deſcribed, cutting EF in G: The Line then AG being drawn, and produced to BE produced in H, BH will be the Tangent of 30 Degrees, and BG the third Part of the Quadrantal Arch deſcribed by AB; then from the Point G to the Side AB, let the Perpendicular GI be drawn. In BA produced, take AN the double of AI, ſo that IN be the Triple of IA; then draw the Secant NG, cutting AF in T, and produce it to BH in *q*.

I ſay, that the ſtrait Line E*q* is equal to the Arch BG.

Upon AT deſcribe a Quadrantal Arch ST, becauſe then the Triangles NIG, NAT are alike; AT will be two Thirds of the ſtrait Line GI, the half Radius; that is, the third Part of the whole Radius.

Becauſe therefore AT is the third Part of the Radius, the Arch ST will be equal to the Arch BG. Let BG be cut in two in the Middle in *i*, and AT in M, and let the Perpendicular *i b* be drawn to AB, and

and AM will be the 6th Part of the Radius AB, and the Arch YM equal to the Arch B *i*; and as N *b* is to N A, ſo ſhall *b i* be to AM: Draw then and produce NM, it will paſs thro' *i*; now let it be produced to BH in Q. Let *b i* be produced to the Concourſe with M *q* in *t*, and *b t* will be the Double of *b i*, and equal to the Chord of the Arch BG. For the right Sine of the Half of any Arch, is the Half of the Chord of the double Arch. Again, let the Arch B *i* be biſected in *e*, and AM in L; and draw the Perpendicular *e a* to AB, and *e a* will be the Half Chord of the Arch B *i*, and the 4th Part of the 2 Chords of the Arches B *i* and *i* G. Now let AL alſo be the 4th Part of the ſtrait Line AT; draw then and produce the Line NL, and it will paſs by *e*. Now let it be produced to BH in P. Therefore the Arch KL is equal to the Arch B *e*. In like manner, if MT be divided in X, draw the Line NX, and produce it, it will cut the Arch BG in *o*, ſo that *o* G is the 4th Part of the Arch BG, and the Quadrantal Arch VX will be equal to the Arch B *o*; and ſo perpetually. The ſtrait Lines then, which are drawn from the Point N, will divide both the ſtrait Lines AT, B *q*, and the Arch BG, into the ſame Proportions.

If then the Biſection were continued as much as may be, we would at length come to the Arch of a Circle, leſs than the Latitude of the Line or Side AB. Alſo according to *Chap.* 5. *Numb.* 9. the Proportion of an Arch to the Chord always decreaſeth. Wherefore the Difference of the Arch to its own Chord decreaſeth alſo, until it become leſs than any aſſignable Difference.

L Let

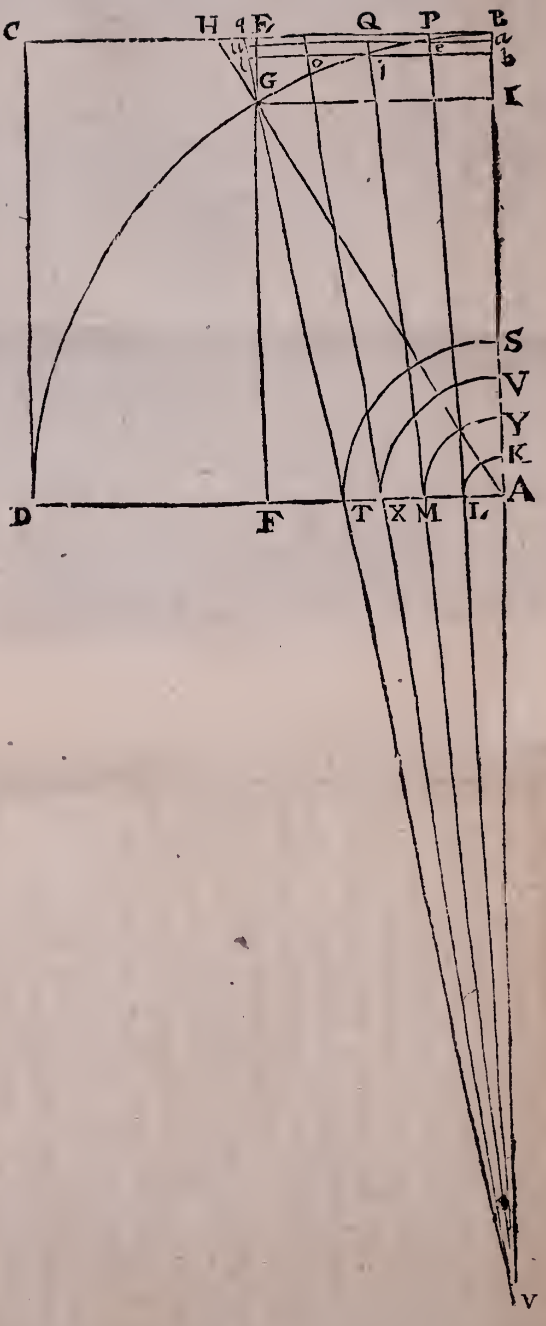
Let us then ſuppoſe the Latitude of the Radius A B to be equal to the leaſt Segment, or leſs than it (for by biſecting, it can never be reduced to nothing) the laſt and leaſt Segment will be in the ſtrait Line BH to *q*, and ſo continually all the equal Segments of the Arch BG will lie extended toward the ſtrait Line B *q*; for B *q* is greater than *b t*, which is equal to the Chord of the Arch BG. Let *a e* be produced till it meet with N *q* in *u*, and *a u* will be equal to 4 Chords of the Arch B *e*, but leſs than the whole B *q*. In like manner, if the Arch *a e* be biſected, and the right Sine of it drawn, that produced till it meet the ſtrait Line N *q*, it will be equal to 8 Chords of the Arch *a e*, but leſs than the ſtrait Line B *q*; and ſo always, until one come to the laſt and leaſt Segment, whoſe Longitude is equal to the Latitude of the Radius A B.

Therefore, if the leaſt Latitude be allowed to the Side A B, the Arch BG, and ſtrait Line B *q* will be equal; but if all Latitude be denied to a Line, not only the leaſt Part of an Arch will be greater than its Chord; but alſo all its Parts to all the others, ſo that no Arch can be equal to a ſtrait Line. Neither can there be any Line drawn, nor Sine, nor Arch, nor Figure. For a Line without Latitude is nothing, and the Half of it as much.

Therefore B *q* is equal to the Arch B G, that is, to the third Part of the Quadrantal Arch deſcribed by the Radius A B.

Now becauſe the Triangles G T F, E G *q*, are Rectangular and alike, and TF the third Part of the Half Radius A F; E *q* will be the third Part of EH (the Exceſs of the Tangent BH above the Half Radius B E). The Triple then of B *q* (that

is,



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is, the Quadrantal Arch deſcribed by the Radius A B) is equal to the Triple of the Semiradius, together with the Triple of E *q*; that is, to the Radius A B, together with the Tangent of 30 Degrees, that is, together with B H, which was to be demonſtrated.

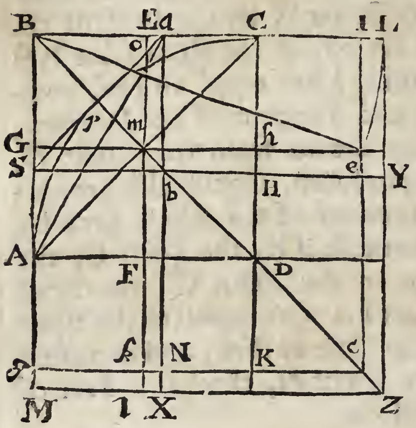
2. If B *q* were produced until it were tripled, and A F until it were doubled, it is manifeſt that a ſtrait Line drawn from the Point N by the End of the doubled A F, would cut off, in the ſtrait Line B H being produced, a ſtrait Line equal to one, compoſed of the Radius and Tangent of 30 Degrees; but that a ſtrait Line drawn from the Point N, paſſing through A D produced, ſhould alſo proportionally divide what remains of the whole Arch by B G, is impoſſible; becauſe if by the Point G, the Tangent were drawn to the Point G; the ſtrait Line G A produced until it were equal to the ſtrait Line B N, would do the ſame as B N; and therefore ſtrait Lines from the Point N, thro' any Part of the Arch, would not do it.

SCHOLIUM.

3. Becauſe I have ſhewed in *Book* 2. (of *Geometrical Roſes*) that a Quadrantal Arch is equal to a right Line, whoſe Square is ten times as great as a Square from the half Radius; and have now made out, that the ſame Arch is equal to a ſtrait Line, made up of the Radius and Tangent of 30 Degrees: In this Place I will, from moſt clear Principles, demonſtrate, that a Square from that ſtrait Line compoſed of the Radius, and Tangent of 30 Degrees, is alſo the Decuple of a Square from the Semiradius.

L 2 Deſcribe

Deſcribe a Square ABCD, and let it be divided 4 Ways, as well by the right Lines EF, GH, as the Diagonals AC, BD, concurring in the Centre of the Square at *m*; and deſcribe the Arch AC cutting EF in *o*; and let it be produced to the Side BC in *a*. B*a* is then the Tangent of 30 Degrees; let the Side BG be produced, and in it being produced, let



CI equal to the half Side BE, and CL equal to the Tangent B*a*, be taken. Then deſcribe a Square from the whole BL, which is BLZM, which is to be demonſtrated to be equal to ten Squares from BE or CI.

The Square from BE is BE*m*G, the Square from the Tangent B*a*, is B*ab*S, to which let CLYH*b* be equal.

Deſcribe alſo a Square from BI, which ſhall be BI*c*g, and let CD, EF be produced to the Side MZ cutting g*c* in K and *k*. Let *ab* alſo be produced to the Side MZ in X, and SH to LZ in Y.

Underſtand now, that the 4 Sides of the Square ABCD, are each divided into 12 equal Parts. So it will be that the Square ABCD does contain 144 equal little Squares.

Becauſe the Side BC is 12, the Side BI is 18, the Square of the former is 144, of this 324, the Square from

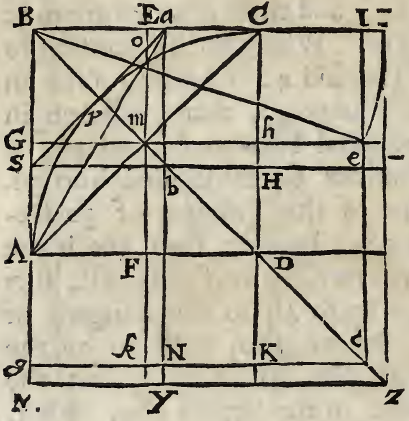
from 19 is 361, the Square from 20 is 400; and theſe ſquare Numbers are in Order next to one another. Wherefore (according to *Chap.* 4. *Numb.* 3.) an Unity being taken from the mean Square 361, the remaining 360 is the mean Proportional betwixt 400 and 324, that is, the Root of the ſquare Number from 400 multiplied into 324.

Again, Becauſe BC is 12, the half Side BE is 6, which ſquared is 36. Now the Square from 7 is 49, the Square from 8 is 64; and theſe Squares are next to one another in Order. Wherefore (according to the ſame *Number* of the ſaid 4th *Chapter*) take an Unity from the mean Square 49, there will remain 48, the mean Proportional betwixt 64 and 36, or the Root of the Number 64 multiplied into 36. Now 48 is the Square of the Tangent of 30 Degrees in the Square 144, becauſe then 360 is the Decuple of the Square from the half Side BE, it is to the Square 144, as 10 to 4; ſo the Tangent of 30 Degrees, in the Square 360, will be to the Square of the Tangent alſo, as 10 to 4; but the Tangent of 30 Degrees, in the Square 360, to wit, the 3d Part of it, is 120, which Number is to 48, the Square of the Tangent in the Square 144, as 10 to 4. Therefore 360 is the Square from the right Line compoſed of the Radius and Tangent of 30 Degrees, which right compounded Line is BL, which was to be demonſtrated.

4. In the right Line I*c*, take I*e* equal to the ſtrait Line CI, which being joined to BE, the Arch of the Circle deſcribed by the Radius B*e* will paſs by L; for BI hath the Power of 9 Squares, whereof I*e* hath the Power of 1; therefore B*e* hath the

Power of 10 half Sides, the Arch therefore deſcribed by it will paſs by L.

5. We muſt not here paſs over a Difficulty, which in the 23d *Chapter* of the *Book* of *Principles*, I did indeed mark, but thought fit to leave the Solution of it to the Readers: The Solution of this will much conduce to the illuſtrating and facilitating of Geometry. This is the Difficulty: The Square CLYH (which is the Square of the Tangent of 30 Degrees, and equal by Conſtruction to B *a b* S) is to the Square C I *e h*, as 4 to 3, as is commonly known: Becauſe therefore the Gnomon I L Z *c* M ought to be equal to the Triple of the Gnomon I L Y *e h* H, and ſo that Gnomon being tripled and placed upon this, ought to agree with it, but it agreeth not, for it is leſs. Hence it may ſeem conclusive, that the Side of the Square, which is the Decuple of the Square from the half Side, is greater than the Square from B L; but why? becauſe it is leſs by ſo much as the double little Square *m b*, or *c* Z, or *e* Y.



For ſuppoſe the right Line BC to be divided into 12 equal Parts, the whole Square A B C D will be 144 of thoſe Parts; the Square C I Y H 48, the Square alſo A X 48, and the Square D Z 48, all

theſe

theſe make 288. Now ſeeing the Square from the half Side C I is 36, the whole Square (which is 10 times as much) ought to be 360; 288 being then taken from 360, the Remainder 72 ought to be equal to the 2 equal Rectangles X D and D Y, and either of them ought to be 36, but it is not ſo. The Square F D K *k* is indeed equal to the Square from BE or CI (that is 36) from which if the Rectangle F *l* be taken, and the Rectangle X K added (ſeeing that is greater than this by the Quantity of the Square *k* X) the remaining Rectangle X D will be leſs than the Square F K, that is, than 36, as much as the Square *k* X or *c* Z; for the ſame Cauſe the Rectangle D Y is alſo leſs than 36 by the Quantity of the Square *e* Y or *c* Z. Therefore the whole Square B L Z M is leſs than the Decuple of the Square from the half Side, which is 36, by the Quantity of the double little Square *c* Z.

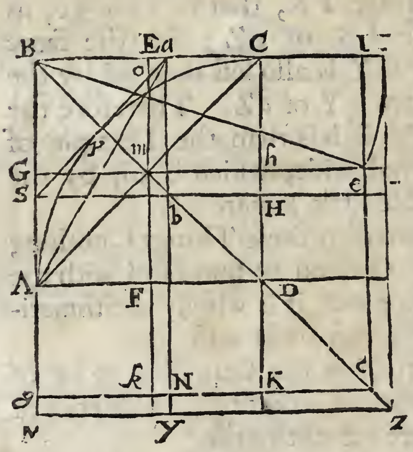
What can be anſwered to theſe Things? nothing truly, if there be any Ground to find fault with the former Demonſtration; but it is wholly Arithmetical, wherein it is not ſo eaſy to be miſtaken.

I will therefore firſt ſhew the Objection to be of no Force; then, whence it is that that Difference of the double little Square *c z* doth ariſe.

That it is of no Force, is manifeſt from hence, that by the ſame Argument it may be proved (it being ſuppoſed, that the right Line B C is divided into 12 equal Parts) that the Square of it alſo is leſs than 144, by the Quantity of the ſame double Square *c* Z. For ſeeing the Square C *m* is 36, the right Angle C *a b* H will be leſs than it by the Quantity of the Square *b m* or *c* Z. For from the Square C *m* on the one Side, the Complement E *b* is taken, on

 the

the other *b b*; but E *b* is greater than *b b* by the Square *b m*. Therefore the Rectangle C *a b* H is leſs than 36, by the ſame little Square *c* Z; for the ſame Reaſon the Rectangle A *b* is leſs than 36, by the ſame little Square *c z*; there remains for compleating the whole Square, the Square *b* D, which is the Half of the Square from the Tangent B *a*. For join *a s* cutting the Diagonal in *p*, it will be as S *a*, or B *b* to B *a*, ſo B *a* to *b p*, that is, to H *b*; *a b* is then the mean Proportional betwixt B *b* and *b* H; therefore the Square *b* D is the Half of the



Square B *a b* S, that is 24; therefore the whole Square 144 is equal to 48 + 24 + 36 + 36, leſs the double Square *c* Z, which is abſurd. For it is not to be doubted, but that the Square A B C D is 144. Therefore this Objection is of no Force, and has this Fault beſides, that the Square from BL is by it made leſs than 360, not by the Double, but the Quadruple of the Square *c* Z.

The Cauſe of this Diſagreement is therefore to be enquired into, and it can be no other, but the reckoning of Lines without Latitude in the Proportions of Superficies; whence it follows neceſſarily, that that which is contained within the 4 Sides B L, L Z, Z M, M B, is leſs than 10 Squares from B E, by

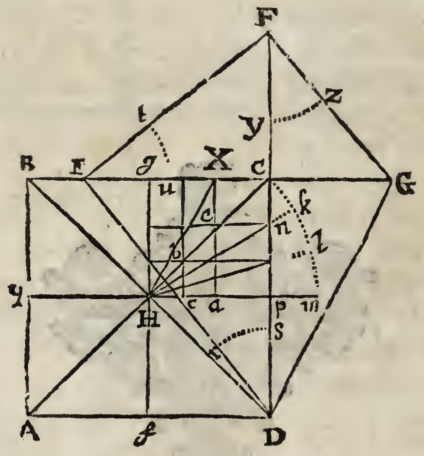
by the Double of *c* Z; but that the Sides themſelves are equal to the Quadruple of *c* Z. Therefore that the Square from B L is the Decuple of the Square from the half Side, ſtands good, and ſhall remain ſo; for it cannot be proved by any Argument, that the Square from B L is leſs than the decupled Square from B E, but that by the ſame it will be proved, that the Square from B C is leſs than 4 Squares of the ſame B E. We have then, by the Solution of this Difficulty, removed one, and perhaps the greateſt Hindrance to Geometry, to wit, the Computation of Superficies by Lines without Latitude.

CHAP. VII.

Of mean Proportionals.

1. HOW to find a mean Proportional betwixt 2 right Lines given, is taught by *Euclid's El. 6.* Prop. 13.

2. To find out 2 mean Proportionals betwixt a right Line given and its Half; let DC be the right Line given, and its Half CG; let them be



difpofed at a right Angle in C; then make a Square DABC, and let it be cut as well by the Diagonals AC, BD, as by the right Lines *gf*, *qp*, meeting 4 Ways in H.

Take

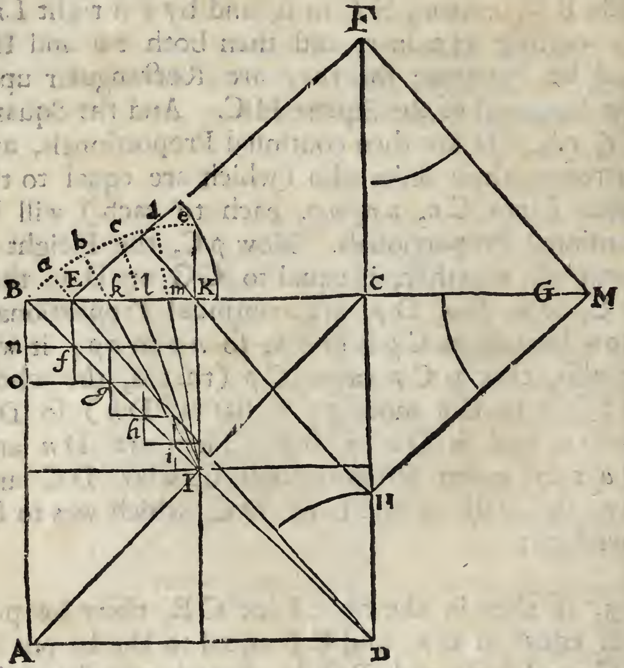
Take the mean Proportional betwixt B C and its Half C G; now that is equal to the Semidiagonal H C, by the Radius H C deſcribe the Arch C *m*, cutting *q p* produced to *m*, and let the Arch C *m* be cut into 3 equal Parts in *k* and *l*, and draw H *k*, H *l* cutting D C in *n* and *o*. Now let C *x* equal to C *n* be taken; draw *x a* parallel and equal to C *p*; cutting H C in *e*, C *e* then is a Square. In like Manner by the Point *o*, draw a Parallel to the Side B C, cutting H C in *b*, and by *b* a right Line *b c* cutting H *p* in *c*, and then both *e b* and H *b* will be Squares; for they are Rectangular upon the Diagonal of the Square H C. And the Squares C *e*, *e b*, *b* H are then continual Proportionals, and therefore their Sides alſo (which are equal to the right Lines C *n*, *n o*, *o p*, each to each) will be continual Proportionals. Now *p* C, the Height of them all together, is equal to C G or D *p*; then D C, D *n*, D *o*, D *p*, are continual Proportionals. Now becauſe as C *p* is to *n p*, ſo *n p* to *o p*; it will be alſo, that as C *p* more C *p* (that is, the whole D C) is to C *p* more *n p* (that is, D *n*) ſo D *n* to D *o*, and ſo D *o* to D *p*. Then are D *n* and D *o* two mean Proportionals betwixt D C and D *p*, the Half of the ſame D C, which was to be found out.

3. If then in the right Line C B, there be put C E equal to D *n*, and C F equal to D *o* be put in D C produced, and F G be joined, and the right Lines D E, E F drawn, the Triangles D E C, E C F, C F G will be Triangles of equal Angles, that is, the Arches *ſ r*, *t v*, *z y* (having equal Shanks) will alſo be equal.

4. How

4. How to find 4 mean Proportionals betwixt a right Line given, and its Half.

Let the right Line given (in the 2d Figure) be DC, and the Half of it CG; make a Square DC BA, the 4th Part of which is the Square IB. So then the ſtrait Line IB is the mean Proportional betwixt the whole BC and its Half CG.



By the Radius IB deſcribe the Arch B*e*, cutting IK produced to *e*; and let it be cut into 5 equal Parts by the ſtrait Lines I*a*, I*b*, I*c*, I*d* cutting BK

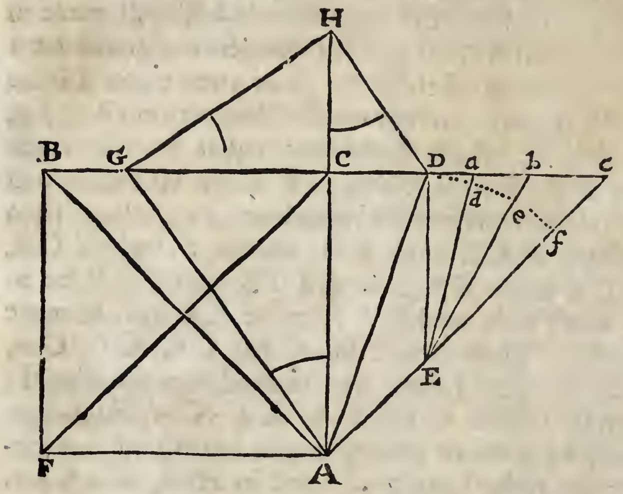
B K in E, *k*, *l*, *m*, and let B *n* equal to B E be taken. The Lines *n f*, E *f* being then drawn, the one parallel to B E, and the other to B *n*, they will meet in the diagonal Line I B at *f*, and B *f* will be a Square. In the ſame Manner, if B *n* be lengthen'd to *o*, that it be equal to B *k*; right Lines drawn by *o* and *k* parallel to the oppoſite Sides, will meet in the Diagonal I B at *g*, and therefore *f g* will be a Square, and ſo of the reſt. Let then upon I B be made 5 Squares in continual Proportion B *f*, *f g*, *g h*, *h i*, *i* I, whoſe Sides are equal to the right Lines B E, E *k*, *k l*, *l m*, *n* I, theſe therefore will be likewiſe continual Proportionals; putting then together, as K B more K B, that is, the whole C B, is to C K more K E; ſo will C K more K E be to C K more K *k*, and ſo C K more K *k* to C K more K *l*, &c. Then are C B, C E, C *k*, C *l*, C *m*, C K, 6 right Lines continuouſly proportional; amongſt which C E, C *k*, C *l*, C *m* intervene. Therefore 4 mean Proportionals are found out betwixt the right Line given and its Half, which was to be done.

5. Confectary. If in the right Line D C produced, C F equal to C *k* be put; and C *m* equal to C *l* be put in B C produced, and in C D there be put C H equal to C *m*: And laſtly, if H K, H M, M F, F E, E D be joined, there will be made 5 equiangled Triangles, D E C, C E F, F M C, M H C, H K C.

6. How to find 2 mean Proportionals between 2 ſtrait Lines whatſoever given.

Let

Let (in this 3 Fig.) B C the greater, and C D the leſſer, be given. Let them be diſpoſed at right Angles in C, and let B C be equal to A C, let A D, A B be joined, and C *c* equal to the right Line B C be taken, and A *c* joined.



From the Point D, draw down D E, making right Angles at D, and cutting A *c* in E: From the Centre E at the Diſtance E D, deſcribe an Arch of a Circle D *f*, ending in A *c* at *f*; let the Arch D *f* be cut into 3 Parts in *d* and *e*, and the Lines E *d*, E *c* being drawn, produce them to D *c* in *a* and *b*. According then to the 2 Art. of this Chap. *c b*, *b a*, *a* D, will be continual Proportionals; and (by El. 6. Prop. 1.) the Triangles A C D, A D *c*, will be as the Baſes C D, D *c*; and the Triangles A C *c*, E D *c* alike; and the Triangle A C *c*, will be to the Triangle E D *c* in the double Proportion of C *c* to D *c*; therefore C *c*, C *b*, C *a*, C D, will be continual Proportionals.

7. Con-

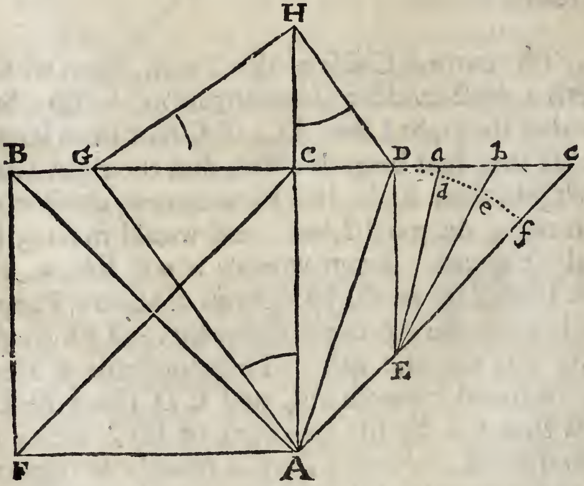
7. Confectary. If then in the right Line C B, there be put C G equal to C *b*, and in A C produced C H equal to C *a* be put; H D being drawn, and A G, G H joined, the Triangles A C G, G C H, H C D, will have equal Angles.

8. In like manner, if C D were put in the Side C B, and the Arch D *f* cut in five Parts, between A C and C D, four mean Proportionals would be found; but if C D were put in A C produced, and the Angle D E *c* cut in feven Parts, fix mean Proportionals would be found betwixt the fame Extreams.

9. The natural Caufe of this Truth, from whence arifeth a moft evident Demonftration, is this; Suppofe that the right Lines A C, B C had been lengthned 'till they had been doubled, and the right Lines, which join their Ends, had been drawn, thefe would be fo many diagonal Lines, and would make 3 like equal Triangles. From whence it will follow, that thofe four Sides would have been continual Proportionals upon the Account of Equality, of which A C is the 1ft, C *c* the 4th. Therefore that 2 Means may be found between A C and C D (becaufe C D is lefs than C *c* by the Quantity of D *c*) feeing the Difference of D *c* arifes from 3 Mulcts or Diminutions of the Side B C, to wit, *c b*, *b a*, *a* D, it is neceffary that the Difference D *c* be divided into 3 Parts, *c b*, *b a*, *a* D continual Proportionals. And becaufe the Angle C A B is half a right Angle, the Angle alfo D E *c* (from which Point E the Divifion of the right Line D *c* into 3 Parts, being continual Proportionals,

Proportionals, taketh its Riſe) ought likewiſe to be half a right Angle.

Now, ſeeing that that Diviſion into 3, is done by a Triſection of a half right Angle, and no otherwiſe; and in taking 4 Means by a Quinquiſection or fivefold Cutting, and no otherwiſe; alſo in taking 6 proportional Means by Septiſection, or a ſevenfold Cutting, and no otherwiſe; and ſo forward: no other Method will ſerve to find out Means that are even in Number, as 2, 4, 6, 8, &c. except the Triſection, Quinquiſection, Septiſection, &c. of a half right Angle.



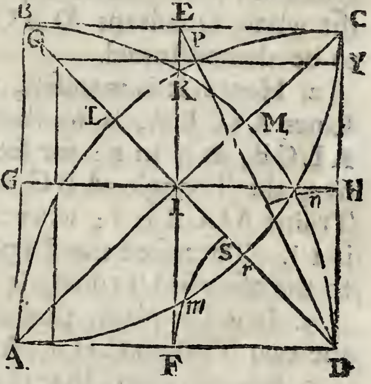
CHAP.

CHAP. VIII.

Of the Proportion of a Square, to the Quadrant of a Circle inſcribed in it.

1. LET the Square ABCD be divided, not only by the right Lines, EF, GH, but alſo by the Diagonals AC, BD meeting fourfoldly at I, in the Centre of the Square, and cutting the Arches AC, BD in L and M; and deſcribe from the Centres C and D, the Quadrantal Arches AC, BD mutually cutting one another in the right Line EF at K.

I ſay, that the Square ABCD is to the Quadrant DAC, as 5 to 4.



Though I have demonſtrated this in another Place, yet for the ſake of thoſe, who cannot ſo eaſily go along in their Thoughts, with long and difficult Demonſtrations, I ſhall in this Place demonſtrate the ſame by a ſhorter and eaſier Method. By the Radius DH, deſcribe the Arch of a Quadrant HF, cutting the Arch BAC in *m* and *n*, and the Diagonal BD in *s* and *r*; the Quadrant then DHF, is the fourth Part of the Quadrant

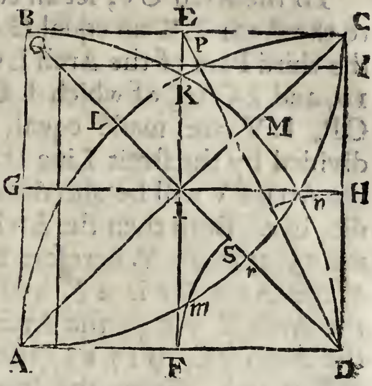
drant DCA, and the Space CLF *s* H, is three Fourths of the ſame. Now the ſame Space CLF *s* H, is three Fourths of the Quadrant B A *r* C; taking then that common Space CLF *s* H from both the Qaudrants, there will remain on the one Part the Quadrant DHF, on the other Part the Triline or 3 Lines D A *r* C leſs than the Biline or two Lines *m n*, more than the two three Lines C H *n*, A F *m*, that is, the ſame three Lines D A *r* C, and the Quadrant D H F equal to one another; and therefore the Space B A F *s* H C, will be equal to the Quadrant B A *r* C. For that Space conſiſts of the Triple of the Square DI, and the fourth Part of the three Lines ALCB, that is, of three Fourths of the Quadrant, and four Fourths of the Triline A L C B. Seeing then the three Lines ALCB, are equal to the fourth Part of a Quadrant; the whole Square ABCD will be to the whole Quadrant D A C, as 5 to 4, which was to be demonſtrated.

2. Hence it is manifeſt, that the Biline or two Lines A I C L A, is to the Triline or three Lines A L C B, as 3 to 2; for ſeeing the whole Square is five, the Triangle A B C will be $\frac{1}{2}$, of which the Triline ALCB is 1; wherefore the Biline AICLA is 1 $\frac{1}{2}$. Therefore the Proportion of the ſaid Biline to the aforeſaid Triline, is as 3 to 2.

3. It is alſo plain, that the Biline *m n* is equal to the two Trilines C H *n*, A F *m*.

4. It is alſo clear, that if a right Line were drawn parallel to the Side B C by the Point P, in which the ſtrait Line D E cuts the Arch C L, and ends in the Side D C at Y, and in the Diagonal B D at Q. The Square from Y Q would be equal to the Quadrant

drant



drant D A C, ſeeing the Square A B C D to the Square from Y Q is as 5 to 4; and the Trilines C Y P, P Q L equal.

5. It is alſo manifeſt, that the half of the Square A B C D, to wit, the Rectangle A E, the Figure A K F being taken away, is the fifth Part of the Square A B C D: For that which is contained within the three Lines A L K, K B, and B A, together with the Triline B E K, is equal to the Triline ALCB, that is, the fifth Part of the Square A B C D; becauſe B E K, C E K are equal, the ſame alſo is true of the Rectangle D E: Whence it follows, that the two Figures F K D, F K A taken together, are equal to three Fifths of the Square A B C D, that is, to three Fourths of the Quadrant D A C.

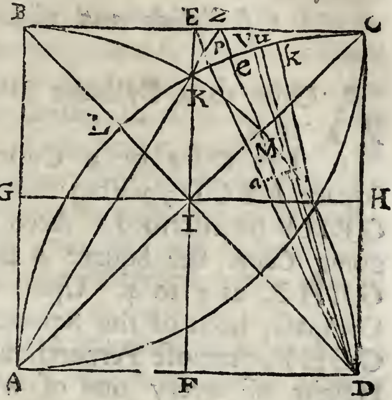
Now ſeeing the Knowledge of theſe Things is of it ſelf of no great uſe, I might have paſſed them over; but that, as I had begun to handle Cyclometry, I was willing alſo to perfect it. Likewiſe becauſe to the Fulneſs of Cyclometry, it belongs to know alſo the Quantities of its Parts, that is, of its Segments or Angles; we muſt now treat of theſe.

6. Deſcribe again (in the ſecond Figure) a Square A B C D, divided as in the firſt Figure.

To the Arch CP, let an equal LV be taken, and to the Arch LK an equal C *k*; becauſe then LK is the third Part of the Arch LC; LK will be 10, K *k* 10, and *k* C 10, of which LC is 30. Again, becauſe CP, LV are made equal, the Arch CL being divided by the ſtrait Line D *e* in two in the Middle, the Arch PV will be alſo divided in two in the Middle, in *e*. Both then the Arch C *e*, and the Arch L *e*, are 15 apiece: Wherefore the Arch V *e*, as well as the Arch P *e* is 2 $\frac{1}{2}$, and the Arch PV 5, and the Arch KV 7 $\frac{1}{2}$, the Arch LV then, as well as the Arch PC, is 17 1 $\frac{1}{2}$, therefore the Arch CV or KP is 12$\frac{1}{2}$. From the ſtrait Line ED, take E *a* equal to the half Side EC, and the Remainder D *a* (according to the 13 *E. Prop.* 1.) will be the greater Segment of the Side DC, or of the ſtrait Line DP divided in extream and mean Proportion. At the Diſtance D *a*, deſcribe an Arch *a b* cutting the Arch DB in *b*; and let D *b* be produced to the Arch CL in *v*; the Arch C *v* then will be 12, of which CL is 30. For (by *El.* 14. *Prop.* 9.) the ſtrait Line D *b* ſubtends the tenth Part of the whole Perimeter, that is a fifth Part of the Semiperimeter, that is, two Fifths of the Arch DB. The Arch then D *b* is two Fifths of the Arch BD; and becauſe the Angle in the Centre, to wit, DA *b* is the double of the Angle in the Circumference, to wit, of the Angle CD *v*, the Arch C *v* will be two Fifths of the Arch CL: Therefore ſeeing CL is 30, C *v* will be 12; now becauſe the Arches LV, CP are equal, CV will be 12 $\frac{1}{2}$, of which LC is 30: Wherefore as well LV as CP ſhall be 17 $\frac{1}{2}$ and P *e* or *e* V 2 $\frac{1}{2}$, and PV or K *e* 5.

If

If we ſuppoſe the Arch CL to be 45, yet the Proportion of the Angles will be found to be the ſame. For LK, K *k*, *k* C, will be each of them 15, and L *e*, *e* C each 22 $\frac{1}{2}$, and K *e* 7 $\frac{1}{2}$, L P 18 $\frac{3}{4}$; wherefore C P will be 26 $\frac{1}{4}$, and LV as much, and C V 18 $\frac{3}{4}$. Now 18 $\frac{3}{4}$ more 26 $\frac{1}{4}$ make 45; but the ſame is the Proportion of 10 to 5, as of 15 to 7 $\frac{1}{2}$. It ſtands then good, that the Angle E D C, to the half right Angle L D C, is as 17 $\frac{1}{4}$ to 30, and to L P as 17 $\frac{1}{2}$ to 12 $\frac{1}{2}$, or as 7 to 5, and to L K (which is 10.) as 7 to 4. We have then the Quantity of the Angle E D C, that was not known before.

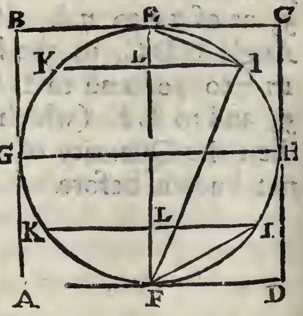


CHAP. IX.

Of Solids and their Superficies.

A Cube to a Cylinder inſcribed in it, is as 5 to 4.

1. Let there be a Cube, whoſe Baſe is the Square ABCD; within this Square let the Circle GEHF be inſcribed. According then to the foregoing *Chap.* the Square ABCD is to the Circle GEHF, as 5 to 4. Upon all the Points of the Compaſs, both of the Square ABCD, and Circle GEHF, ſuppoſe Perpendicular Lines erected, the Height of every one of which is equal to the Height of EF. So a Cube will be deſcribed, together with a Cylinder inſcribed in it, and the Baſe of the Cube will be to the Baſe of the Cylinder, as 5 to 4. Now the Plane that cuts the Cylinder and Cube Parallel-ways to the Baſe, will every where make a Section of the Square, to the Section of the inſcribed Circle, as 5 to 4. Therefore as the Square ABCD to the Circle GEHF (that is, as 5 to 4) ſo (according to *El.* 6. *Prop.* 1.) all the Sections of the Cube together (that is, the Cube) will be (to all the Sections of the Cylinder together, that is, to the Cylinder) as 5 to 4.



2. The Superficie of a Cube, is to the Superficie of

of a Cylinder inſcribed in it, as 5 to 4: For the Superficie of a Cube is equal to ſix Squares from the Side ot Height of the Cube. Now *Archimedes*, in his Book of *a Sphere and Cylinder*, hath demonſtrated, that the Superficie of a Cylinder without the Baſes, is equal to the four greateſt Circles in a Sphere, and with the Baſes, to ſix Circles. The whole Superficies then of a Cylinder, is equal to the ſix greateſt Circles in a Sphere: Therefore the Superficie of a Cube, to the whole Superficie of a Cylinder, is as ſix Squares to ſix greateſt Circles, that is, as one Square to one Circle inſcribed in it, that is, as 5 to 4.

3. A Cube to a Sphere inſcribed in it, is as 15 to 8: For a Cube to a Cylinder (as is already ſhewed) is as 5 to 4. Now a Cylinder to a Sphere (as *Archimedes* hath demonſtrated in the firſt Book of a Sphere and Cylinder) is as 3 to 2: Therefore the Proportion of a Cube to a Sphere, is made up of the Proportions of 5 to 4, and 3 to 2; be it then as 3 to 2, ſo 4 to another; now that ſhall be $2\frac{2}{3}$. The Proportion then of 5 to $2\frac{2}{3}$, is made up of the Proportions of a Cube to a Cylinder, and of a Cylinder to a Sphere: Therefore a Cube to a Sphere is as 5 to $2\frac{2}{3}$, that is, (both being multiplied by three) as 15 to 8.

4. A Sphere is equal to the half Cylinder in which it is inſcribed, together with the half of the Cone which is inſcribed in the ſame Cylinder: For ſeeing it is demonſtrated by *Archimedes*, that a Cylinder is to a Sphere inſcribed in it, as 3 to 2, and to the inſcribed Cone, as 3 to 1, a Cylinder, Sphere, and Cone will be as 3, 2 and 1. But 2 (that is, a Sphere) is equal to the Half of the Aggregate of

3 and 1, (that is, to the half of the Cylinder, and half of the Cone put together) as is propofed.

5. *Archimedes* alfo, in his fecond Book of the *Sphere and Cylinder*, *Prop.* 1. fhews how a Sphere equal to a Cylinder may be found, to wit, that a Cylinder be taken, which to the Cylinder propofed is as 3 to 2, that is, as the propofed Cylinder to a Sphere; then that two mean Proportionals be found betwixt the Height of the taken Cylinder, and the Diameter of the propofed Sphere: But to find out two mean Proportionals betwixt two right Lines, was not then found out. How that is to be done, I have demonftrated in the feventh Chapter of this Treatife.

6. The Superficie of a Sphere is equal to the four greateft Circles in the fame Sphere.

For if the Arch of the Semicircle E F be turned round upon the *Axis* E F, it will be the Superficies of the Sphere. From the Point F, let the ftrait Line F I be how you will drawn to the Circumference, and from the Point I, draw the right Line I K parallel to the Side B C, cutting the Circumference in K, and the Axis in L; the Angles at L will be right; and as F L to L I, fo will L I be to L E, and as F I is to F L, fo will E I be to E L, becaufe of the like Triangles F L I, E L I. Now whilft the Semicircle turns upon the Axis E F, let the Circle I K be defcribed from the Point I in the Superficie of the Sphere, and it will fo fall out in

whatfo-

whatſoever Point of the Circumference E F the Point I be placed ; but as F E is to L E, ſo is the Circle upon E F to the Circle upon L E ; for Circles are in the double Proportion of Rays or Radius's.

Becauſe then all the Perimeters I K make the Superficie of the Sphere, and all the right Lines I K make the Circle GEHF ; the Superficie of the Sphere will be to the Circle GEHF in the double Proportion of the Axis E F to the Radius of the Circle GEHF the half of the ſame EF : But the Circle deſcribed on the Axis EF, is alſo to the Circle GEHF in the double Proportion of the Axis E F, to the Semidiameter of the Circle GEHF ; therefore the Superficie of the Sphere is equal to the Circle, whoſe Semidiameter is the Axis E F. Now the Circle, whoſe Radius is the Axis of the Sphere, is the Quadruple of the Circle GEHF : wherefore the Superficie of the Sphere is alſo the Quadruple of the ſame.

7. Hence it follows, that the Superficie of any Segment, or Portion of a Sphere, is to the Superficie of the remaining Portion, as the Portion of the Axis cut off by I K is to the remaining Portion ; to wit, the Superficie of the Portion FIK, is to the Superficie of the Portion EIK, as the Portion of the Axis F L, to the remaining Portion E L.

8. Laſtly, Becauſe *Archimedes* hath demonſtrated, that the Convex Superficie of a Cylinder is the Quadruple of the Circle, which is the Baſe of the Cylinder, it follows, that it is the ſame thing, as to the Quantity of a Cylinderical Superficie, whether it be made by the Revolution of the Perimeter of the Baſis about the Axis, or by the Motion of the Baſe by the Sides of the Cylinder, or by the circular Motion of the Side. For all theſe beget equal Quantities of Superficie ;

ficie; neither is it hard to be underſtood, even without a Demonſtration, that there is no Difference in making a Superficie, whether the Circle that is the Baſe of the Cylinder be carried ſtrait by the Side, or whether in every Point of the Side Perimeters be ſeverally deſcribed, or whether from the whole Side one Perimeter be deſcribed. Now the Superficie of a Cylinder (that is, the thin Coat of the Cylinder ſtript off, its Skin being diſplayed and extended in a Plane) becomes a Triangle, of which one Side is the Diameter of the Baſe, the other a ſtrait Line, whoſe Square is equal to ten Squares from the Diameter of its own Baſis.

CHAP. X.

Of a new Method of treating of Solids, and their Superficies, by the efficient Cauſes.

LEMMA.

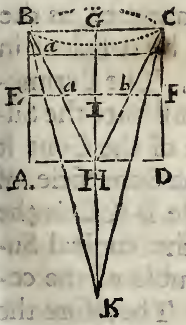
AN Agent working uniformly in a given Time, for perfecting a deſigned Work, if in ſome Parts of that Time he work, and in ſome others reſt; the Work that is done, will be to the Work left undone, as the Time wherein he worketh, to the Time wherein he reſteth. For Example, If a given Piece of Land may, by continued Labour, be wholly ploughed in three Days; then, if it be only ploughed two Days, and one Day ſpent idly; that which ſhall be ploughed, will be to that which remains unploughed, as 2 to 1; and what was deſigned to be ploughed, will be to what is ploughed, as 3 to 2, and to theu nploughed, as 3 to 1. In like manner,

ner, how much an Agent, by continued and uniform Labour can do in any time, a double Agent, by the like Labour, can do as much, if one half of the Time be ſpent in Work, and the other half idly.

Moreover, how much the Agent loſeth in Time of working, ſo much it muſt be thought he reſted; and theſe are manifeſt by the Light of Nature.

1. The Convex Superficie of a ſtrait Cylinder is equal to the Superficie of a ſtrait Cone, having the ſame Baſe with the Cylinder, but double the Height.

Here (Fig. 1.) take the Cylinder A B C D, ſuch as is propoſed, whoſe Baſe is the Circle B C. Now A B C D is a Square; let the Square A B C D be divided in two in the Middle by the right Line G H, parallel to the Side AB, and lengthen G H to K; let then G K be the Double of G H. Join BK, KC, and B H, C H, then the Square A B C D, and the Triangle B K C have the ſame Baſe, but the Height of the Triangle B K C, is double the Height of the Square A B C D. Suppoſe the right Angle D G to turn round about the Axis G H, and by that Circumvolution from the Rectangle D G a right Cylinder will be deſcribed; but from the Triangles CHG, CKG, two right Cones, and from the Side D C, the Superficie of the Cylinder, and from C H, C K, the Sides of the Cones, two conical Superficies, and from the Point C, the Circumference of the Baſe. Theſe Things ſuppoſed, we are to demonſtrate, that the conical Superficie of the Cone BKC, is equal to the whole Superficie of the Cylinder A B C D.



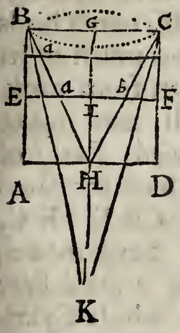
Draw

Draw the right Line *a b* as you pleaſe, but parallel to the Side B C, cutting the ſtrait Lines B H, CH in *a* and *b*. The Proportion then of B C to *a b*, will be the ſame with the Proportion of H *a* to H B; and becauſe *a b* is placed any where, the ſame will the Proportion be every where; and becauſe the right Line B C by an uniform Motion, and to right Angles by the Sides B A, C D, will compleat both the Cylinder A B C D, and its whole Superficie; but the Motion failing according to the Proportions of Times, they will deſcribe the Triangle BHC; the right Line BC reſts as much from working, as it worketh. The Superficie B H C will then be equal to the two Superficies B A H, C D H, that are not made (according to the foregoing Lemma.) Now becauſe the Diameters are in the ſame Proportion one to another, as the Perimeters; the Perimeters alſo will be deficient in the ſame Proportion with the Times. The Perimeters therefore of the Circles, which conſtitute the Superficie B H K, that is, the Superficie of the Cone it ſelf B H C, is equal to the Half of the Superficie of the whole Cylinder. But the Triangle B K C is the double of the Triangle B H C, and the conical Superficie of the Cone B K C, is the double of the conical Superficie of the Cone B H C: Therefore the Superficie of the Cone B K C, is equal to the Convex Superficie of the Cylinder A B C D, which is the Thing propoſed.

2. The Convex Superficie of a Cylinder, is the Quadruple of the Baſe of the ſame Cylinder; for ſeeing the convex Superficie of a Cylinder is made by the Perimeter of the Circle, which is the Baſe of the Cylinder working uniformly; if a Perimeter, which is the double of the Perimeter of the Baſe, ſhould

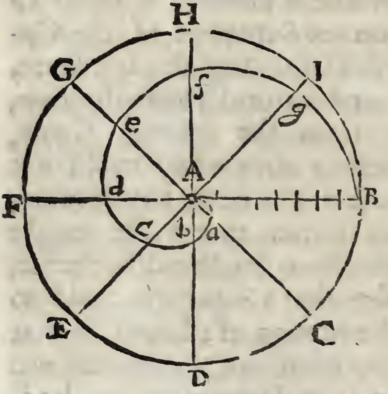
work

work one half of the ſame time, and reſt another half, it will perform as much (according to the forcgoing Lemma.) But the Perimeter of the Circle, whoſe Radius is HG, is the double of the Perimeter of the Baſe BC; and the Circle from HG is made by the Radius HG equal to BC, by a circular Motion, that is, the Radius BC ceaſing as much as it operateth, to wit, failing in operating according to the Proportion of the Times wherein it doth operate. Therefore the Circle, whoſe Radius is HG (which is the Quadruple of the Circle that is the Baſe of the Cylinder) is equal to the Convex Superficie of the Cylinder ABCD. And this is a ſhort, clear, and natural Demonſtration, drawn from the efficient Cauſe, but ſuch as *Archimedes* could not make uſe of; it being the Opinion, in thoſe Times, that a Line ought always to be conſider'd without any Breadth, that is, as nothing; and therefore that a Superficie could no ways be deſcribed by the Motion of a Line; ſo that becauſe of the Prejudices of others, *Archimedes* was forced to make uſe of a long Demonſtration, leading *ad impoſſibile*, which no Man's Wit but his own could have overcome.



3. A right Cylinder is the Triple of a right Cone inſcribed in it: For a Cylinder is made by the uniform Motion of the Circle BC (I ſay, of the Circle, not of the Perimeter) which is the common Baſe both of the Cylinder ABCD, and the Cone BHC. Now Circles have a double Proportion of that which their Diameters have: Therefore the Circle BC, to the Circle

Circle *a b*, is in a double Proportion of the Diameter BC to the Diameter *a b*, and ſo every where. Now the Circle, whilſt it maketh the Cone BHC, loſeth every where of its Magnitude in Working, the double Proportion of the Diameter BC to the Diameter *a b*. But (as has been ſhewed, *Chap.* 2. *Art* 9.) when mean Proportionals, as well Geometrical as Arithmetical, are taken every where, all together, they are the ſame; therefore the Circle BC loſeth of its Magnitude, whilſt it maketh the Cone BHC two Thirds of its own intire Magnitude: Now how much of its Magnitude it loſeth, ſo much it ceaſeth from work.



It will then make the third Part of that which the ſame entire Circle BC, would have made; that is, a third Part of the whole Cylinder: Therefore a Cone is the third Part of a Cylinder, that is, a Cylinder is the Triple of a Cone inſcribed in it.

4. It may be ſhewed, by the ſame Method, that the ſpiral Space, which ſprings from the firſt Revolution of a Circle, is the third Part of the ſame Circle.

From the Centre A, by the Radius AB, let a Circle be deſcribed, and eight Ways cut by the ſtrait Lines AC, AD, AE, &c. Divide likewiſe the Semidiameter eight Ways, of which let one Eighth A *a* be marked in the ſtrait Line AC, then two Eighths in the Radius AD at *b*, three in the Radius AE

at

at *c*, four in the Radius AF at *d*, five in the Radius AG at *e*, ſix in the Radius AH at *f*, ſeven in the Radius AH at *g*. Now you muſt ſuppoſe that every eighth Part is divided, as the whole Radius, in as many equal Parts, as it can be ſuppoſed it can be divided into; ſo that of what Parts AB is 8, A *a* is 1, A *b* 2, A *c* 3, A *d* 4, A *e* 5, A *f* 6, A *g* 7; then you muſt ſuppoſe a ſpiral Line, (whoſe Beginning is A, and End B) drawn through all the Points, *a*, *b*, *c*, *d*, *e*, *f*, *g*, B; and this will be the ſpiral Line deſcribed by *Archimedes*. We muſt then ſhew, that the Space concluded by this ſpiral Line, and the Semidiameter AB, is the third Part of the Circle BCD——

Becauſe the Circle BCD, is made by the Circumduction of the entire Radius A B, and the ſpiral Space by the ſame Radius, but failing, that is, reſting every where according to the double Proportion of the Times (for all the Circles through *a*, *b*, *c*, *d*, &c. have a double Proportion of the ſtrait Lines A *a*, A *b*, A *c*, &c. the Space which is left without the ſpiral Line unmade, will be the double of that which is made by the ſpiral Converſion; and therefore both the Spaces, made and unmade together, are the Triple of the ſpiral Space.

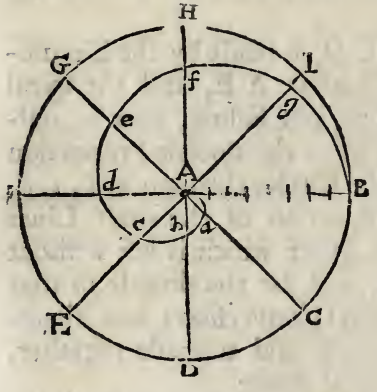
5. Now it is manifeſt, that ſince the ſpiral Line A, *a* *b*, *c*, *d*, *e*, *f*, *g*, B, is continually deficient in the ſame Proportion with the Times, it is equal to the half Perimeter of the Circle BCD.

6. That a Solid alſo, whoſe Baſe is the ſpiral Space, is the third Part of a Cylinder whoſe Baſe is the Circle BCD.—

7. Alſo that a Square circumſcribed by the Circle BCD—is to the ſpiral Space as 15 to 4. Now (as we

we demonſtrated before) a Square is to a Circle inſcribed in it, as alſo a Cube from the Radius, to the Cylinder, as 5 to 4. The Proportion then of a Square to the ſpiral Space, is compounded of the Proportions of 5 to 4, and 3 to 1; ſo that if it be, that as 3 to 1, ſo 4 to another, which ſhall be $\frac{4}{3}$: The Proportion of a Square to the ſpiral Space will be made up of the Proportions 5 to 4, and 4 to $\frac{4}{3}$; the Square then is to the ſpiral Space, as 5 to $\frac{4}{3}$; that is (both being multiplied by 3) as 15 to 4.

8. It likewiſe follows from the ſame, that if the Circle BCD— be the Baſe of a Cylinder, whoſe Baſe is equal to the Diameter of the ſame Circle, and that there be a Solid, whoſe Baſis is indeed the ſpiral Space, but the Height equal to the Height of the Cylinder; the Sphere wherein the greateſt Circle is BCD, will be the Double of the ſaid Solid. Now we



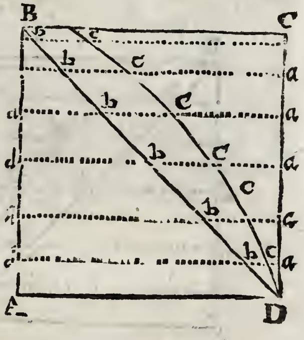
have ſhewed before, that a Square into which the Circle BCD is inſcribed, as alſo a Cube from the Diameter, is to a Cylinder whoſe Baſis is the Circle BCD, as 5 to 4, and to a Sphere, as 15 to 8, and to the ſpiral Space, as 15 to 4; for the Proportion of 15 to 4 being ſubtracted from the Proportion of 15 to 8, there remains $\frac{60}{120}$, that is, $\frac{1}{2}$. The Solid then whoſe Baſis is the ſpiral Space, is the half of the Sphere wherein BCD is the greateſt Circle.

9. By

9. By the ſame Method it will be found, that any Rectangle is to the Parabole inſcribed in it, as 3 to 2.

Deſcribe then any right Angle ABCD, in whoſe Side DC take any where D *a*, to which in the Side A B, take an equal A *d*, cutting the Diagonal B D in *b*, and let that be ſo every where; then as *d a* every where to *d c*, ſo let *d c* be to *d b*, and draw the crooked Line B *c c* D through all the Points *c*; that this crooked Line is Parabolical, and the Figure AB *c c* D the half of a Parabole, all Mathematicians do agree. It is then to be ſhewed, that the Rectangle ABCD is to the half Parabole AB *c c* D, as 3 to 2.

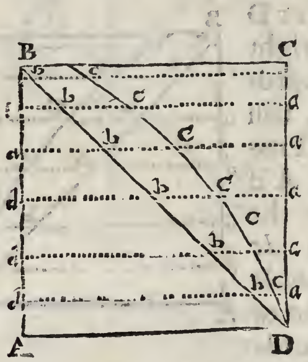
For deſcribe the Rectangle ABCD from the right Line AB uniformly, and parallelways moved to the oppoſite Side DC. Now the Semiparabole AB *c c* D is deſcribed by the right Line CB moved towards the oppoſite Side AB, ſtill loſing in the double Proportion of the time AB to the time D *a*; for the Proportion of *d a* to *d c* is every where the double of the Proportion of *d a* to *d b*; and therefore (according to Ch. 2. Art. 9.) all the *d a* together, are to all the *d c*, in a double Arithmetical Proportion of all the *da* to all the *db*. So then that which is made uniformly from the whole AB (that is, the whole Rectangle ABCD) to that which is made by the Motion of the Side DC towards AB failing or loſing (that is, reſt-



ing

ing from work) according to the ſubduplicated Arithmetical Proportion of Time, will be that which is left unmade, to wit, CD *cc* B one Part of the whole Rectangle ABCD, of which that which is perfected, to wit, the half Parabole AB *cc* D, is two.

And ſo the Rectangle ABCD, is 3 of the ſame Parts: The Rectangle ABCD is therefore to the Semiparabole inſcribed in it, as 3 to 2, and the double Rectangle to the whole Parabole inſcribed in it, as 3 to 2.

10. We may alſo uſe the ſame Method in finding out the Proportion of a right Angle to the Parabolaſter (which is called a Cubical Parabole) for if it be every where that as *da* to *dc*, and *dc* to *db*, ſo *db* to a fourth *de* (ſo that there be four continual Proportionals *da*, *dc*, *db*, *de*) the crooked Line of that Parabolaſter will paſs through all the Points *e*; and the Proportion of the Rectangle ABCD will be to the half of that Parabolaſter, as 4 to 3; and ſo you may proceed to the ſecond, third, &c. Parabolaſter, whoſe Proportions to the Rectangle in which they are inſcribed (as alſo of Cylinders to the Cone and Conoides inſcribed in them) are reduced into the Table of Chap. 17. of my Book *De Corpore*. Such is then the natural Aptitude of the *Lemma* prefixed to this Chapter, to the Demonſtrating of the greateſt Problems of pure Geometry

Geometry, that he who knows by what Proportion of Proportions (for ſeeing Proportion is a Quantity, there will be Proportions of Proportions, as well as of other Quantities) every compared Figure is framed, he cannot be ignorant of their Proportions one to another. But what (may ſome body ſay) is that pure Geometry? Is that Geometry true which is impure? Let him term it mix'd, not impure. But how can a thing pure, mixed to a thing pure, become impure? But neither muſt that be ſaid; but that then it is mixed, when it is applied to Matter. For it is indeed manifeſt, that if it be not applied to Matter, it is uſeleſs, and no more but mere hard Words. I call that pure Geometry, with which nothing of Arithmetick is blended, but what may alſo agree to a continual Quantity, which is the proper and adequate Subject of Geometry. Therefore the chief things that corrupt Geometry, *are the Surdity of Numbers, Longitude without Latitude, and Latitude without Thickneſs*, and the lately introduced Doctrine, that Fraction is Proportion, as if the half were the Proportion of the half to the whole.

CHAP. XI.

Of DEMONSTRATION.

IN the firſt Place it may be doubted, whether Demonſtration goes before, or comes after Knowledge. Moſt ſay, that it goes before; for a Demonſtration is either ſcientifick or frivolous: It is therefore the efficient Cauſe of Knowledge, and for that reaſon goes before its Effect. It muſt then be granted, that the Maſter's Demonſtration goes before the Knowledge of the Scholar; but no more. But if any Concluſion be demonſtrated by any Man, he muſt firſt have known the Truth of it, before he could demonſtrate it either to himſelf or others. For no Man can demonſtrate, that which he knows not whether it be falſe or true: It is then manifeſt that Knowledge in its own nature goes before Demonſtration.

In the next Place it may be asked (ſince Geometry is very uſeful and ornamental to Mankind) who are chiefly the Men to whom we are indebted for ſo great a Good, the Demonſtrators, or not Demonſtrators. It is certain that long before *Euclid*, many large and artful Fabricks were built, the Tower of *Babylon*, the Pyramids of *Egypt*, the wonderful Walls and Gardens in *Babylon*, the Palace of *Perſia*, and others; alſo a Sun-dial was firſt ſhewed at *Lacedemon*. Theſe Works without doubt required Knowledge; yet the Authors of them demonſtrated nothing, but by natural Logick foreſaw the Reality of their future Works, tho others afterward were curious to do it, with that purpoſe

purpoſe, that they might excite the Minds of many to the Invention of uſeful Things. The Proportions alſo of the five regular Bodies (as you may read in the old *Greek Epigram)* were found out indeed by *Pythagoras*, tho' *Plato* demonſtrated them, and after him *Euclid El.* 3. *Plato*'s Demonſtration is not extant; ſo that we owe thoſe good Inventions to ſeveral Inventors, whoſe Names (except a few) are now loſt, and not to Demonſtrators. How then is Demonſtration uſeleſs? Not indeed to thoſe who are taught. Now to be taught I think not very laudable, though to teach, provided it be rightly done, and without Hire, is honourable. But doth not a Demonſtration of ſome ample Science, hitherto unknown, deſerve great Thanks? Yes, but he that ſhall do that, is to be reckoned among the Inventors of profitable things. And therefore we are indebted to *Euclid*, who firſt of all taught the World the Method of Demonſtrating, that is, of ſound Reaſoning. All agree that moſt ample Thanks are due to thoſe who firſt adviſed Men to aſſociate, and to unite together under the Obedience of one Supreme Power. The next I think is due to them, who ſhall perſuade Men that they ſhould not violate the Compacts and Agreements that once they have made.

Thirdly, Some may ask what a Demonſtration is: Moſt Men, and not without good Ground uſe to call a Demonſtration, an evident Probation of the Truth, in any dubious Queſtion. Every one thinks that he has enough of Demonſtration, when his Mind does fully acquieſce to thoſe things that are alledged for Probation. But the ancient Philoſophers nurſed up in perpetual Diſputations, as often as there was any Debate concerning the Compariſon of Motions and

Magnitudes, for the moſt part they deſcrib'd Figures, and put before the Eyes of their Diſciples, as if ſhew'd by them to the Sight (that is ſomewhat more as they thought than prov'd) call'd their Arguments ἀποδείξεις, (in *Engliſh*) Demonſtrations. But now when any Man is a little too vehement and poſitive in aſſerting, he affirms that he has demonſtrated.

Now a Demonſtration is, when the Truth of the Concluſion hath its firſt Foundation, in thoſe things which are already known to them, to whom he that proves them, ſpeaks. Now theſe Foundations are Definitions, and beſides Axioms, which are not indeed demonſtrated, yet ought to be true and well underſtood; for from Truths nothing but what is true can be inferred, nor can any thing though never ſo true be known, that is not underſtood.

Fourthly, It may be demanded what the Uſe of Definitions and Axioms is: This is the plain Uſe of Definitions, that he to whom a Demonſtration is made, may underſtand what certain an univerſal Signification of the Word the Demonſtrant would have taken; for the Signification that changeth, deceiveth. Therefore a Definition conduceth to the Underſtanding, and without Underſtanding there is neither any Demonſtration, nor in him that learneth can there be any Knowledge, but from thence unſeaſonable and obſcure Diſtinctions are introduced; ſo that of what is ſaid, nothing maketh Impreſſion on the Imagination. There are indeed, ſome Truths which can no Ways be known by Man, but nothing is demonſtrated that cannot be underſtood by him.

The Uſe of Axioms conſiſts in this, that they abbreviate the too long Series of Demonſtrations, to wit, by removing unneceſſary Demonſtrations. For the Truth

Truth of Axioms ought to appear ſooner, and more clearly than the Means themſelves, by which they are proved.

In every Demonſtration, the Cauſe of the Concluſion ought to be in the Antecedents, by Virtue of which it is inferred; that is, in things before demonſtrated, or known by the Light of Nature. Therefore where the Words cohere and hang together, there will be a Demonſtration; for tho' the Cauſe of the ſubject Matter be not known, yet the Concluſion will retain the Truth of the Principles from whence it is derived. Now Demonſtrations of this kind are eaſy, tho' they little advance Knowledge.

That is the chiefeſt of all Demonſtrations, which is drawn from the Production of the ſubject Matter, according to the Order of Nature; and ſo theſe are the moſt uſeful Definitions for Knowledge, wherein the Generation of the ſubject Matter is explained; that is, by what Motion, what Concourſe of Motions, what Proportions of Motions and Times, all Spaces and Magnitudes are determined.

The next Demonſtration to this, is, when (from the Negation of Truth) ſomething impoſſible is inferred. Now this kind of Demonſtration has its Force from this, that from a Truth nothing but Truth can be deduced.

CHAP. XII.

Of FALLACIES.

THat a Man who underſtands his Words, that is, his own Speech, and ſets upon the Proof of it from true Principles (I mean) natural Definitions, proceeding ſlowly in Mathematical Matters, ſhould be long and often deceived, and being admoniſhed, ſhould perſiſt in his Error, is a thing almoſt impoſſible. Speaking properly, there can be no Error in the Intellect; for to err in the Intellect, is the ſame as not to underſtand. The right Uſe of the Tongue, Feet, and Hands, is not demonſtrated by a Maſter, but by Exerciſe we learn it; and though all our Words almoſt change their Signification, according to the Variety of the things whereof we ſpeak or write; yet we uſe them at home, in the Fields, and in the Market, without any hurt, becauſe it is enough for Civil Society, if one underſtand aright what another ſays.

It is otherwiſe in Philoſophy, where nothing but Truth is ſought after; but eſpecially if Glory attend Invention. It is one thing to walk, another thing to walk upon a Rope; the one is eaſy, and without great Damage, tho' a Man ſhould trip; ſo that Negligence is there pardonable: It is not ſo to a Rope-dancer. Truth walks upon a very ſmall Thread, without that metaphorical Latitude of common Converſation; and eſpecially Mathematical Truth, which unleſs it be poiſed by the Weight of Definitions and Axioms, it tumbles headlong, to the Laughter of Spectators.

The

The chief and most frequent Cause then of Fallacies in the Mathematicks, is, that they build their Reasoning upon Definitions not understood, or false or ambiguous ones, from which no Truth can be deduced. The *Greek* Philosophers were of the same Mind also, when to this Science they gave the Name *Mathematicks*, from the *Greek* Word that signifies to understand; for commonly when any one, speaking to another, doubted if he was understood or not, he asked, μανθάνεις, *Do you understand?* To which it was answered, μανθάνω, or ὀ μανθάνω; that is, *I do*, or *do not understand*. So natural it was to denominate the Mathematicks from the Understanding. Another Cause of Fallacies is, not to know what Motion is, and its Properties; that is, to be ignorant of the immediate natural Cause of all things. Those known Fallacies reckoned up by *Aristotle*, by which a Child can hardly be deceived, I purposely pass over, as the greatest Bane of Geometry. In the first Place, I condemn a *Line without Latitude*, a thing unconceivable. Secondly, The Side of a square Figure, supposed for the Root of a Number. Thirdly, The nature of Proportion not understood. Fourthly, All consideration of *infinitum*, whether Geometrical or Arithmetical. And here I should have made an end, had not there been one who affirms, That he can also demonstrate those things, which are neither intelligible, explicable, nor conceivable; which is indeed to say, that all Sciences are not worth a Rush.

Nothing (says he) *is more obvious in Nature than continual Quantity, and local Motion. Now these either are, or are not divisible* in infinitum: *Can this Disjunctive be denied? Can it be said, that they neither are, nor are not thus divisible? Let any choose what Member of this he pleases; shall he remove the Difficulties that*

that are in it? Or ſhall he anſwer the Objections that can be made to the contrary?

Though theſe be Queſtions, yet in this Place they have the Force of Negations; let him then object his Difficulties. I ſhall explain the Matter itſelf; to wit, that which Mathematical Writers underſtand, when they ſay, that Quantity is diviſible *in infinitum.* They do not ſay nor underſtand, that a finite Quantity (ſuppoſe a Line) is diviſible into Parts that are infinite in Number; but that a Line never ſo little, is of its own nature capable of Diviſion. Neither by Diviſion do they underſtand a material Separation, that is, a Separation of one Part from another; but that in all continual Quantity, a Quantity leſs, and that aſſignable, is ſtill to be ſuppoſed; and that the Signification of this Diviſion is nothing elſe but the unlimitted Conſideration of a Part, in a Whole never ſo little. For of whatſoever it may be ſaid, it is a Whole, it may very well be ſaid, that there are Parts in it; but that a mortal Man can divide any thing eternally, or if he could do it, that yet the Parts ſhould not be of a finite Number, it is impoſſible. Now what is ſaid of a Line, ought likewiſe to be underſtood of Motion, Time, and every thing elſe that is diviſible, except Number. Let an Objection now be brought againſt this (I confeſs I do not remember that ever I read any) that we may ſee, if it may not eaſily be underſtood, whether it be ſtrong or weak. Moreover,

I ſuppoſe (ſays he) *that moſt know that famous Argument of* Zeno, *which is called* Achilles, *and that how that great Diſputer, whilſt that, by apparent Impoſſibilties and Abſurdities on each Side, he demonſtrated local Motion to be impoſſible, he was confuted by one of the Hearers, who roſe and walked through the School.*

Of

Of which Words this is the Sum, That *Zeno* was, indeed, confuted by the Walker, but that his Sentence was rightly inferred by a Demonſtration leading to an Abſurdity. What *Zeno*'s Argument is, that the Reader may judge of it, I will ſhew. Seeing the leaſt Space that is, cannot be paſſed over by Motion, but that the half of that Space is firſt to be paſſed over, and that the leaſt Space again has alſo its half, and ſo perpetually; *Zeno* concludes (ſubſuming that no Space can be paſſed over in an Inſtant) that it requires an infinite Time to paſs over the leaſt Space whatſoever. Now it is manifeſt, that if a Space cannot be paſſed over without an eternal Motion, it cannot at all be paſſed over.

The Stoicks exemplified this Argument taken from *Zeno* their Maſter, in *Achilles* ſwift of Foot, and a ſlow-paced Tortoiſe; and they ſaid (putting *Achilles* and the Tortoiſe into the Race) that *Achilles* could never overtake it, if it were but the leaſt Diſtance before him. But why? becauſe whilſt *Achilles* ran over that half Diſtance, the Tortoiſe alſo advanced a little. I ſhould be aſhamed to repeat thoſe Trifles, did I not perceive, that they who leaſt ought to be, may yet be deceived by ſuch childiſh Fallacies: But what is to be anſwered, I ſhall anſwer, Firſt, That it is no wonder, if he that is unwilling, never overtake a ſlow Runner, that is got before him. For ſuch is the nature of Motion, that he who always will, or is forced to ſlacken his Swiftneſs in proportion to the Space that is left, will never be able to paſs over even the ſmalleſt Space whatſoever. Secondly, What is made by any whole, and the half of it, and the half of that half, and ſo on, will be leſs than two; they are ſo far from making an infinite Time. By that Argument of *Zeno*'s then, it

it cannot be inferred, that ſwift *Achilles* could not in running overtake the Tortoiſe; but only this, that if he pleaſed he would not. Therefore the Example of *Zeno*'s Sophiſm does not prove, that there can be a Demonſtration of a thing that is not intelligible. He goes on;

The pure and moſt ſimple Mathematicks are the moſt eaſy, evident, and comprehenſible of all human Sciences. Yet in Geometry and Arithmetick, how many Propoſitions are there firmly demonſtrated, which nevertheleſs are inexplicable, unconceiveable, and incomprehenſible? I ſhall give a few Inſtances, &c.

Is not this abſurd? Can he be ſaid to demonſtrate a Truth, that does not make it apparent and intelligible? How can I know whether it be true or falſe, if I cannot imagine it in my Mind? But let us read the Examples, which he ſays he will ſubjoin.

1. *That the leaſt Space imaginable may be equal to another Space upon the ſame Baſe, and of the ſame Height, whoſe Sides are drawn out* in infinitum.

That a Parallelogram and Triangles, indeed, of the ſame height, upon the ſame Baſe, are all equal to one another, is intelligible to every one. What he adds, *whoſe Sides* are drawn out *in infinitum*, is abſurd; as a Bench is not called a Bench before it be perfected, ſo neither is a Space before it be finiſhed.

For it is downright impoſſible and abſurd; nor from *Torricellius* does it follow, that a Finite is equal to an Infinite.

2. *One Infinite may be greater than another Infinite.* Who ſaid ſo? He is ſilent; for it is abſurd.

3. *That all the circular Angles of Contact are equal.* What is a circular Angle of Contact? The Expreſſion itſelf is unintelligible.

4. *The*

4. *The eternal Appropinquation of two Lines, but the Concourſe impoſſible, to wit, Aſymptotes.* The Cauſe of this is before explain'd in the Sophiſm of *Zeno.*

5. *The Affections of ſurd and irrational Quantites; all theſe are ſo demonſtrated, as that they cannot be denied, and many others, which nevertheleſs are inexplicable, incomprehenſible, and inconceivable.* But what are theſe ſurd and irrational Quantities? That there are ſome continual Quantities, which have not the Proportion of Number to Number, and are called incommenſurable, there is no Mathematician but knows; and of theſe alſo that are commenſurable one with another, many are irrational; becauſe though they be commenſurable one with another, yet becauſe they are not commenſurable to any Quantity taken at Pleaſure, they are ſaid to be irrational.

6. *In Numbers can the Affections of a Unity and Ternary be fully comprehended, expreſſed and explained, that in Nature there ſhould be one Quantity, and no other, which with its infinite Powers aſcending, and Roots deſcending, all ſhould be equal among themſelves, or rather one and the ſame?* The Words themſelves are pretty obſcure; but I think this is it that he would have ſaid: Seeing the ſquare Power of a Number, and a ſquare Number, alſo the ſecond Power and cubical Number, &c. is the ſame thing to Arithmeticians; and that that Number is called the Root of a ſquare Number, which being multiplied by itſelf makes any Number, and the Root of a cubick Number the ſame with that Number, which being multiplied by itſelf, and again by the Product, maketh any one, and that one multiplied by itſelf never ſo often, makes no more but one; it is manifeſt that all the Products and their Roots are the ſame Unities. Now though this Secret be only proper to an Unity, yet

†

yet it is not hard to be underſtood, becauſe nothing can be multiplied by one. Laſtly,

He adds, *Can ſo much as any one of the infinite potential Roots of the Number 3 be explained, thought, or comprehended?*

It cannot, I know; and why? Neither that, nor that of many other Numbers, can, I know, and can briefly explain, and that is, becauſe they have no Root.

CHAP. XIII.

Of an INFINITE.

TO the Word *Infinitum* ſomething is underſtood, as Work, Time, Space, *&c.* If it ſignify a Work, then, in its Latin Acceptation, it ſignifies a Work that is indeed begun, but not as yet brought to the End that the Artiſt deſigned in his Mind; and ſo in that Senſe a thing infinite is the ſame as a thing unfiniſhed or imperfect, but what may be finiſhed; ſuch as a Houſe begun, but not perfected. If any then ſhould ſay of an Artiſt, that he had in his Mind to perfect an infinite Work, he would ſpeak abſurdly, as if he ſhould ſay, that it was in his Mind to do that which he never intended: For no Man thinks of doing more than what he can accompliſh; ſo that no Man can judge of a Work that is not his own, without conſulting the Artiſt, whether it be finiſhed and perfect, or infinite, that is, unfiniſhed and imperfect.

An

An *Infinite*, if Space be underſtood, ſignifies a Space greater than can be equalled by the greateſt Number of Meaſures, as of Feet, Paces, Miles, or even of the Diameters of the Earth, or of the Orb of the fixed Stars; that is to ſay, which cannot be included within Bounds. In like manner, an infinite Time is that which no Number of Hours, or Days can equal.

Therefore of an *Infinite*, according to this Senſe, it cannot be ſaid, that one is greater than another: Draw then the finite right Line A B, and ſuppoſe it produced beyond D A C B E |—|—|—|—| B by E *in infinitum*. Therefore both the right Line B E — and A B E — are infinite in Length. Now A B E — is greater than B E — by the whole length of A B, be it ſo. Let A B be divided in C, and put A D equal to AC; and let it be ſuppoſed lengthen'd ſtraitways by F *in infinitum*. A D F will be then greater than D F by the finite Longitude C D, that is, by the Quantity of A B.

Wherefore C B E — and C A D — are not unequal; therefore there is ſome certain Point, that is a mean in the infinite Line — D B, ſo that the Centre of an infinite Sphere will be the Point C, and (becauſe the Points A and B are taken at Pleaſure) in every Point of an infinite Sphere, will the Centre of the Sphere be; and ſo the Semidiameters of an infinite Sphere from any Centre, whether A, B, or C, are not unequal one to another. Therefore one infinite Line is not greater than another.

By

†

By the ſame Reaſon, if the Line AB be put for an infinite Time, it may be proved that two Eternals cannot be unequal.

Now the Cauſe why a Finite may be conſidered in an Infinite, conſiſts in this, that neither the ſubject Body is in the Thinker, nor Space, the Image of the Body, in the Thing thought of, but only in the Memory: And in the Memory and Senſe there can be no Infinite.

But Mathematicians uſe often the Word *infinite* for *indefinite*. Now indefinite is the ſame as *never ſo great*, and ſometimes it is taken for *infinitely little*, provided it be not *nothing*. Sometimes alſo *infinite* goes for *as much as is poſſible*. But nothing is properly *infinite*, unleſs it exceed all aſſignable Number of given Meaſures. But it is ſaid to have been demonſtrated by *Torricellius*, that a certain acute hyperbolical Solid is, even in this Senſe of *infinite*, equal to a certain Cylinder, whoſe Baſe hath a Diameter equal to the half Baſe of the Hyperbole, but a Height equal to the tranſverſe Axis of the ſame Hyperbole. I have often and attentively read the Demonſtration of this Problem, and never found any Sophiſm in it. Yet I found that the Diſtance, which *Torricellius* ſuppoſes infinite, is meant of an indefinite Diſtance; nor could it be otherwiſe underſtood by himſelf, who in very many Demonſtrations uſeth the *Cavallerian* Principle of Indiviſibles; which Indiviſibles of *Cavallerius* are ſuch, that their Aggregate may be equal to any given Magnitude. So that ſo abſurd a Propoſition as this, *an Infinite is equal to a Finite*, ought not to be aſcribed to *Torricellius*; for there can be no Solid ſo ſmall, which, being infinite, doth not exceed every finite Solid, as is manifeſt by the Light

Light of Nature; that is, the Abſurdity of Arithmeticians Reaſoning about an Infinite, and meaſuring Superficies and Solids by Lines without Latitude; who obſerving no Difference betwixt Arithmetick and Geometry, took the Root of a Number (which is part of its ſquare Number) for the ſame thing with the Side of a ſquare Figure, though they confeſs that the Side is no Part of its own Square. So much of Mathematicks. I now expect what the Algebraiſts will ſay to the contrary.

The End of the THIRD BOOK.

MEASURING

OF

GLAZING,	JOINERY,
PAINTING,	CARPENTERS,
PLAISTERING,	and
MASONRY,	BRICKLAYERS

WORKS,

BY

VULGAR ARITHMETICK,

WITHOUT

Reducing the *Integers* into the leaſt Denomination.

The FOURTH BOOK.

By *VEN. MANDEY.*

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MEASURING OF CARPENTERS WORK.

CHAP. I.

MENSURATION,

OR Measuring, is a Science, whereby we are certified (in superficial Measure) either how many Inches, or how many Feet and Inches, or how many Yards, Feet and Inches, or how many Squares, Feet, and Inches, there is in any Dimension, whether it be of Glass, Board, Stone-paving, Plaistering, Painting, Tyling, Carpenters or Joiners Work, &c. And so consequently it certifies us of the Content of many Dimensions, by adding the Products of the several Dimensions together.

It doth the same in solid Measure; there is only this Difference between superficial and solid Mensuration; to wit, in superficial Measure there is only two Sums in a Dimension, *viz.* Length and Breadth, to be multiplied one by the other: But in solid Measure

ſure there are three Sums in a Dimenſion; to wit, Length, Breadth, and Thickneſs, to be multiplied each by the other; and this kind of ſolid Menſuration ſerveth for the meaſuring of Timber, Stone, Digging, Bricklayers Work, for it is commonly reduced to a Thickneſs, and all manner of ſolid Bodies whatſoever.

The Inſtruments that are uſed in taking of the Lengths and Heights (or Breadths) in meaſuring of the following Works, are a ten Foot Rod, and a five Foot Rod, and a two Foot Ruler, and ſometimes a Line.

I ſhall begin with the Meaſuring of Carpenters Work, it being moſt eaſy to learn, and ſo proceed through the other Trades, leaving tne Meaſuring of Bricklayers Work until the laſt, it being moſt difficult.

CHAP. II.

THE Flooring, Roofing, Partitions, and Ceiling Joiſts, being Carpenters Work, is generally meaſured by the Square (as it is vulgarly ſaid) or more properly is reduced into Squares.

Which Square conſiſts of (or contains) 100 ſuperficial Feet, being the Product of a Square Superficies multiplied in it ſelf, being ten Feet in Length, and ten Feet in Breadth.

Note, That a Superficies is that which hath only Length and Breadth, as a Square incloſed with four Lines on Paper, is called a Superficies (as appears more at large in the precedent Treatiſes of Geometry.)

And from thence the meaſuring of Flooring, Roofing, Partitioning, Ceiling-Joiſting, Boarding, Plaiſtering, Painting, &c. is called ſuperficial Meaſure, becauſe their Thickneſſes are not conſidered, nor taken Notice of in Meaſurement.

For generally Boards, Glaſs, Painting, Plaiſtering, are ſeverally for the moſt part of one Thickneſs.

And although ſome Floors, and Roofs, and Partitions, do require to be made ſtronger than other ſome, by reaſon of the Greatneſs of the Building, and by this means the Timbers are larger and thicker; yet there is no reſpect had to (nor cognizance taken of) the Largeneſs and Thickneſs of the Timbers, in the Meaſurement; but an Allowance is made in the Price, by adding ſo much *per* Square more than is uſually given for ordinary Work.

Thus 12 Inches in Length, and 12 in Breadth, is called a ſuperficial Foot, of Roofing, Flooring, Boarding, Partitioning, Glazing, Painting, Plaiſtering, &c.

Note, When a Carpenter's Bill of Meaſurement is made, there is ſet down,

For ſo many Squares of Roofing (at what Price they agree upon *per* Square) ſo much Money.

Likewiſe for ſo many Squares of Flooring and Boarding, at ſo much *per* Square, ſo much Money.

Alſo for ſo many Squares of Partitioning, at ſo much *per* Square, ſo much Money.

And for ſo many Squares of Ceiling-Joiſts, &c.

The Windows they ſet down either at ſo much *per* Light, or ſo much *per* Window.

The Door-caſes at ſo much a-piece, either with or without Doors.

The

The Mantletrees and Tarſels at ſo much a-piece.

The Lintolling, Guttering, Cornice, and Window Boards, at ſo much *per* Foot.

Stairs at ſo much a Pair, or ſo much *per* Step, *&c.*

CHAP. III.

IT is ſuppoſed that he that intends to learn the Science of Meaſuring is an Arithmetician.

For although ſeveral Propoſitions may be anſwer'd, ſome Geometrically, and ſome on the Line of Numbers (otherwiſe called *Gunter*'s Line) and ſome on the Lines of Superficies and Solids on a Sector; yet without Arithmetick it is impoſſible to meaſure exactly all kinds of Plains and Bodies.

Therefore to the Arithmetician, I ſay, multiply the Length of any Dimenſion (that is, either Square or Oblong) by the Breadth thereof, and the Product is the ſuperficial Area or Content.

I ſhall begin with a Dimenſion of Inches, and ſo proceed gradually to Feet, and from Feet, to Feet and Inches (or Parts.)

Note, That two Sides of a Superficies being meaſured and expreſs'd by Arithmetical Figures (one Sum being the Length, the other the Breadth) is called a Dimenſion.

Example in Inches.

Suppoſe a Piece of Board or Glaſs, or flat Stone, or Painting, or Plaiſtering, or any thing, be it what it will, that is to be meaſured ſuperficially, (vulgarly

ly called flat Meaſure) be 11 Inches in Length, and 9 Inches in Breadth (which two Sums 11 and 9, are called a Dimenſion) and you would know the Area or Content thereof;

	In.	
Set down your Dimenſion thus—	11	the Length.
	09	the Breadth.
Then multiply the Length by the Breadth.		
And the Product is————	99	Inches.

Which 99 Inches is half a Foot and 27 Inches, or very little more than $\frac{2}{3}$ of a Foot.

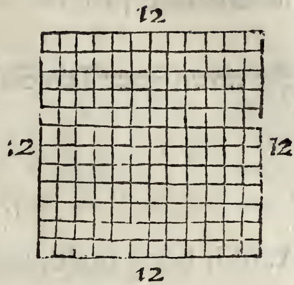
So that the Product of 11 Inches, multiplied by 9 Inches, which is 99 Inches, contains 8 ſuch Inches, whereof 12 make a ſuperficial Foot, that is to ſay, it contains 8 Inches, whereof each Inch is one Inch in Breadth, and 12 Inches long: For 8 times 12 is 96, which is $\frac{2}{3}$ Parts of a Foot; then the 3 Inches which are remaining, (for the Product was 99 Inches, take 66 from 69, and there remains 3; theſe three Inches, I ſay) are $\frac{3}{12}$, or, as I call them, 3 Parts of ſuch an Inch as aforeſaid; namely an Inch in Breadth, and 12 in Length.

For, by the way, you muſt note, That in a ſuperficial Foot, there is contained 144 Inches, which is the Product of 12 Inches multiplied by 12, thus————————

12	Length.
12	Breadth.
144	Product.

Or

Or as you may ſee in the Geometrical Figure hereunto annex'd.



By this Geometrical Figure, you may plainly perceive, that there is in a ſquare Superficies, being 12 Inches on each Side, 144 Superficial Inches; for every little Square in the Geometrical Figure repreſents an Inch: So conſequently in half a Foot there is 72 Inches, being produc'd from an Oblong, being 12 Inches in Length, and 6 in Breadth.

Which Oblong you may perceive, or imagine in this ſquare Geometrical Figure, by taking the whole Length one Way, and half the Length, or 6 of the little Squares, the other way: Therefore I ſhall not need here to deſcribe the Figure of an Oblong.

Alſo half a ſuperficial Foot, which is 72 Inches, is produced from an Oblong very near a Square, being 9 Inches in Length, and 8 Inches in Breadth, for 9 times 8 is 72.

Likewiſe a Quarter of a ſuperficial Foot contains 36 Inches, which is one half of 72 Inches, or the Product of 6 Inches, multiplied by 6 Inches, or 9 by 4.

I ſhall now proceed to Dimenſions in Feet.

Suppoſe there is a Timber Floor (or a Stone Pavement, or a Plaiſter'd Ceiling, or a Piece of Wainſcot, or any other Superficies) that is 22 Feet in Length, and 18 Feet in Breadth, and you would know

know in Carpenters Work, how many Squares there is contained therein.

Feet.

Set down your Dimenſion thus ——— ——— { 22 / 18

176
22

Which being multiplied, the Product is —— 396 ſuperficial Feet.

Now to bring theſe Feet into Squares, you muſt divide 396, the Content in Feet by 100 (the Content of a Square whoſe Side is 10 Feet) and the Quotient is 3 Squares, and 96 Feet remaining.

See the Example ——— —— $\frac{\cancel{3}\cancel{9}\cancel{6}}{\cancel{1}\cancel{0}\cancel{0}}$ (96 Feet. / (3 S uares.

Here you ſee by the Example, that there is 4 Squares of Flooring, wanting 4 Feet; or 3 Squares and three Quarters of a Square, and 21 Feet.

If the Dimenſion had been Stone Pavement of Maſonry, then there had needed no Diviſion, for they work by the Foot; ſo there had been 396 Feet of Pavement with broad Stone, at ſo much *per* Foot.

But if it had been Plaiſtering, or Painting, or Wainſcoting, or Hangings, or ſuch like; then you muſt have brought the Feet into Yards, which is done by dividing 396 Feet by 9 Feet, which is the Number of Feet contained in a ſuperficial Yard (or Quadrate) being 3 Feet in Length, and 3 Feet in Breadth.

See

$$\begin{array}{l} \quad\; 3 \\ \text{See the Example ———— } \cancel{3}\cancel{9}\cancel{6}\ (44 \text{ Yards.} \\ \quad\; \cancel{9}\cancel{9} \end{array}$$

Thus having divided 396 Feet by 9 Feet (or one Yard) you have 44 Yards in the Quotient, which is the Number of Yards contained in a Superficies of Plaiſtering, or Painting, or Wainſcot, or Hangings, or ſuch like, being 22 Feet in Length, and 18 Feet in Breadth.

Another Example in Feet.

Suppoſe a Floor or Partition be 146 Feet in Length, and 97 Feet in Breadth.

Set down the Dimenſion thus —— Feet.

	Feet.
	146
	97
	1022
	1314
Being multiplied, the Product is ——	14162 Feet.

Which being divided by 100, produceth 141 Squares, and 62 Feet: Or you need not divide it, but read it thus, One hundred forty, one hundred and ſixty two Feet, which is 141 Squares and an half, and 12 Feet.

If you were to bring the ſame Dimenſion into Yards, then you muſt divide the Product of the Multiplication, being 14162, by 9 Feet (or one Yard) as you did in the foregoing Example.

$$\begin{array}{l} \quad\; 563\ (5 \text{ Feet.} \\ \text{See this Example ———— } \cancel{1}\cancel{4}\cancel{1}\cancel{6}\cancel{2}\ (1573 \text{ Yards.} \\ \quad\; \cancel{9}\cancel{9}\cancel{9}\cancel{9} \end{array}$$

So

So you have in the Quotient 1573 Yards and 5 Feet remaining, which is half a Yard and half a Foot. Thus much may ſerve for meaſuring of Dimenſions of whole Feet.

I ſhall now proceed to Dimenſions in Feet, and Inches, which is ſomething more difficult.

Suppoſe a Floor, or Partition, or any other ſuperficial thing, as Plaiſtering, or Painting, &c. be 21 Feet, and 6 Inches in Length, and 15 Feet, and 6 Inches in Breadth; and it is required to know how many ſuperficial Feet there is contained therein, and conſequently how many Squares, or Yards.

	Feet	In.
Set down your Dimenſion thus ——	21	06
	15	06
	105	
	21	
Then multiply the Feet, and the Product is —— —— ——	315.	

Then for the 6 Inches in Length, and 6 in Breadth, multiply them Diagonal or Croſs-wiſe into the Feet, ſaying 15 times 6 Inches, (or 6 times 15 Inches) is 7 Feet and 6 Inches: Or thus more briefly, (6 Inches being half a Foot) ſay, The half of 15 Feet is 7 Feet and 6 Inches, which you muſt add to the former Product 315. This being done, in the next place you muſt multiply the 6 Inches in the Breadth into the 21 Feet in the Length; ſaying, The half of 21 Feet is 10 Feet and 6 Inches, which you muſt likewiſe add to the reſt.

Then

Then laſt of all, you muſt multiply the Inches of the Breadth into the Inches of the Length, ſaying, 6 times 6 is 36, which 36 is accounted but for 3 Inches, the Reaſon whereof, in the next following Page but one, ſhall be ſhewn.

For Note, How many times 12 you have in the Product of the Inches, being multiplied in themſelves, ſo many Inches you muſt add to the former Work, as in this Example, you have 3 times 12 in 36: As you may ſee in the following Example.

Whence Note, That it is uſual to begin the Multiplication towards the left Hand firſt, namely, the Feet into the Feet, and afterwards the Inches into the Feet, &c.

Example of Operation.

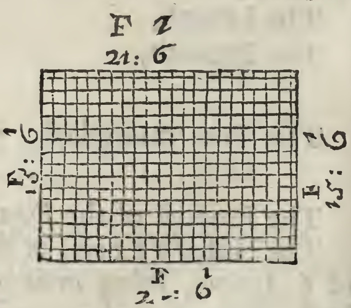
	Feet	In.
The Length,	21	6
The Breadth,	15	6
The Feet multiplied —— ——	105 21	
The Product of the Feet —— ——	315	
The half of 15 Feet, or the Product of 6 Inches, being multiplied by 15 Feet —— —— ——	7	6
The half of 21 Feet, or the Product of 21 Feet, being multiplied by 6 Inches	10	6
The Product of 6 Inches multiplied by 6 Inches, which produces 36, or 3 times 12, which, as you read before, is 3 Inches —— ——		3
	333	3

Then

Then add your Sums together, beginning at the Inches on the right Hand, ſaying, 3 and 6 is 9, and 6 is 15 Inches, which is 1 Foot and 3 Inches; then ſet down 3 under the Column of Inches, and carry 1 to the Feet, and ſay, 1 that I carry and 7 makes 8, and 5 makes 13; then ſet down 3 in the place of Units, under Feet, and carry one to the next place, ſaying, 1 that I carry and 1 is 2, and 1 makes 3, which I ſet down on the left Side of the former 3 (to wit) under the place of Tens: Then I proceed in the next place to 3, which being in the place of Hundreds is 300, and ſet that 3 down on the left Side of the two other.

So the whole Product of 21 Feet and 6 Inches, being multiplied by 15 Feet and 6 Inches, is 333 Feet and 3 Inches, as you have it above in the Example: I think it convenient here to add a Geometrical Figure for the better underſtanding of what hath been ſaid.

By this Geometrical Figure, you may perceive the Truth of what hath been ſaid.



Firſt, Here you ſee that the whole Squares or Feet are 21 in Length, and 15 in Height or Breadth, which produce 315 Feet.

Secondly, In the upper Part of the Figure, you ſee 21 half Squares or half Feet, which is, the 6 Inches which you multiplied by the 21 Feet, the Product whereof was 10 Feet and an Half; ſo in the Figure, accounting two half Squares (or half Feet) for one whole one, you will find 10 Feet and an Half, as before by Arithmetick.

Thirdly,

Thirdly, On the right Side of the Figure, you ſee 15 Oblongs or half Feet, which are the ſix Inches that you multiplied by the 15 Feet, which did produce 7 Feet and an half; ſo in the Figure, accounting two halfs for one whole, you will have 7 Feet and an Half, as you had before by Arithmetick.

Fourthly, In the uppermoſt Angle, on the right Side of the Figure, you ſee a little Square, which is 6 Inches in Length, and 6 Inches in Breadth: This is the 6 Inches that you multiplied by 6 Inches, which produced 36 Inches, for which there was ſet down but 3 Inches: The Reaſon whereof is this;

Theſe 3 Inches are the $\frac{1}{4}$ of 12 Inches, ſo is 36 the $\frac{1}{4}$ of 144 Inches, of which I have declared before, that there is 144 ſquare ſuperficial Inches in a ſuperficial Foot.

Therefore for Conveniency and Brevity in adding ſeveral Dimenſions together, the odd Inches being multiplied in themſelves, 144 Inches are accounted as 12 Inches and 12 of 144 are accounted as one Inch.

Now if you will give your ſelf the Trouble to tell the Squares in the Geometrical Figure, which Squares repreſent Feet, and the half Squares or Oblongs which repreſent half Feet, and add them together, you will find that they make 333 Feet beſides the little Square in the upper Angle of the Figure on the right Side, which repreſents 3 Inches or $\frac{1}{4}$ of a Foot; ſo you will have 333 Feet and 3 Inches, as was produced by Arithmetick before.

By what hath been declared, I think it eaſy for a mean Capacity to underſtand how to caſt up the foregoing Dimenſion, and bring it into Feet.

Now to know how many Squares of Carpenters Work there are in this Dimenſion, whoſe Product is 333 Feet and 3 Inches; you muſt either divide 333 by 100, or elſe cut off the two laſt Figures with a Stroke thus 3|33 and read it thus, 3 Squares, and 33 Feet and 3 Inches.

So in 14162 Feet, cut off the two laſt Figures thus; 141|62, and then you read it 141 Squares and 62 Feet.

So in other Numbers, conſiſting of three Figures or more, cut off the two laſt Figures to the right Hand, and the remaining Figures will be Squares.

But if you would know how many Yards there is in 21 Feet and 6 Inches in Length, and 15 Feet and 6 Inches in Breadth; you muſt, as before is taught, divide the Product of the Dimenſion, being 333 Feet, by 9, being the Number of Feet in a ſquare Yard ſuperficial.

As in Example,

$$\begin{array}{l} \cancel{6} \\ \cancel{3}\cancel{3}\cancel{3}\,(\,37 \text{ Yards.} \\ \cancel{9}\cancel{9} \end{array}$$

So your Quotient is 37 Yards; the odd 3 Inches muſt be added after you have divided.

Take another Example.

Suppoſe you have a Floor or Partition, that is 35 Feet and 10 Inches in Breadth, and 46 Feet 9 Inches in Length. Set down your Dimenſion as before, and multiply the whole Feet 46, by the whole Feet 35; and the Work will ſtand thus,

Feet.	In.
46	09
35	10
230	
138	

Secondly, Multiply the Inches into the Feet croſs-ways, ſaying 9 times 35 is

is 315, which muſt be ſet apart by it ſelf, as you may ſee in the Example in this Page.

Thirdly, You muſt multiply the 10 Inches by the 46 Feet, which produces 460; this you muſt add to the 315, which before you ſet apart, and they make 775 Inches, which you muſt divide by 12, to bring the Inches into Feet, which being divided, the Quotient is 64 Feet, and 7 Inches remaining, which 64 you muſt add to the Multiplication of the Feet, and the 7 to the Inches.

See the Example of the Inches multiplied into the Feet.

46	09
35	10

The Product of the 10 Inches being multiplied into 46 Feet is — 460 Inches

The Product of 9 Inches multiplied by 35 Feet is — 315 Inches

Which being added, is — 775 Inches

The 775 Inches being divided by 12, produces, as in the Margin, 64 Feet and 7 Inches, which muſt be added to the Multiplication of the Feet in the preceding Page, and then the Work will ſtand thus:

```
  1
 15(7
775(64
 122
  1
```

46	09
35	10
230	
138	
64	7
	7

Fourthly, You muſt multiply the 10 Inches belonging to the Breadth, by the 9 Inches belonging to the Length, and the Product will be 90 Inches of 144 Inches; or 7 Inches and an Half of 12 Inches, for if you di-

vide 90 by 12, you will have 7 in the Quotient, and 6 remaining, which 7 in the Quotient is 7 Inches, and muſt be added to the Place of Inches; and the 6 that remains over and above the Quotitient, being the $\frac{1}{2}$ of 12, is half an Inch, which is ſeldom accounted in Carpenters Work: But although it is not uſually done, yet if you are minded to be exact in Meaſuring, you may ſet down theſe odd Parts of Inches by themſelves, and at laſt add them all together, and for every 12 of theſe Parts you may add 1 Inch, and for 144 of them you muſt add 1 Foot, as you read before.

Then adding all your Sums together, you find that 46 Feet 9 Inches, multiplied by 35 Feet 10 Inches, is 1675 Feet 2 Inches.

Which if you are to bring into Squares, you cut off the two Figures next the right Hand thus 16|75 and then you read it 16 Squares and 75 Feet (or 16 Squares and $\frac{3}{4}$ of a Square.)

An Example of the whole Work.

	Feet	In.
The Length,	46	09
The Breadth,	35	10
The Product of 46 by 5,	230	
The Product of 46 by 3,	138	
The Product of all the Inches multiplied by all the Feet,	64	07
The Product of the 10 Inches multiplied by 9,		07
The whole Product	16|75	02

Alſo

Alſo by the way, take Notice that Girders Ends, and Ends of Breſt-Sommers, and Plates, and ſuch like, muſt be remember'd and ſet down in your Bill of Meaſurement.

Alſo you muſt allow in your Meaſure, for the Ends of the Joiſts that lie in the Walls; if it be Flooring unboarded that you are to meaſure, then you ought to allow 9 Inches into each Wall that way the Joiſt Ends are laid.

But if you meaſure Flooring and Boarding, then if you allow 6 Inches, it is reaſonable, becauſe though the Timbers go into the Walls 9 Inches, yet the Boarding goeth but home to the Wall.

And as you add theſe Things, you muſt alſo remember to deduct the Stairs and Chimneys where the Workman findeth Materials, elſe not.

Thus much will ſerve as to the Meaſuring of Flooring, Partitioning, and ſuch like; In the next Place I ſhall treat

Of the Meaſuring of Roofs.

Suppoſe a Building to be 30 Feet in Breadth, from the Outſide of one Wall to the Outſide of the other, and 65 Feet in Length, from out to out of the Walls, and you would know how many Squares of Roofing there are in the Roof of this Building.

If the Roof be true Pitch, you need not meaſure the Length of the Rafter, but take this for a general Rule:

That the Length of the Rafter is $\frac{3}{4}$ of the Breadth of the Building, therefore the Breadth being 30 Feet, the Length of the Rafter will be 22 Feet and 6 Inches, which is $\frac{3}{4}$ of 30) which being doubled

(for ſo it muſt be for both Sides) makes 45 Feet: Then multiply 45 by the Length of the Building being 65, and the Product will be 29|25, which is 29 Squares and a Quarter.

But the uſual way, which is ſomething briefer, is thus: Multiply the Length 65 by the Breadth 30, and that produces 1950 Feet for the flat of the Building; then add half the Number of Feet in the flat (to wit) 975 to 1950, and the Product is 2925 as before.

You may add the half thus, ſaying, The half of 19 is 9, then becauſe twice 9 is but 18, and there remains 1; you muſt carry that one to the next Figure being 5, and for that 1, which remained of the 19 after it was halfed, you muſt add 10 to 5, which makes it 15.

Then ſay, The half of 15 is 7 and 1 remaining, add that 1 to the Cypher, and it makes 10.

Then ſay, The half of 10 is 5.

See both the Examples.

	Feet	In.
The Length of the Rafter on one Side of the Roof	22	6
The Length of the Rafter on the other Side	22	6
The Length of both Rafters being added	45	0
The Length of the Building to be mulplied by 45.	65	0
	225	
	270	
The Product	2925	0

An

An Example of the other way, called Menſuration on the Flat.

	Feet	In.
The Length,	65	0
The Breadth,	30	0
The Product of the Length by the Breadth on the Flat,	1950	0
The half Flat to be added to the Flat,	975	0
The Product of the Flat and half	2925	0

Note, When you meaſure Roofing this laſt way, being called Meaſuring on the Flat, you muſt write over your Dimenſions, *Flat and Half*; or 1 $\frac{1}{2}$ to ſignify, that after you have multiplied the Length by the Breadth, you muſt add half that Product to it ſelf.

Note, If the Building that you are to meaſure be broader at one End than it is at the other, you muſt meaſure the Breadth in the Middle, both for the Flooring and the Roofing; or elſe meaſure each End, and add the Sums together, and take half the Product: As, ſuppoſe a Building to be 14 Feet wide at one End, and 12 Feet wide at the other, they being added, make 26, the half whereof is 13 Feet for the Width of the Building.

How to meaſure a Gable-end.

Suppoſe a Timber Building be 40 Feet in Breadth, and you have meaſured all the Carcaſs of the Building, except the Gable-end, or Ends;

To meaſure this Gable-end, multiply half the Baſe by the whole Perpendicular; or half the Perpendicular by the whole Baſe, gives you the Area, or Content of it.

I call the Plate, whereon the Rafters ſtand, the Baſe.

The Baſe being 40 Feet, the Perpendicular is 22 Feet and 4 Inches.

See the Example.

	Feet	In.
The half of the Baſe,	20	0
The whole Perpendicular,	22	4
The Product of 2 multiplied by 20,	40	0
The Product of 20 by 20,	400	0
The Product of the 4 Inches multiplied by the 20 Feet,	6	8
The total Product,	446	8

The other Example.

	Feet.	In.
The whole Baſe,	40	0
The half of the Perpendicular,	11	2
The Product of 40 by 1,	40	0
The Product of 40 by 10,	400	0
The Product of the 2 In. by the 40 Feet,	6	8
The total Product,	446	8

Thus you ſee both ways agree, and the Gable-end contains 4 Squares and 46 Feet 8 Inches. If

If there be two Gable-ends equal, or both alike in the Building, then you muſt ſet down the total Product twice.

If there be four Gable-ends, and each of them of one Bigneſs, then you muſt ſet down the total Product four times.

Note, If the Building be ſquare (*viz.*) hath four right Angles, and the Breadth 40 Feet; and the Roof true Pitch;

The Length of the Rafter, and alſo of the Hip Rafter, and the Angles which they make; alſo the Length of the Diagonal Line, and of the Perpendicular, are as in the following Table.

As you may ſee by the Figure A B C D hereunto annexed.

	Feet	In.
The Breadth of the Building,	40	00
The Length of the Rafter,	30	00
The Length of the Hip Rafter,	36	00
The Length of the Diagonal Line,	56	06 $\frac{2}{5}$
The Length of the Perpendicular,	22	04 $\frac{1}{4}$

		Deg.	Min.
Rafter Angles	At Foot	48	10
	At Top	41	50
Hip Angles	At Foot	38	22
	At Top	51	38

Explanation of the Fig. A B C D.

A B. The Breadth of the Building.
A P. The half Breadth.
A E, or E B, The Length of the Rafter.
E P The Perpendicular.

AF, or FB, The Length of the Hip Rafter.

AGH, The Angle that the Hip makes at Top, and K *i*, the Meaſure of it.

HAG, The Angle that the Hip makes at the Foot, and LI the Meaſure of it.

PAE, The Angle that the Rafter makes at Foot, and P *a* the Meaſure of it.

AEP, The Angle that the Rafter makes at Top, and *a*, *n*, the Meaſure of it.

LIP, The Quadrant of a Circle divided into 90 Degrees, for the meaſuring of the Angles.

AC, The Diagonal.

LM, The Line over which the Rafters muſt be placed, that the upper Ends of the Hip Rafters are fixed to.

The Length of the Hip Rafter is found, by taking the Length of the Rafter A E, and ſetting it on the Perpendicular from P to F, draw F A the Hip Rafter.

If you have a Roof of any other Width, you may deſcribe it from this Figure, or you may find the Length of the Hip Rafters, and the Length of the Diagonal, and likewiſe of the Perpendicular by the Rule of Three, and the foregoing Table.

The Angles are always equal to thoſe in the Table, let the Width be what it will, provided the Building be right angled, and the Roof the uſual true Pitch (that is, the Rafters Length to be $\frac{3}{4}$ of the Breadth of the Building.

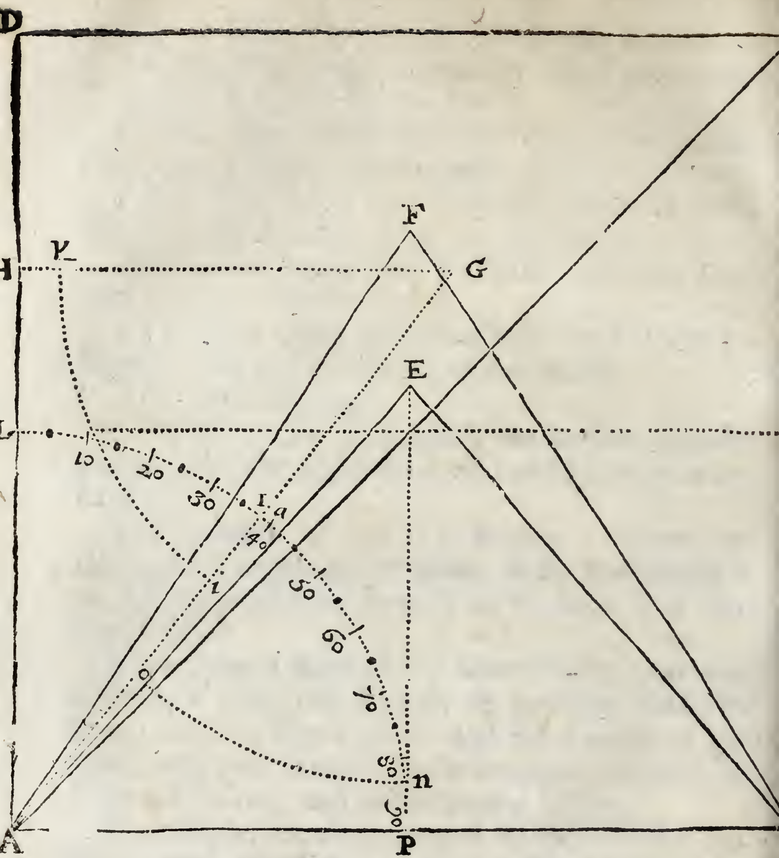
To Meaſure a Hipt Roof.

Suppoſe a Building to be 60 Feet in Length, and 40 Feet in Breadth, ſtanding alone, being Hipt from each

the Ends being a Triangle, which you mea-
ſure, as you did the Gable-end.

Multiply 40 Feet being the Baſe, by 15 Feet being one half of the Rafter (or Perpendicular as you may call it, although it lean from an upright) and the Product is 600.

The



Place this between Page 208 and 209.

... of the Building.

To Measure a Hipt Roof.

Suppose a Building to be 60 Feet in Length, and 40 Feet in Breadth, standing alone, being Hipt from each

each Angle, the Angles being right, and the Roof true Pitch, and it is required to know how many Squares of Roofing there is in it.

Multiply the whole Length, being 60 Feet, by 30 Feet, the Length of the Rafter, for one Side of the Roof.

Or add 30 and 30 together, they make 60 for both Sides of the Roof, then multiply 60 by 60, the Product is 3600, or 36 Squares in that Hipt Roof.

Which I thus demonstrate.

The Length of the Roof on the Ridge, from Hip to Hip is 20 Feet, and the Length of the Roof at the Plate is 60 Feet.

Add these two Sums together, and they make 80 Feet, whereof take half, that is 40 Feet, which is the mean Length of the Roof, if you measure in the Middle between the Top and Bottom; then multiply this 40 Feet by 60 Feet, being the Length of both Rafters on each Side the Roof, and the Product is 2400, which is the Measure of the two Sides of the Roof.

Proceed we, in the next Place, to the two Ends;

Being each of them 40 Feet in Length, and the Rafter of each End 30 Feet in Length; each of the Ends being a Triangle, which you must measure, as you did the Gable-end.

Multiply 40 Feet being the Base, by 15 Feet being one half of the Rafter (or Perpendicular as you may call it, although it lean from an upright) and the Product is 600.

The

The other End of the Roof being the ſame as this, add 600 to 600 it makes 1200, which being added to 2400, the Product of the two Sides, make 3600, or 36 Squares as before, which was to be demonſtrated.

Suppoſe you were to meaſure the Roofing of a corner Houſe, being 60 Feet long, and 40 Feet broad, and joining to other Houſes each Way, then there will be but one Hip, and one Sleeper, yet the Dimenſions will be the ſame, and the ſame Number of Squares in it, as in the former that was Hipt, provided the Building be Square, and the Roof true pitch.

See the Examples.

	Feet	In.
The whole Length of the Roof,	60	00
The Length of both Rafters,	60	00
The Product or Number of the Squares	3600	00

The Demonſtration.

	Feet	In.
The Length of the Roof on the Top is	20	00
To which add the Length of the Roof at the Plate	60	00
The Sum is	80	00
The Length of both Rafters	60	00
Being multiplied by 40, the Mean between 60 and 20	40	00
The Product is Which is the Content of the two Sides.	2400	00

Now

Now for the two Ends.

	Feet	In.
The Length of the Baſe,	40	00
Multiplied by the half Length of the Rafter,	15	00
	200	00
	40	
Produces	600	00
To which add the Meaſurement of the other End,	600	00
The Sum is the Content of both the Ends,	1200	00
To which add the Meaſure of both Sides,	2400	00
The Sum is the Area or Content of the whole Roof, as before,	3600	00

How to find the Length of the Hip for a Roof that is 20 Feet in Breadth, by the Rule of Three.

You ſee by the foregoing Figure A B C D, that a Roof being 40 Feet in Width or Breadth, gives a Hip Rafter that is 36 Feet in Length; therefore ſtate your Queſtion thus: If 40 give 36, what will 20 give?

Multiply and Divide, and you will find 18

36		3	
20		720	18
720		400	
		4	

After the ſame Manner you may find any of the reſt. You muſt remember to add the Rafters Feet, and Eves Boards, in the Bill of Meaſurement.

CHAP.

CHAP. IV.

Meaſuring of Glaziers Work.

THE Meaſuring of Glaziers Work is the ſame as of Carpenters Work; only Carpenters Work is not meaſured any nearer than two whole Inches.

But Glaziers Work is ſometimes meaſured to $\frac{1}{4}$ of an Inch; and the beſt and readieſt way to meaſure Glazing, is to take the Dimenſions with a ſliding Ruler, ſuch as Glaziers generally uſe, which Ruler is divided Decimally, a Foot into 100 Parts, ſo that a whole Foot contains 100 Parts or Diviſions; $\frac{3}{4}$ of a Foot contains 75 Parts; $\frac{1}{2}$ of a Foot contains 50 Parts,, and $\frac{1}{4}$ of a Foot contains 25 Parts.

Thoſe that deſire to meaſure the Decimal Way, I refer them to a Treatiſe put forth by my Friend *Tho. Hammond* (Entituled, *A new and exact Way of Menſuration*) which I writ for him when I was Apprentice, and indeed it is a very ready Way of Meaſuring, provided our two Foot Rules were divided into 20 equal Parts, or a Foot Rule into 10 Parts (or, as I may call them, Inches) and every one of thoſe Parts divided into 10 equal Parts.

But to come to the Thing intended (to wit) to meaſure Glazing by vulgar Arithmetick, and with a two Foot Ruler divided into 24 Parts, and each of thoſe into 8 Parts, as they are commonly made and uſed.

Suppoſe you have a Pane of Glaſs which is 5 Feet 8 Inches and an half in Length, and 5 Feet 7 Inches and $\frac{1}{4}$ in Breadth, and you would know the Area or Content thereof:

Set

	Feet	In.	Parts.
Set down your Dimensions thus,	5	8	6
	5	7	3

And read it, 5 Feet 8 Inches and 6 Parts of 12, which is the ½ Inch; by 5 Feet 7 Inches, and 3 Parts of 12, which is the ¼ Inch.

Then multiply the Feet and Inches one by the other, as you are taught in the foregoing Pages, saying, 5 times 5 is 25 Feet; which being set down under the Feet, multiply cross-ways 5 by 8, which is 40 Inches, or 3 Feet and 4 Inches. This being set down,

In the next Place, multiply 7 by 5, which is 35 Inches, or 2 Feet and 11 Inches. This being set down,

In the next place multiply the 7 Inches by the 8, it is 56 Inches, of which 144 make a Superficial Foot, for which you must set down 4 Inches, because you can have but 4 times 12 out of 56, and there remains 8. Therefore I set down 4 Inches and 8 Parts, of which Parts 12 make an Inch, 12 of which Inches are accounted a superficial Foot.

In the next place multiply the Parts of Inches cross-ways; first into the Feet, saying, 6 times 5 make 30, which is 2 Inches and 6 Parts. This being set down,

Say 5 times 3 is 15, or 1 Inch and 3 Parts, which you must put also under the rest.

Then multiply the Parts of Inches cross-ways into the Inches, saying, 6 times 7 is 42, which is 3 Parts and an half, for which I set down 3 under the Place of Parts of Inches, and 6 against it towards the right Hand, which signifies ½ Part of a Part (for 12 of these Parts make one such Part, whereof there are

12

12 in an Inch) Line Meaſure. This being done, multiply 3 by 8 (which is 3 Parts by 8 Inches) that makes 24 Parts, for which you muſt ſet down 2 under the Place of Parts of Inches.

Note, That as in multiplying the Inches by the Inches, you account every 12 of the Product for 1 Inch, ſo in multiplying the Parts of Inches into the Inches, you muſt account every 12 of the Product for 1 Part; as in multiplying 3 Parts by 8 Inches produceth 24, which is twice 12, therefore I ſet down 2 under the place of Parts.

Laſtly, you muſt multiply the Parts of Inches by the Parts (if you will meaſure ſo near) ſaying, 3 times 6 is 18, which is 1 Part and a half of a twelfth Part of an Inch Line Meaſure; ſee the Dimenſion caſt up.

Example.

	Feet	In.	Parts.	
	05	8	6	
	05	7	3	
The Product of 5 Feet by 5 Feet is	25	0	0	
The Product of 8 Inches by 5 Feet is	03	4	0	
The Product of 7 Inches by 5 Feet is	02	11	0	
The Product of 7 Inches by 8 In. is	00	4	8	
The Product of 6 Parts by 5 Feet is	00	2	6	
The Product of 3 Parts by 5 Feet is	00	1	3	Pa.
The Product of 6 Parts by 7 Inch. is	00	0	3	6
The Product of 3 Parts by 8 Inch. is	00	0	2	0
The Product of 3 Parts by 6 Parts is	00	0	0	1 ½
The total Product is	31	11	10	7 ½

Thus

Thus you ſee that the Area or Content of 5 Feet 8 Inches and an Half (or 6 Parts of 12) in Length, being multiplied by 5 Feet 7 Inches and $\frac{1}{4}$ (or 3 Parts of 12) in Breadth, is 31 Feet, 11 Inches and 10 Parts of an Inch, accounted to have 12 Parts, and 7 and $\frac{1}{2}$ of one of thoſe Parts; but as for the laſt 7 Parts and $\frac{1}{2}$, they are not worth the ſetting down nor taking notice of, the Value of thoſe Parts being ſo ſmall.

The Truth of this Area or Content may be proved three ſeveral Ways.

Firſt Decimally, Second Geometrically, Thirdly by Vulgar Arithmetick.

Firſt Decimally thus; the Decimal of 8 Inches and $\frac{1}{2}$ is 71, and the Decimal of 7 Inches and $\frac{1}{4}$ is 60 and ſome ſmall matter more: Therefore in Decimals, I ſet down the Dimenſion thus;

F. P.
571
560
34260
2855
31|97|60

And having multiplied it, I find the Product to be 319760, then cut off the 2 Figures next to the Left Hand with a Stroak, as you ſee in the Example, and they repreſent 31 Feet.

Then the Remainder is 2760 of 10000, and by cutting of 2 Figures more thus 97|60 it is 97 of 100 and ſomething more, it is near 98 Parts of 100, 92 of which Parts, is the Decimal of 11 Inches, and the 6 Parts which remain are $\frac{6}{10}$ of an Inch; ſo that by this you ſee the Decimal Way of Meaſuring and this Vulgar doth agree.

Secondly, you may prove it Geometrically, if you deſcribe a Geometrical Figure (after the ſame Manner that the Figure is deſcribed in P. 189 of *Carpenters Work*, being drawn according to a Scale;) 5 Feet 8 Inches

8 Inches and $\frac{1}{2}$ in Length, and 5 Feet 7 Inches and $\frac{1}{4}$ in Breadth; and then divide the Feet and Inches, and Parts; and you will find it to agree with the Area or Content before produced.

Thirdly, it may alſo be proved by Vulgar Arithmetick thus.

Reduce the Feet and Inches into the leaſt Denomination (to wit) Quarters of Inches, and then multiply them one by another.

To do this, multiply 68 Inches (which are the Inches in 5 Feet and 8 Inches) by 4, and that brings them into Quarters of Inches Line Meaſure, then add 2 to the Product, for the half Inch in the Length.

$$\begin{array}{r} 68 \\ 4 \\ \hline 272 \\ 2 \\ \hline \end{array}$$

Thus in the Length you have 274 Quarters of Inches.

The Breadth being 5 Feet 7 Inches and $\frac{1}{4}$, multiply 67 (the Inches in 5 Feet and 7 Inches Line Meaſure) by 4, being the Quarters in an Inch Line Meaſure, and to the Product add 1, which is the $\frac{1}{4}$ Inch in the Breadth.

$$\begin{array}{r} 67 \\ 4 \\ \hline 268 \\ 1 \\ \hline \end{array}$$

And in the Breadth is contained 269 Quarters of Inch.

Thus

Thus having brought theLength and Breadth into the leaſt Denomination (to wit, Quarters of Inches) I ſet down my Dimenſion thus, and multiply it:

```
          274
          269
        -----
         2466
        1644
        548
        -----
The Product is 73706 Quarters.
        -----
```

Of which Quarters there are 16 in an Inch ſuperficial Meaſure (to wit, 4 in Length, and 4 in Breadth) and of which Inches there are 144 in a Foot ſuperficial Meaſure (to wit, 12 in Length, and 12 in Breadth.) Therefore I multiply 144, the Inches in a ſuperficial Foot, by 16, the Quarters in a ſuperficial Inch, and

```
          144
           16
         ----
          864
          144
         ----
In a ſuperficial Foot there is 2304 Quarters.
         ----
```

Then divide 73706, the ſuperficial Quarters of Inches in the whole Dimenſion, by 2304, the ſuperficial Quarters of Inches in a Foot.

228
04582
73706 (31 Feet.
23044
230

And the Quotient is 31 Feet, and the Remainder is 2282 Quarters, which are not a Foot.

Therefore divide 2282 by 192 (which is the Number of Quarters contained in 12 Inches in Length, and 1 Inch in Breadth, which 192 is the Product of 12 by 16.)

12
16
——
72
12
192 Quar.

17
360
2282 (11 Inches.
1922
19

And the Quotient is 11 Inches, and 170 the Remainder, which is 170 Parts, or Quarters of an Inch, in Breadth, and 12 Inches in Length; which Inch, as I told you before, contains 192 Quarters.

Wherefore divide this 170 by 16 (to wit, the Quarters that are contained in an Inch in Length, and an Inch in Breadth.)

1
170 (10 Parts.
166
1

And the Quotient is 10, and the Remainder 10.

Thus you see the Product of 5 Feet, 8 Inches, and $\frac{1}{2}$ in Length, being multiplied by 5 Feet, 7 Inches, and

and $\frac{1}{4}$ in Breadth, is 31 Feet, 11 Inches, 10 Parts, and $\frac{10}{16}$ of a Part, which $\frac{10}{16}$ of a Part, is in Value the ſame that 7 and $\frac{1}{2}$ is, of a Part containing 12, as you found it by the firſt Way of working in Vulgar Arithmetick.

Another Example.

Suppoſe a Pane of Glaſs be 5 Feet, 3 Inches and $\frac{1}{2}$ long, and 2 Feet, 4 Inches and $\frac{1}{2}$ broad, and you deſire to know the Content thereof.

	Feet.	In.	Parts.
Set down your Dimenſion thus	5	3	6
	2	4	6

Then multiply the Feet and Inches one into another, ſaying, 5 times 2 is 10 Feet; ſet that down under the Feet; then multiply croſswiſe, ſaying, 3 times 2 is 6 Inches; ſet that under the Inches.

Then ſay, 4 times 5 is 20 Inches, or 1 Foot and 8 Inches; ſet the 1 under the Feet, and the 8 under the Inches.

Then ſay, 4 times 3 is 12, which is 1 Inch.

Next multiply the Parts croſsways into the Feet, ſaying, 6 times 2 is 12, which is 1 Inch, which muſt be ſet under the Place of Inches.

Then multiply the 6 Parts by 5 Feet, it makes 30 Parts, which is 2 Inches, and 6 Parts; which being ſet down,

Multiply the Parts into the Inches, ſaying, 6 times 3 is 18, which is 1 Part and an half; for which I put 1 under the Place of Parts, and 6 againſt it towards the Right Hand, which ſignifies $\frac{1}{2}$ or $\frac{6}{12}$ of a Part.

Then multiply the 6 Parts by the 4 Inches, it makes 24, which is two Parts; therefore add 2 under the Place of Parts.

Lastly, multiply the Parts by the Parts, saying, 6 times 6 is 36, which is $\frac{3}{12}$ of a Part; set this 3 under the 6 on the Right Hand.

Then adding them all together, the Content is 12 Feet, 6 Inches, 9 Parts, and $\frac{9}{12}$ of a Part; which $\frac{9}{12}$ you need not set down as you read before, they signifying very little; then it will be 12 Feet, 6 Inches, and 9 Parts, or $\frac{3}{4}$ of an Inch.

See the whole Work.

	Feet.	In.	Parts.
The Length	05	3	6
The Breadth	02	4	6
The Product of 5 Feet by 2 Feet is	10	0	0
The Product of 3 Inches by 2 Feet is	00	6	0
The Product of 4 Inches by 5 Feet is	01	8	0
The Product of 4 Inches by 3 Inches is	00	1	0
The Product of 6 Parts by 2 Feet is	00	1	0
The Product of 6 Parts by 5 Feet is	00	2	6 Pa.
The Product of 6 Parts by 3 Inches is	00	0	1 6
The Product of 6 Parts by 4 Inches is	00	0	2
The Product of 6 Parts by 6 Parts is	00	0	0 3
The Total is	12	6	9 9

Note, That in measuring of Glazing, many times in a Building there are several Window-frames of one bigness, and in one Window-frame there are several Panes of Glass of one Bigness or Dimension; as in a six

ſix Light Window there are 6 Panes of Glaſs, 3 of them of one Dimenſion, and 3 of another; ſo in a four Light Window there are 4 Panes of Glaſs, 2 of one bigneſs, and 2 of another.

In a ſix Light Window you need meaſure but one Pane of a Sort, and ſet down each Dimenſion; and becauſe there are 3 of a Sort of one bigneſs, ſet down 3 againſt your Dimenſion, which ſignifies 3 times that Dimenſion.

If it be a four Light Window, and have 2 Panes of Glaſs of one bigneſs, and two of another, then ſet down 2 againſt your Dimenſion.

Example.

If you were to meaſure a ſix Light Window, and the upper Panes were each of them 2 Feet 6 Inches in height (or length) and 1 Foot 2 Inches in Breadth; and the lower Panes 6 Feet 4 Inches in Length, and 1 Foot 2 Inches in Breadth; you may make but two Dimenſions of theſe 6 Panes of Glaſs.

	Feet.	In.	Parts.	
Setting them down thus	2	6	0	(3)
	1	2	0	

Feet.	In.	Parts.	
6	4	0	(3)
1	2	0	

The 3 in the Circle thus (3) ſignifies 3 times the Dimenſion that it ſtands againſt; therefore when you have caſt up the Dimenſion, the Product thereof

you must multiply by 3, and set that down for the 3 Panes.

Example.

	Feet	In.	Parts.	
	2	6	0	(3)
	1	2	0	
	2	0	0	
		6		
		4		
		1		
The Product of one Pane is	2	11	0	
Which multiply by 3, because there are 3 Panes of that bigness		3		
And the Product is	8	9	0	

Being the Content of the 3 Panes.

Then the other 3 Panes, being each of them 6 Feet 4 Inches in Length, and 1 Foot 2 Inches in Breadth,

	Feet	In.	Parts.	
Set down thus	6	4	0	(3)
And multiply	1	2	0	
The Product of 6 Feet by one, is	6	0	0	
The Product of 2 Inches by 6 Feet is	1	0	0	
The Product of 4 Inches by 1 Foot is	0	4	0	
The Product of 2 Inches by 4 Inches is	0	0	8	
The Total Product or Content is	7	4	8	
Which multiply by			3	

Saying

	Feet	In.	Parts.
	7	4	8
			3
Saying 3 times 8 is 24 Parts or 2 Inches, which carry to the Inches, and ſet a Cypher under the Parts, thus			0
Then ſay 3 times 4 Inches is 12 Inches, and 2 that I carry is 14 Inches, that is, 1 Foot, and 2 In. ſet down the 2 In. and carry the 1 Foot to the Feet		2	
Then ſay 3 times 7 is 21 Feet, and 1 that I carry is 22 Feet; ſet that down	22	0	0
Add them, and the Product is	22	2	0

For the Area, or Content of the 3 Panes.

	Feet	In.	Parts.
The Content of the firſt 3 Panes is	08	9	0
The Content of the laſt 3 Panes is	22	2	0
Being added together is	30	11	0

Which is the Content of all the ſix Panes in the ſix-light Window.

Then ſo many ſix-light Windows of the ſame bigneſs as you have in the Building to meaſure, multiply the Number of them by 30 Feet and 11 Inches, and the Product is the Content of them all.

To meaſure a four-light Window.

A four-light Window having 2 Panes of one bigneſs, and 2 Panes of another; you may ſet down but 1 Pane of each bigneſs, and (2) againſt it thus.

Suppoſe the upper Panes to be 2 Feet 1 Inch in length, and 1 Foot 8 Inches and ½ in breadth, a-piece;

And the lower Panes to be 4 Feet in length, and 1 Foot 8 Inches and $\frac{1}{2}$ in breadth a-piece.

	Feet	In.	Parts	
Begin with the two upper Panes. And set them down thus	2	1	0	(2)
	1	8	6	
	2	0	0	
	1	4	0	
		1		
		0	8	
		1	6	
The Content of 1 Pane is	3	7	2	
The other Pane being the same, add	3	7	2	
The Content of both the upper Panes is	7	2	4	

Secondly, Set down the Dimension of the two lower Panes thus,

	Feet	In.	Parts	
	4	0	0	(2)
	1	8	6	
	4	0	0	
	2	8	0	
		2		
The Content of one Pane is	6	10	0	
And because there is (2) stands against the Dimension, you must multiply it by 2, or else add	6	10	0	
The Content of both Panes is	13	8	0	
To which add the Content of the upper Panes	7	2	4	
And the Content of the 4 Panes in the 4 Light Window is	20	10	4	

How

How to ſet down Dimenſions in your Pocket-Book.

Note, Before you begin to ſet down your Dimenſions, it is convenient to divide the Breadth of your Page or Leaf into ſo many ſeveral Columns as you think convenient, with Lines drawn of Ink: The Leaves of your Pocket-Book being of the Breadth of this Book, you may divide a Leaf into four Parts or Columns.

You muſt likewiſe before you ſet down any Dimenſions, expreſs the Workmaſter, and the Workmens Names, alſo the Place where, and the Day of the Month, and Date of the Year that you meaſure. Likewiſe if the Work that you are to meaſure, be Glazed with ſquare Glaſs, you muſt write *Squares* above your Dimenſions, and over thoſe Dimenſions which are glazed with Quarries, you muſt write *Quarries*; that when you come to make the Bill of Meaſurement, you may expreſs them ſeverally, becauſe they are of ſeveral Prices.

In the next Page I will ſet down all the Dimenſions which you have been taught to caſt up in Glazing, and ſome others, with the Product to each Dimenſion.

Glazing done by A. B. *for* C. D. *in* Long-Acre, *and measured the 22d of* January, 1727.

Quarries. F. I. P.	Products. F. I. P.	Squares. F. I. P.	Products. F. I. P.
5 8 6 5 7 3	31 11 10	4 3 0 1 2 0	04 11 06
5 3 6 2 4 6	12 06 09	2 0 0 1 6 0	03 00 00
2 6 0 (3) 1 2 0	08 09 00	3 0 0 2 3 0	06 09 00
6 4 0 (3) 1 2 0	22 02 00	6 0 9 5 0 3	30 05 03
2 1 0 (2) 1 8 6	07 02 04	1 2 0 (2) 3 0 0	07 00 00
4 0 0 (2) 1 8 6	13 08 00		52 01 09
	96 03 11		

Explanation of the Columns.

In the first Column towards the Left Hand, are the Dimensions (which you have been taught to cast up) of Glazing done with Quarries.

In the second Column you have the Product of each Dimension just against it.

In

In the third Column you have 5 Dimenſions of Glazing done with Square Glaſs.

In the laſt Column you have the Product of each Dimenſion juſt againſt it; and at the Bottom you have the total Sum of all the Products in that Column, being 52 01 09.

Likewiſe at the Bottom of the ſecond Column you have the total Sum of all the Products of the Dimenſions done with Quarries, which is 96 Feet 3 Inches and 11 Parts. As for the odd Parts, you may leave them out when you make your Bill of Meaſurement.

When you are meaſuring, and ſetting down the Dimenſions in your Book, whether it be of Glazing, or any other Trade, you muſt leave every other Column vacant or empty, that ſo having ſet down all your Dimenſions in your Book (which you generally do before you caſt up any) when you caſt them up (which muſt be in another Book or Sheet of Paper) you may enter the Product of each Dimenſion juſt againſt it, as you ſee in the Page before.

If there be another to meaſure againſt you, you muſt, after you have done ſetting down all your Dimenſions, compare your Dimenſions together, to ſee if the Dimenſions in both your Books agree.

The Reaſon why you ſet down the Product of each Dimenſion juſt againſt it in the next Column towards the Right Hand, is,

If you meaſure againſt another Meaſurer, and there ſhould be a Miſtake in either of your caſtings up of the Dimenſions (as it often happens thro' Security or Negli-

Negligence) then one by reading over the Dimensions in his Book, with the Product to each Dimension as he goes on; and the other looking in his own Book, the Miſtake will ſoon be found, which muſt be rectified between you.

Therefore, to be certain in caſting up your Dimenſions, you ought to caſt them up twice, if not three times; (to wit) after you have caſt them all over once, begin and caſt them over again, and ſee whether it agrees with your firſt caſting up; if not, then caſt them up again.

When you make your Bill of Meaſurement, you muſt ſet your Name to it at the lower End of the Bill.

An Example of a Bill.

Glaziers Work done by *A. B*, for *C. D*, in *Long-Acre*, and meaſured the 22d of *January*, 1727.

	l.	*s.*	*d.*
For 96 Feet, and 3 Inches of Glazing done with Quarries, at 5 d. *per* Foot	2	00	1 $\frac{1}{4}$
For 52 Feet, and 1 Inch of Glazing with Squares, at 7 d. *per* Foot	1	10	4 $\frac{1}{2}$
The Sum is	3	10	5 $\frac{3}{4}$

Meaſured the Day and Year above-written by V. M.

You may ſometimes happen to meet with Panes of Glaſs of various Forms; as ſometimes the Top of a Pane of Glaſs is concluded with a Semicircle, ſometimes

times with a Scheme or Segment of a Circle, ſometimes Triangular, and ſometimes Ovalar, and ſometimes an Oval in Glaſs, or a round Pane.

For the Meaſuring of each of theſe, and ſuch like, I refer you to the Meaſuring of Planes in the Fifth Book, which doth ſucceed this Treatiſe.

Thus much may ſuffice the ingenious Practicer, as to the Meaſuring of Glazing.

CHAP. V.

THE Meaſuring of Joiners Work is the ſame that is taught before, only there is this Difference; Carpenters Work is brought into Squares, and Glaziers into Feet; and Joiners Work muſt be brought into Yards.

Which to do, after you have caſt up all your Dimenſions, and brought them into Feet, divide the whole Product by 9, and that gives you the Number of Yards.

You muſt divide by 9, becauſe 3 Feet in Length, and 3 in Breadth, contain 9 Feet, which is a ſuperficial Yard.

I will ſuppoſe ſome Dimenſions to be caſt up, whoſe Contents or Products are theſe following.

For it would be needleſs to ſpend time to teach the caſting up of Dimenſions in Feet and Inches again, having ſpoken largely and plainly thereto in the precedent Pages.

Wainſcot

Wainſcot.

Feet.	Inches.
110	08
304	07
065	06
934	02
041	00
073	01
1529	00

~~68~~ (8
~~1529~~ (169 Yards, and 8 Feet.
~~999~~

The Total; which divide by 9 to bring the Feet into Yards, as you ſee above, and in 1529 Feet, you find 169 Yards, and 8 Feet;

Which muſt be ſet down in the Bill of Meaſurement at ſo much *per* Yard.

Then, for Cornices, and Baſe, and Sub-baſe, Joiners do them by the Foot, Line Meaſure, or Running Meaſure, as it is called, if they do no other Work with it: But if they do other Work joining with it, they meaſure it all together by the Yard.

Likewiſe Architrave and Frieze they do by the Foot Line Meaſure.

The Chimney-Pieces at ſo much a-piece, and ſometimes meaſure them in to the other Work; deducting nothing for the Vacancy of the Chimney.

CHAP.

CHAP. VI.

PAinters Work is meaſured like Joiners Work, and brought into Yards, only there is this Difference:

When you meaſure Painting upon Cornices, or Moldings of any Sort, you muſt have a Line, and girt the round Moldings, and bend the Line into the hollow Moldings. Thus when you are to meaſure a Room, you begin with your Line at the Top of the Cornice, where you may faſten it with a ſmall Nail, and girting the round Moldings, and bending the Line into the Hollows and Angles, bring the Line down to the Bottom of the Painting, then meaſure the Length of the Line with your Rule, and ſet down that Sum.

Then begin in one Angle of the Room, and meaſure the Length of that Side whereof you took the Height, and ſet down the Length under the Height which before you ſet down, and that is one Dimenſion.

And if the oppoſite Side of the Room be like unto that which you have already meaſured and ſet down, and the Room ſquare, you may either ſet down that Dimenſion twice, or elſe ſet a Figure of (2) encloſed againſt the Dimenſion, which ſignifies that Dimenſion is 2 times: Alſo you muſt take the Length of the other 2 Sides, and ſet them down by the ſame perpendicular Height; or you may take the Length of each Side of the Room, and add them all together, and ſo make but one Dimenſion of all the Sides by the perpendicular Height.

Then after you have ſet down all your Dimenſions, and brought them into Feet and Inches by caſting them

them up, you muſt divide the whole Product by 9, and that brings the Feet into Yards, as you read before: For you muſt not think to learn to meaſure any one of theſe Trades only, but you muſt begin at the beginning of Menſuration (to wit) of Carpenters Work, and ſo proceed gradually; and be ſure to be perfect in caſting up one Dimenſion before you enter upon another.

Thus proceeding with Care and Diligence, until you have attained to the meaſuring of Glazing, you may with eaſe afterwards meaſure all the reſt of the Works that are ſuperficial.

Indeed, the meaſuring of Bricklayers Work is ſomething more difficult, becauſe it is reduced to a thickneſs, and ſo becomes ſolid Meaſure.

As for the Painting of Windows, they are generally ſet down at ſo much *per* Light, and Caſements at ſo much a-piece.

The next Work to be meaſured is Plaiſtering, of which I need ſay no more than this, that it muſt be brought into Yards; and where a Sommer or Girder lies below the Ceiling, it is uſually deducted where the Workman finds Materials; but not elſe.

Alſo in rendring where Materials are found by the Workman, ſometimes $\frac{1}{4}$ is deducted for the Quarters; but not where Workmanſhip only is found, becauſe it might be rendred as ſoon if there were no Quarters.

The meaſuring of Maſon's Work (to wit, flat Paving, and ſuch like) is meaſured the ſame Way as all the reſt. There is no need of Diviſion, for Maſon's Work by the Foot.

I come now to treat of the Menſuration of Bricklayers Work.

CHAP.

CHAP. VII.

BRickwork, as you read before, muſt be reduced to a certain or uſual Thickneſs, in caſting of it up, for which Reaſon it becomes ſolid Meaſure.

Which uſual Thickneſs is 14 Inches, or, as it is generally called, one Brick and half.

By one Brick and half, is meant the Length of a Brick, and half the Length; or the Length of one Brick, together with the Breadth of another.

For the Breadth of two Bricks, with a Joint of Mortar between them, is anſwerable to the Length of a Brick, the Length whereof ought to be, and generally is 9 Inches, and the Breadth 4 Inches and $\frac{1}{4}$, which being added together with a Joint of Mortar makes 14 Inches, the aforeſaid uſual Thickneſs.

And as Carpenters Work is brought into Squares, Bricklayers Work muſt be brought into Rods, which Rod contains 272 Feet and 3 Inches, and is produc'd from 16 Feet and 6 Inches in Length, being multiplied by 16 Feet and 6 Inches in Height or Breadth; or a Quadrate whoſe *Side* is a Pole or Perch.

When you come to meaſure the Brickwork of a Building, you will find the Walls thereof to be of ſeveral Thickneſſes; ſome thicker than a Brick and half, and ſome thinner, as 1 Brick Walls, or Walls 9 Inches in Thickneſs.

Which ſeveral Thickneſſes muſt all be brought, or reduced to the uſual Thickneſs of 1 Brick and half, which to do, obſerve the following Rules.

1. Set down each Thickneſs in a Column by it ſelf, (to wit) ſet down your Dimenſions that are

1 Brick (or 9 Inches) in Thickneſs, in a Column by themſelves.

Alſo thoſe Dimenſions that are 1 Brick and ½ in Thickneſs, muſt be ſet in a Column by themſelves.

Likewiſe thoſe of 2 Bricks in Thickneſs by themſelves, and thoſe of 2 Bricks and ½, in a Column by themſelves.

And thoſe of 3 Bricks in a Column by themſelves.

So likewiſe if your Walls be thicker than 3 Bricks, (in Length) ſet each Thickneſs by it ſelf.

By ſetting the Dimenſions by themſelves, is meant to ſet them in a Page or Column alone ſeverally, and not to intermix Dimenſions of ſeveral Thickneſſes in one Page or Column.

Notwithſtanding, ſome Meaſurers ſet down their Dimenſions as they take them of ſeveral Thickneſſes in one Column; but I do not think it ſo convenient for a Learner, as to ſet them down ſeverally.

2. After you have taken your Dimenſions and caſt them up, and by adding the ſeveral Products together (of one Thickneſs) have produced the total Product, then to reduce it to 1 Brick and ½, work thus;

If the Dimenſions that you have caſt up, be 1 Brick's Length in Thickneſs, that is the Breadth of 2 Bricks; then you muſt multiply your total Product by two, which is the Thickneſs of the Wall in the leaſt Denomination (to wit) two 4 Inches, or the Breadth of 2 Bricks, and divide that Product by 3, (becauſe there is the Breadth of 3 Bricks in a Brick and ½) and the Quotient is the Sum of the 1 Brickwork, reduced to 1 Brick and ½ Brickwork.

Example.

Example.

Suppose you have cast up several Dimensions of 1 Brick's Length in Thickness, and the Products (or Contents) of each Dimension being added together, produceth a total Product of 4976 Feet of Brick-work, being one Brick thick.

To reduce this to 1 Brick and ½ thick, multiply 4976 by 2 (to wit, the Breadth of 2 Bricks, and that brings it into the least Thickness or Denomination, namely, 4 Inch-work, or Brick-work ½ a Brick thick.)

	4976
	2
So the Product of 4 Inch-work is	9952

Which divide by 3 (because there are the Breadth of 3 Bricks in 1 Brick and ½) and in the Quotient you have 3317 Feet, of 1 Brick and ½ Work, and 1 remaining.

	21	
See the Division	9952	(3317
	3333	

The 1 that remains is 1 Foot of 4 Inch-work, or 4 Inches of 1 Brick and ½ Work being reduced.

For if you divide 12 (which are the Inches in that Foot) by 3 (the Number of 4 Inches or ½ Bricks in a Brick and ½) the Quotient will be 4, which is 4 Inches or ⅓ of a Foot.

So likewise your Dimensions that are thicker than 1 Brick and ½) as suppose 2 Bricks in Length, which

is called 2 Brick-work, which 2 Brick-work contains the Breadth of 4 Bricks, you muſt in reducing 2 Brickwork to 1 Brick and $\frac{1}{2}$ work, firſt multiply the Sum by 4, and divide the Product by 3.

Example.

If you have 5845 Feet of 2 Brickwork, multiply it by 4.

See the Multiplication	5845
	4
The Product is	23380 Feet

Of 4 Inch Work, which you muſt divide by 3.

	221 1
See the Diviſion.	23380(7793
	3333

And the Quotient is 7793 Feet of 1 Brick and $\frac{1}{2}$ Work, and 1 remaining, which is 1 Foot of 4 Inch Work, which being reduced, as you are taught in the foregoing Page, is 4 Inches (or $\frac{1}{3}$ of a Foot) of 1 Brick and $\frac{1}{2}$ Work. Thus in 5845 Feet of Brickwork being 2 Bricks thick (or the Walls being the Length of 2 Bricks in Thickneſs) you have, when it is reduced, 7793 Feet, and 4 Inches of 1 Brick and $\frac{1}{2}$ Work, (the uſual Thickneſs.)

If your Dimenſions be 2 Bricks and $\frac{1}{2}$ in Thickneſs, you muſt multiply the total Product by 5, and divide the Product of that Multiplication by 3.

If your Dimenſions be 3 Bricks in Thickneſs, you may ſave the Labour of dividing and multiplying, becauſe

becauſe a Wall of 3 Bricks thick is juſt ſo much more as a Wall of 1 Brick and ½ thick; therefore if you ſet down the Sum of your Product in 3 Bricks twice, it will be the ſame as if you ſhould multiply by 6, and divide the Product by 3, for this Reaſon, that 6 is as much more as 3.

In meaſuring of Brickwork you muſt ſet down your Dimenſions with the Number of their Thickneſſes over them, or adjoining to them; namely, over Dimenſions of 1 Brick in Thickneſs ſet 1 B. ſignifying 1 Brick; and over Dimenſions that are 1 Brick and ½ in Thickneſs, write ½ B, and over Dimenſions 2 Bricks in Thickneſs, write 2 B, *&c.*

And caſt up your Dimenſions as you are taught in meaſuring of Carpenters Work and Glaziers Work.

But in Brickwork as well as in Carpenters Work, it is uſual to meaſure no nearer than whole Inches, that is to ſay, ½ Inches and ¼ of Inches, nor ¾ of Inches are not ſet down.

For which Parts of Inches an Allowance is made when you come to divide your Number of Feet to bring them into Rods.

To bring or reduce Feet into Rods.

After you have reduced all your ſeveral Thickneſſes, and brought them to the uſual Thickneſs of a Brick and ½, you muſt divide your whole Product by 272, (being the Number of Feet contained in a Rod) to bring or reduce the Feet into Rods.

Taking no notice of the 3 Inches belonging to the 272 Feet, being both contained in a Rod.

For you read before, that in a Rod is contained 272 Feet, and 3 Inches, which 3 Inches is allowed to

make good for the Parts of Inches, which is not ſet down when you ſet down the Dimenſions.

But if you are minded to be exact and curious in meaſuring, you may ſet down the odd Parts of Inches, and caſt them up as you were taught in meaſuring of Glazing, and divide the total Product by 272 Feet and 3 Inches, but this is ſeldom or never done in Brick-work.

Folded between Page 240, 241, you have deſigned the Ground-plat of a Building; which together with the Inſtructions that follow, will much aſſiſt you in making an Eſtimate for Building from a Deſign given.

It will alſo inſtruct you how to take your Dimenſions, and to ſet them down in your Book, better than by a Multitude of Words.

The Explanation of the Deſign.

This Deſign is the Ground-plat of a Building, being 25 Feet in the Front, and 40 Feet in the Flank or Depth.

The Front and Rear-Front Walls are 2 Bricks and ½ thick.

The Flank Walls are 2 Bricks thick, as you may perceive by the Scale annexed to the Deſign.

You may ſuppoſe this Deſign to be the Ground Floor, having no Cellar beneath it.

And the Story to be 11 Feet and 6 Inches in Height from the Foundation to the Top of the next Floor.

The Windows are 4 Feet in Breadth, and 6 Feet 6 Inches in Height or Length.

The Door-caſes in Front and Rear are 4 Feet wide, and 9 Feet in (Length or) Height.

The

The Chimneys, you ſee, ſtand back to back, the Jaums being 2 Bricks thick.

The Encloſure of the Stairs, and the Partition next the Entry, are of Timber.

What hath been ſaid is ſufficient for Explanation.

I ſhall, in the next place, inſtruct you how to meaſure the Brickwork of this Story in the Ground-plat, which being well underſtood, you may with eaſe meaſure all the reſt of the Stories over it.

I ſhall begin with the Front.

And becauſe all Walls are (or ſhould be) made with a Baſis, therefore we will ſuppoſe the Front Wall and Rear Front-Wall to be 3 Bricks thick, 6 Inches high, which is 2 Courſes of Bricks; for 3 Inches is generally allow'd for a Courſe (or the Thickneſs of a Brick laid in Mortar.

So the firſt Dimenſion will be 25 Feet in Length, by 6 Inches in Height, 3 Bricks thick.

The Thickneſſes muſt be always ſet over the Dimenſions (as you will ſee in the next Page.)

And becauſe the Front and Rear are both of one Length, you need not ſet down the Dimenſion twice, but ſet a Figure 2 encloſed in a Circle thus (2) which ſignifies the Dimenſion to be 2 times.

I think it will be beſt for Inſtruction, to caſt up the Dimenſions as they are taken, (or ſet down) but after you underſtand the Way, you muſt ſet down all your Dimenſions firſt, and caſt them up afterwards.

I ſhall alſo ſet the ſeveral Thickneſſes of the Dimenſions (of this Story of the Ground-plat) in one Column, becauſe there will be but a few Dimenſions in this Story, and not above a Dimenſion or two of one Thickneſs, and it is not worth while to ſet down a Dimenſion or two in a Column by it ſelf.

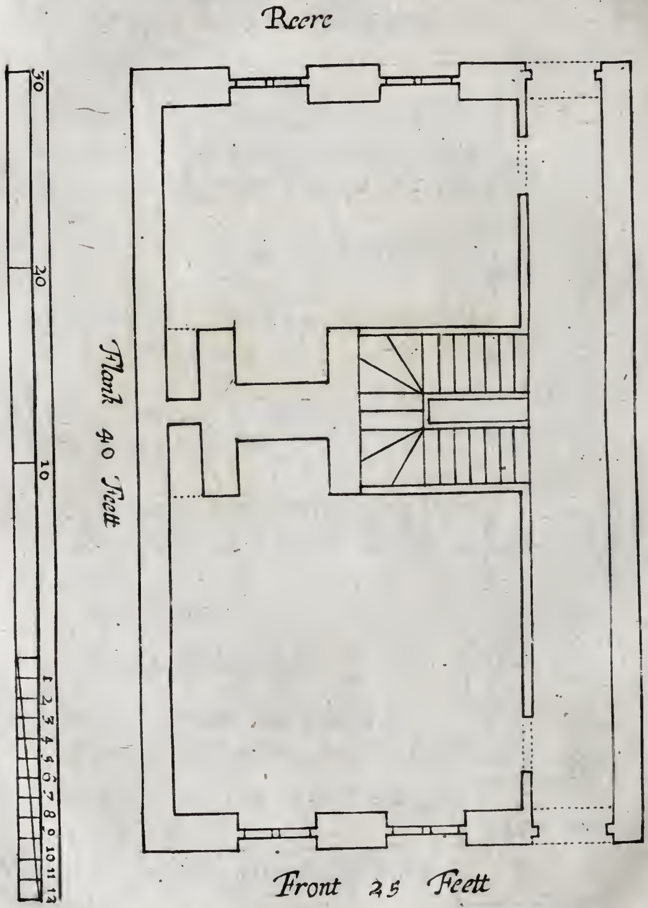
R 4 *Example.*

Example.

	3 B. Feet.	In.	
The Length of the Front is	25	00	(2)
The Height of the Brickwork is 2 Courses, or	00	06	
The Product is	12	06	
The Rear Front being the same Dimension, and being joined in one Dimension by the Figure (2) you must add	12	06	
Which being added, the Content of the 3 Brickwork is	25	00	
And because 3 Bricks is the Double of 1 ½ B. add	25	00	
	1 ½ B.		
And the Content of this first Dimension being reduced into 1 ½ B. is	50	00	
	2 ½ B.		
2*dly*. I proceed to the remaining Part of the Front and Rear, being in Height	11	00	(2)
In Length	25	00	
Being multiplied	55		
	22		
The Content of the Front is	275	00	
The same being added for the Rear	275	00	
The Content of 2 ½ B. work in Front and Rear is	550	00	
Which to reduce, you must multiply by	5		
The Prod. of 4 Inch. or ½ B. work is	2750		

Which

Betwen Pag. 240 & 241.



Which muſt be divided by 3, to reduce it to 1 $\frac{1}{2}$ B. in Thickneſs.

$$\begin{array}{r} 22 \\ \cancel{2}\cancel{7}\cancel{5}\cancel{0}(916 \\ \cancel{3}\cancel{3}\cancel{3} \end{array}$$

And the Quotient is 916 and 2 remaining, which is 916 Feet of 1 $\frac{1}{2}$ B. and 2 Feet of $\frac{1}{2}$ B. which is 8 Inches of 1 $\frac{1}{2}$ B. being reduced.

Having done with the Fronts, we proceed to take the Dimenſions of the Flank-Walls.

And becauſe we meaſured the Fronts from Out to Out, as we call it in brief, (to wit, from the Outſide of one Flank-wall, to the Outſide of the other) we muſt meaſure the Length of the Flank-walls within (to wit, from the Inſide of the Front-wall, to the Inſide of the Rear Front-wall) ſo the Thickneſs of the two Front-walls, which is 3 Feet and 10 Inches, being taken out of the 40 Feet in Depth,

	$\frac{1}{2}$ B.	
	Feet.	Inches
The Length of one Flank-wall is	36	02 } (2)
The Height of the Baſe is	00	06 }
And becauſe both Walls are alike, or equal in Length and Height, you muſt put (2) againſt the Dimenſion, and it ſerves for both Walls	18 18	01 01
The Cont. in 2 $\frac{1}{2}$ B. work of both Walls is	36	02
Being multiplied by	5	
The Product of $\frac{1}{2}$ B. work is	180	10
Being divided by 3 to reduce to $\frac{1}{2}$ B.	$\cancel{1}\cancel{8}\cancel{0}(60$ $\cancel{3}\cancel{3}$	1 $\cancel{1}\cancel{0}(3$ $\cancel{3}$

It is 60 Feet, 3 Inches and $\frac{1}{2}$ of an Inch of 1 $\frac{1}{2}$. B. Work.

The

	2 B.	
The next Dimension of the Flank-wall is	36 02 11 00	(2)
	36	
Being multiplied	361 10	
The Cont. in 2 B. work of one Wall is	397 10	
To which add the other Wall	397 10	
The Cont. in 2 B. work of both Walls is	795 08	
Which multiply by	04	
The Con. of both Walls in $\frac{1}{2}$ B. work	3182 08	
Being reduced to 1 $\frac{1}{2}$ B. the Content of both Walls are 1060 Feet and 10 Inches.	3182(1060 3333	

Although I cast up and reduce each Dimension as I proceed, yet after you understand the Way, you must reduce all your Dimensions of one Thickness at one time.

Having cast up and reduced both the Fronts, and both the Flank-walls, the next Dimensions to be taken are the Chimneys: Which are usually agreed for, at so much *per* Fire-place (or Chimney). Also sometimes they are done by the Rod, at the Rate of the other Work, and then they must be measured.

These two Chimneys in the Design, standing Back to Back, are 5 Feet a-piece between the Jaums.

The Jaums are 2 Bricks thick.

The Wall between the Chimneys is 14 Inches thick; to which is added 9 Inches, for a falling Back to each Chimney. There-

Therefore we will take the Wall first by it self, which is 9 Feet 10 Inches in Length, and 11 Feet 6 Inches, the aforesaid Height of the Story, and set it down.

	$1\frac{1}{2}$ B.	
The Heighth	11	06
The Length	09	10
	99	
	4	06
	9	02
		05
The Cont. of the Wall between the Chim.	113	01

Altho' heretofore I have put Feet or F. over the place of Feet; and In. or I. over the Place of Inches. Supposing now that you know one Place from another, I shall desist doing it, as in the last Dimension; and ouly set the Thicknesses over the Dimensions.

In the next place, suppose the Mantle-tree to lie 5 Feet high, and the falling back to begin at 3 Feet high.

The Mean between 5 and 3 is 4, which is the Height of the 9 Inch Work that is added to each Chimney for falling back, and the Length is 5 Feet.

	1 B.		(2)
The Dimension of the falling back of both Chimneys.	05	00	
	04	00	
	20	00	
	20	00	
The Content in 1 B.	40	00	

Multi-

	1 B.	
	40	00
Multiply it by	2	
The Product is	80	

	22	
Which divide by 3	~~8~~~~0~~	(26
	~~3~~~~3~~	

Having reduced it, you find 26 Feet and 8 Inches of 1 ½ B. work in both fallings back.

The 4 Jaums being 3 Feet 6 Inches deep apiece, above the falling back; being added together, make 14 Feet of 2 B. work, which muſt be multiplied by the Height of the Story.

For although the Jaums in the next Story, be but 1 ½ B. and from the Mantle-tree in this Story it be wrought but a Brick and half, yet the Wings being added, makes two Bricks.

	2 B.	
The Breadth of the 4 Jaums being added together is	14	00
The Height of the Story is	11	06
	14	
	147	
The Cont. of the 4 Jaums in 2 B. work is	161	00
Being multiplied by	4	
The Cont. of the 4 Jaums in ½ B. work is	644	

	12
Which divide by 3 in the Margin	~~6~~~~4~~~~4~~(214
	~~3~~~~3~~~~3~~

And

And the Content of the Jaums being reduced to 1 $\frac{1}{2}$ B. is 214 Feet, and $\frac{2}{3}$ of a Foot, or 8 Inches.

The 2 Breaſts of the Chimneys are next to be meaſured, which are 5 Feet apiece between the Jaums; and altho' they are but 9 Inches thick apiece, yet conſidering the croſs Withs, and the Peers that are wrought from the falling back till the Wings gather to them, they are uſually meaſured at the ſame Thickneſs that the Jaums are, and at the Height of the Story, abating nothing for the Vacancy between the Floor and the Mantle-tree.

	2 B.		
The Length of the Breaſt between the Jaums is	05	00	(2)
The Height of the Story is	11	06	
	05		
	52	06	
	57	06	
The Dimenſion being (2) add	57	06	
The Content of the 2 B. work in both Breaſts is	115	00	
Which multiply by	4		
Being brought into the leaſt Denomination, or $\frac{1}{2}$ B. it is	460	00	

$$\begin{array}{l} \cancel{1}\,1\,1 \\ \cancel{4}\cancel{6}\cancel{0}\,(153 \\ \cancel{3}\cancel{3}\cancel{3} \end{array}$$

Which divide by 3

Being reduced, the Content is 153 Feet and $\frac{1}{3}$ of a Foot (or 4 Inches) of 1 $\frac{1}{2}$ B.

Thus having taken all the Dimenſions in one Story, and caſt them up,

The

T

The next Thing to be done, is to take the Deductions of the Windows and Doors in the Fronts.

Note, That the Deductions (to wit, Windows and Doors, and all Vacancies whatſoever that you meaſure) muſt be ſet in Columns by themſelves, with this Note *Ded.* (ſignifying Deductions) over them, together with the Thickneſs of the Wall wherein the Windows or Doors ſtand that you are to deduct.

Example.

	Ded.	2 ½ B.	
The Height or Length of a Window is	06	06	(4)
The Breadth is	04	00	

And becauſe there are 4 Windows in both Fronts, and all of one Bigneſs; therefore put (4) againſt the Dimenſion, ſignifying that *that* Dimenſion is 4 times (or that there are 4 Windows of that Bigneſs.)

	Ded.	2 ½ B.	
The Height of a Window	06	06	(4)
The Breadth	04	00	
	24	00	
Being multiplied	2		
The Prod. of 2 ½ B. work in 1 Win. is	26	00	
Which multiply by	4		and
the Prod. of 2 ½ B. work in the 4 Win. is	104		
Which multiply by	5		
To bring it into the leaſt Denomination, and it is	520		
Which divide by 3; and	2 1 1		
Being reduced to 1 ½ B. there	5 2 0 (173		
is 173 Feet and 4 Inches.	3 3 3		

There

There are alſo in Front and Rear 2 Door-caſes to be deducted (deduct ſignifies to take out) of the ſame Thickneſs of 2 $\frac{1}{2}$ B.

	Ded.	2 $\frac{1}{2}$	
The Height of a Door is	09	00	(2)
The Breadth	04	00	
Being multiplied	36		
	2		
The Product of 2 $\frac{1}{2}$ B. work in both Doors is	72		
Which multiply by	5		
Being brought into $\frac{1}{2}$ B. work is	360		

Which divide by 3, as in the Margin ~~360~~ (120
~~333~~
being reduced to 1 $\frac{1}{2}$ B. there is 120 Feet, as in the Margin.

Note, Tho' I reduce the Deductions here for Inſtruction; hereafter you need not reduce them, but take the Product of the Deductions of one Thickneſs out of the Product of Dimenſions of the ſame Thickneſs.

In the following Page I will ſet down all the Dimenſions as they were taken, with the Product of each Dimenſion in a Column juſt againſt it.

And although I ſaid before, that you might divide a Page or Leaf of your Meaſuring Book into 4 Parts or Columns; yet in Meaſuring of Bricklayers Work, it will be neceſſary to divide a Page only in two Parts, (as you ſee in the next following Page) that ſo you may have room to ſet a Name to each Dimenſion for Diſtinction-ſake.

Dimen-

Dimensions of the Design.			*Products.*
	3 B.		3 B.
Basis of the Front and Rear	25 00 06	(2)	25 00
	2 ½ B.		2 ½ B.
Front and Rear	25 00 11 00	(2)	550 00
	2 ½ B.		2 ½ B.
Basis of both the Flank Walls	36 02 06	(2)	36 02
	2 B.		2 B.
Both the Flank Walls	36 02 11 00	(2)	795 08
	½ B.		1 ½ B.
The Wall between the Chimney	11 06 09 10		112 01
	1 B.		1 B.
The falling back of both Chimneys	05 00 04 00	(2)	040 00
	2 B.		2 B.
The 4 Jaums	14 00 11 06		161 00
	2 B.		2 B.
The Forepart or Breasts of both Chimneys	11 06 05 00	(2)	115 00

Having set down all the Dimensions with their Products, in the next place we must set down the Deductions of the Windows and Doors with their Products.

Deductions

Deductions in the Design.	*Ded.*			*Products.*	
	$2\frac{1}{2}$ B.				
The 4 Windows	06	06	(4)	$2\frac{1}{2}$ B.	
	04	00		104	0
	$2\frac{1}{2}$ B.				
The 2 Doors	09	00	(2)	$2\frac{1}{2}$ B.	
	04	00		072	0

The next Work is to add the Products of each ſeveral Thickneſs into one Sum.

Products of ſeveral Thickneſſes.

3 B.		$2\frac{1}{2}$ B.		2 B.		$1\frac{1}{2}$ B.		1 B.	
25	00	550	00	795	08	113	1	40	0
		36	02	161	00				
		586	02	115	00				
				1071	00				

The ſeveral Products of each Thickneſs being added,

In the firſt Column of the left Hand, you have 25 Feet of 3 B.

In the ſecond Column you have 586. 2. of $2\frac{1}{2}$ B.

In the third Column you have 1071. 8. of 2 B.

In the fourth Column you have 113. 1. of $1\frac{1}{2}$ B.

In the fifth Column you have 40 Feet of 1 B.

The next Work muſt be to take the Products of the Deductions out of the Products of the Dimenſions.

	Products of Ded. in 2 ½ B.	
	104	0
	72	0
The total Product of Ded. in 2 ½ B. is	176	0

Which 176 Feet of 2 ½ B. work being contained in the Windows and Doors, muſt be ſubtracted from 586 Feet and 2 Inches, being the total Product of all the Dimenſions that are 2 ½ B. in Thickneſs.

For this Reaſon.

Becauſe when we meaſured the Front and Rear, we meaſured the whole Length and Breadth over the Windows and Doors, allowing no Abatement for them.

Note, That whatſoever Windows or Doors, or other Vacancies you meaſure over when you take the Dimenſions, you muſt remember to deduct them out of the total Product of the Dimenſions of the ſame Thickneſs wherein they are ſituate.

Example.

The Doors and Windows being in 2 ½ B. work; I ſet down the total Product of all the Dimenſions of that Thickneſs, which is	586 02
The total Product of all the Deductions of that Thickneſs which are to be ſubtracted is	176 00
The Remainder is	410 02

So likewiſe if there had been Deductions in the other Thickneſſes, you muſt have ſubtracted them before you begin to reduce your ſeveral Thickneſſes to the uſual Thickneſs of 1 ½ B.

But ſeeing we have no other Ded. in this Deſign, to ſubtract out of any other Thickneſs, the next Work will be to reduce each Thickneſs to the uſual Thickneſs of 1 ½ B.

Beginning with the greateſt Thickneſs (to wit 3 B.) we find 25 Feet, which, as you read before, becauſe it is juſt as thick again as 1 ½ B. you need not multiply it by 6, and divide

divide it by 3, but add just so much more to the Dimension as it is (to wit 25 to 25) and so you will have 50 Feet, which must be set down in a Column with 1 $\frac{1}{2}$ B. over it.

Products reduced.
1 $\frac{1}{2}$ B.
0050 0

The next Thickness to be reduced is 2 $\frac{1}{2}$ B. the total Product whereof is (the Deductions being taken out) 410 Feet and 2 Inches, which multiply by 5, and divide the Product by 3.

Example.

	2 $\frac{1}{2}$ B.		211
	410	2	2050 (683
		5	333
$\frac{1}{2}$ B. Work	2050	10	

The 1 that remains in Division is 4 Inches, and the 10 Inch. in the $\frac{1}{2}$ B. work is 3 Inches of 1 $\frac{1}{2}$ B. work, which being added is 7 Inches; so the 2 $\frac{1}{2}$ B. work being reduced is 683. 7. which must be set in the Column. — 0683 7

The next Thickness to be reduced is 2 B.
2 B, the total Product whereof is 1071 8
Which multiply by 4
And the 4 Inch (or $\frac{1}{2}$ B.) work is 4286 8

The 2 remaining in Division is 8 Inches, and the 8 Inches of $\frac{1}{2}$ B. is 2 Inches of $\frac{1}{2}$ B. which added is 10 Inches, so the 2 B. work being reduced is — 1428 10

1022
4286 (1428
3333

The next Thickness being 1 $\frac{1}{2}$ B. needs no reducing, because it is the usual Thickness, the Product whereof is — 0113 1

The next Thickness to be reduced is 1 B.
1 B. the total Product whereof is 40
Which multiply by 2
And the Product of $\frac{1}{2}$ B. work is 80

22
80 (26
33

Which divide by 3, and the Quotient is 26 Feet and 8 Inches, being reduced; — 0026 8

The Product of all the Thicknesses being reduced is — 2302 2

Having

Having reduced all the Thickneſſes to 1 $\frac{1}{2}$ B. in Thickneſs, you find the total Product to be 2302 Feet and 2 Inches.

Which 2302 muſt be divided by 272, the Number of Feet contained in a Rod, as you read before.

Example.

126
2302(8
272

Being divided, the Quotient is 8, and the Remainder is 126, which is 8 Rods and 126 Feet and 2 Inches (if you add the two Inches that belonged to the 2302 Feet which you divided.)

Note, The whole Rod containing 272 Feet, the half Rod contains 136 Feet, and the Quarter of a Rod contains 68 Feet; for twice 136 is 272, and four times 68 is the ſame.

Then take 68 Feet, or $\frac{1}{4}$ of a Rod, out of the 126 Feet that remain, by ſubtracting 68 from

126
68

And the Remainder is 58 Feet.

So there is 8 Rods and $\frac{1}{4}$ and 58 Feet of 1 $\frac{1}{2}$ B. work in one Story of the Deſign.

It is convenient, before I proceed farther, to add ſomething more concerning meaſuring of Chimneys.

Firſt *Note*, If you are to meaſure a Chimney ſtanding alone or by it ſelf; without any Party-wall being adjoyned;

Then

Then girt it about for the Length, and the Height of the Story is the Breadth.

The Thickneſs muſt be the ſame that your Jaums be, provided the Chimney be wrought upright from the Mantle-tree to the Ceiling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, becauſe of the Gatherings of the Breaſt and Wings to make room for the Hearth in the next Story.

Secondly Note, If the Chimney-back be a Party-wall, and the Wall be meaſured by it ſelf, then you muſt meaſure the Depth of the 2 Jaums, and the Length of the Breaſt, which being added together, will be your Length, and the Height of the Story your Breadth, at the ſame Thickneſs your Jaums be.

Thirdly Note, When Chimneys are agreed for at a Rate by the Fire-place or Chimney-building, and the Back of the Chimneys ſtand againſt a Wall (that you meaſure beſides) you muſt deduct 4 Inches in Thickneſs out of the Menſuration of the Wall againſt the Chimneys, which 4 Inches is allowed for the Backs of the Chimneys.

Fourthly Note, When you meaſure Shafts of Chimneys, girt them with a Line round about the leaſt Place of them for the Length, and their Height ſhall be your Breadth; and if they be 4 Inch-work, then you muſt ſet down your Thickneſs at 1 B. work: But if they be wrought 9 Inches thick (as ſometimes they are when they ſtand high and alone above the Roof) then you muſt account your Thickneſs 1 $\frac{1}{2}$ B. in Conſideration of Withs and Pargetting, and Trouble in Scaffolding.

Observe these following Rules, which will much help you in casting up your Dimensions of any Kind, and save you the Labour of Division, to know how many Feet there is in any Number of Inches that you multiply into Feet.

Let the first Example be

30	02
18	03
240	
303	
7	06
550	06 ½

First, Having multiplied the whole Feet, in the next place you multiply the Inches into the Feet, saying 2 times 18 is 36 Inches; or thus, 18 Inches being a Foot and an half, say, 2 Feet and 2 half Feet are 3 Feet, which being set down:

In the next Place you come to multiply 3 into 30, which you need not multiply; for 3 being one fourth Part of 12, say, The one ¼ of 30 Feet is 7 Feet and 6 Inches; which is a quicker Way than to multiply 30 by 3, and divide the Product by 12.

Let the second Example be

25	04
42	05
50	
100	
14	
10	5
	1
1074	6

The Feet being multiplied and set down, next you are to multiply 4 by 42; but 4 being ⅓ Part of 12, say, the one Third of 4 is 1, and the one Third of 12 is 4, which is (being added) 14 Feet.

In the next Place you come to multiply 5 25, which you may say is 25 half Feet lacking 25 Inches, that is, 10 Feet and 5 Inches.

Or

Or thus, 24 Inches being 2 Feet, ſay 5 times 24 Inches is 10 Feet, then add the 5 Inches, it makes 10 Feet and 5 Inches. Laſtly, 5 by 4 is 20, which is 1 Inch and 8 Parts, which 8 Parts is not accounted, nor ſet down, (as you have read before.)

100	7
94	6
400	
900	
50	
47	
7	10
	3
9505	1

Let the third Example be

The Feet being multiplied, you proceed to multiply 6 by 100, but 6 being the half of 12, ſay the half of 100 is 50 Feet, which being writ down, you proceed to multiply 7 into 94, then ſay the half of 94 is 47 Feet, which being ſet down, there is ſtill remaining 94 Inches, which you may ask your ſelf how many times 12 you can have in 94? Your Anſwer will be 7 times, and 10 Inches remaining, therefore you muſt ſet down 7 Feet and 10 Inches. Laſtly, you proceed to multiply the Inches by the Inches, ſaying 7 times 6 is 42, which is 3 Inches and $\frac{1}{2}$; ſo the total Product is 9505 Feet and 1 Inch, as you ſee in the Margin above.

230	8
310	9
2300	
690	
206	8
115	
57	6
	6
71679	8

Let the fourth Example be

The Feet being multiplied and ſet down, you proceed to multiply 310 by 8; but inſtead of multiplying, take the $\frac{2}{3}$ of 310, becauſe 8 is $\frac{2}{3}$ of 12; ſaying thus, the $\frac{2}{3}$ of 3 is 2, but the $\frac{2}{3}$ of 1 you cannot take in Integers, therefore you add a Cypher next to the Figure 2, and proceed to take the $\frac{2}{3}$ of 10, which is 6 Feet and 8 Inches; ſo you find the

the $\frac{2}{3}$ of 310 Feet to be 206 Feet and 8 Inches; which being ſet down,

The next is 230 Feet to be multiplied by 9 Inches; which 9 Inches is $\frac{3}{4}$ of 12 Inches; therefore you muſt take $\frac{3}{4}$ of 230 Feet, ſaying, the half of 230 is 115, and $\frac{1}{2}$ of 115 is 57 Feet and 6 Inches, which being added to 115 make 172 Feet and 6 Inches, as you ſee by caſting up the Dimenſion,

Let the fifth Example be

The Feet being multiplied and ſet down, inſtead of multiplying 10 by 94, you muſt divide 10 into 2 Parts (to wit) 6 and 4, 6 being $\frac{1}{2}$, and 4 being $\frac{1}{3}$ of 12, and then ſay, the $\frac{1}{2}$ of 94 is 47, and the $\frac{1}{3}$ of 94 is 31 Feet 4 Inches, which being ſet down, you proceed to know the Product of 48 Feet multiplied by 11 Inches, which to do ſay thus, It is 48 Feet lacking 48 Inches, or 4 Feet, therefore I ſet down 44 Feet; behold the Example or Dimenſion caſt up in the Margin above.

94	11
48	10
752	
376	
47	
31	4
44	9
4635	1

Thus I have ſhewn you from 1 Inch to 12 Inches (beginning at 2 Inches, and ending at 11 Inches) how to work them into Feet, to ſave Multiplication and Diviſion; which way, after you are expert in it, will ſave you ſome Trouble and Time in caſting up many Dimenſions.

Some

Some Things to be obſerved in meaſuring of Brickwork.

Sometimes Walls are wrought 2 Inches thicker than any of theſe Thickneſſes before ſpoken of.

Which 2 Inches ſerve for a Water-Table to the Wall; which is uſually ſet off, about 2 Feet above the Ground; therefore you may meaſure the Wall at the ſame Thickneſs that it is above the Water-Table, and add the 2 Inch-work to it thus:

Suppoſe a Wall 20 Feet in Length, and 2 B. thick above the Water-Table;

After you have taken the Dimenſion of the Wall from the Bottom to the Height that you are to take it, at 2 B. thick, then add 20 Feet in Length by the Height of the 2 Inch-work (to wit) from the Bottom to the ſetting off, or Water-Table, which being halfed is ſo much 4 Inch-work, and then reduce it to 1 $\frac{1}{2}$ B. work.

Secondly Note, That all kinds of ornamental Work in Brick, are generally done at a Rate by the Foot, except there be an Allowance of a Sum of Money over and above the Rate by the Rod, or except a good Rate is allowed by the Rod, and all the ornamental Work to be meaſured into the Rodwork.

By ornamental Work you are to underſtand ſtrait or circular Arches over Windows or Doors, Facio's with or without Moldings, Architrave round the Windows, or rubbed Returns, Friezes and Cornices, Ruſtick Quines, *&c.* In fine, all kind of Work that is hewed with an Ax, or rubbed upon

a

a Stone, is ornamental Work, and ought to be paid for besides the Rod-Work.

Thirdly Note, When you meaſure Arches, either ſtrait or circular, you muſt meaſure them in the Middle (that is to ſay) if a ſtrait Arch be 12 Inches in Height or Depth, you muſt meaſure the Length of it in the Middle of the 12 Inches, which Length will be longer than if you meaſure it on the under Side next the Head of the Window, by ſo much as one Side of the ſpringing of the Arch is skewed back from the Upright of the Jaum.

Alſo in circular Arches, obſerve that the upper Part of the Arch is more in Length (being girt about) than the under Part, becauſe it is a Segment of a greater Circle, cut off by the ſame right Line that the leſſer is, therefore it muſt be girt in the Middle.

Fourthly Note, That Facio's, Architrave Friezes, Cornices, Baſe Moldings and Plinths, are meaſured by the Foot, Line-meaſure (or Running-meaſure, as it is commonly called.)

Fifthly Note, You muſt take notice of the Jaums of Windows and Doors being ſplayed on the Inſide of Buildings, at the Rate by the Foot Running-meaſure.

Sixthly Note, Meaſuring of Tyling is the ſame as meaſuring of Roofing, which is taught at the beginning of this Tract in meaſuring of Carpenters Work; only there is this Difference, you muſt meaſure the Length of all the Vallies, and add ſo many Feet as they are in Length, to the Number of Feet in the Meaſurement of the Roof; you muſt alſo add the Eyes and Barge-courſes, Tyling being done by the Square like Roofing.

You

You may alſo meaſure Tyling by the Flat and half of the Building, after the ſame Manner that you are ſhewed to meaſure Roofing (in meaſuring of Carpenters Work) adding the Barge-courſes and Eves; alſo there you may ſee how to meaſure Hipt Roofs.

Seventhly Note, That Brick-paving is done by the Yard, and is brought into Yards by dividing the Product of the Feet by 9, as you have read in meaſuring of Painting, Plaiſtering, *&c.*

Eighthly Note, The meaſuring of Gable-ends in Brickwork, is the ſame as in the meaſuring of Carpenters Work, wherein you are taught to meaſure them.

Ninthly, Remember in the meaſuring of Walls joining to each other anglewiſe, that you take the Length of one Wall to the Outſide of the Angle, and the Length of the other to the Inſide of the Angle.

Tenthly, If you ſhould have a Gable-end, to meaſure, and you know the Width of the Houſe, or the Length of the Baſe Line of the Gable-end, and you deſire to know the Length of the Perpendicular, you may find it by the Proportions of the Table that is in the meaſuring of Carpenter's Work, and the Rule of Three.

Or briefer thus, Suppoſe the Width of the Gable-end to be 20 Feet, and you would know the Length of the Perpendicular; take the Length of the Rafter which is 15 Feet, being $\frac{3}{4}$ of the Width (for if the Roof be true Pitch, $\frac{3}{4}$ of the Breadth is the Length of the Rafter) to which add half the Length of the Rafter (*viz.*) 7 Feet and $\frac{1}{2}$, the Product is 22 Feet and $\frac{1}{2}$; the half whereof (to wit) 11 Feet and 3 Inches is the Perpendicular.

Although

Although this is a Way commonly uſed, yet it is a ſmall Matter more than the Length of the Perpendicular, as by the former Rule will appear.

Thus much may ſuffice a diligent and practical Reader concerning Meaſuring, and of which you may ſee more in the *Appendix*.

As to the Meaſuring of Vaults and Arches, the following Treatiſes, intituled, *The Meaſuring of Plains* and *Bodies*, will inſtruct you, as alſo the *Appendix*.

And becauſe Brickwork is done at ſeveral Rates *per* Rod, and many times there happens a Number of Feet to remain, after you have divided your whole Product by 272.25 (to bring your Number of Feet into Rods) it will be convenient to ſhew how to find the Price of 1 Foot of Brickwork, or of any Number of Feet, at any Rate *per* Rod.

Having the Price per *Rod given, and the Number of Feet that you deſire to know what they come to, work thus:*

Firſt, Reduce the Price of a Rod into Shillings, by multiplying the Pounds by 20 (the Number of Shillings in one Pound) and if the Price of a Rod be Pounds and Shillings, then add the Shillings to the Product of the Pounds, being multiplied by 20, ſo you will have the Price of a Rod in Shillings: Next multiply the Number of Shillings by 12 (the Pence in one Shilling) and that brings the Price of a Rod into Pence. Then work according to the Rule of Three, ſaying,

Example

Example 1.

If I have 5 *l.* for a Rod of Brickwork, what must I have for 34 Feet, according to that Rate?

State the Question thus.

If a Rod, which is 272 Feet and $\frac{1}{4}$, or 272 $\frac{25}{100}$, give 5 *l.* what will 34 Feet give?

```
                              l.    s.
Multiply                      5     00
                         by         20
                                ---------
The Product is                     100 Shillings
Which multiply by                   12 Pence
                                ---------
And the Product is                1200 Pence
Which multiply by                   34 Feet
                                ---------
                                  4800
                                 3600
                                ---------
And the Product is               40800
                                ---------
```

Which divide by 272.25, adding 4 Cyphers to the Product or Dividend.

```
        17
       2340
      268580
    1357500 0        d.
   408000000 0  (149. 86/100
   272255555
    27222222
     27222
      277
       2
```

And the Quotient is 149 *d.* and $\frac{86}{100}$ of a Penny, being more than 3 *q.* for the 34 Feet, at 5 *l.* per Rod.

Which

Which 149 Pence divide by 12, and the Quotient is 12 *s.* and $\frac{5}{12}$, or 5 Pence 3 Farthings, the Rate of the 34 Feet desired.

2	(5 *s.*	*d.*	*q.*
~~149~~	(12	5	3
~~122~~			
~~1~~			

Example 2.

If I have 5 *l.* 5 *s.* for a Rod, what comes 20 Feet to at that Rate?

State the Question thus, and it saves you the Labour of multiplying by 20.

	F.	S.	
If	272.25. give	105	what will 20 Feet give?
Multiply by		12 Pence	
		210	
		105	
The Product is		1260	the Rate of a Rod in Pence,
which multiply by		20	the Number of Feet, And
the Product is		25200	: To this Product add 4

Cyphers, and divide by 272.25.

```
       1535
      69800
    2520000 (92  56
    2722255
      27222
        272
```

The Quotient is 92 *d.* and $\frac{56}{100}$ of a Penny, which 92 *d.* being divided by 12 *d.* gives you 7 *s.* 8 *d.* and the

the Fraction remaining namely $\frac{56}{100}$ is somewhat above 2 Farthings.

So at 5 *l.* 5 *s.* *per* Rod, 20 Feet comes to 7 *s.* 8 *d.* 2 *q.*

Example 3.

If I have 5 *l.* 5 *s.* for one Rod of Brickwork, what must I have for 1 Foot?

	l.
Multiply	05
By	20 Shillings
The Product is	100
Add the	5 Shillings
It is	105 Shillings
Which multiply by	12 Pence
	210
	105
The Price of a Rod is	1260 Pence

Which should be multiplied by the Third Number in the Question (to wit) 1 Foot; but because multiplying any Number by 1 doth not encrease it,

Therefore divide 1260 *d.* by 272.25 Feet.

220
7655
1710000 *d.*
12600000 ($\frac{62}{100}$ of a Penny, being more
2722555 than 2 *q.* and not 3 *q.*
27222
272

And the Quotient is 4 *d.* $\frac{62}{100}$ of a Penny.
Then to know the Value of the Fraction $\frac{62}{100}$,

Multiply

Multiply the Numerator 62 by 4 (the Number of Farthings in one Penny) the Product is 248, which divide by the Denominator 100, and the Quotient is 2 Farthings, and the Remainder $\frac{48}{100}$, of a Farthing; which Numerator 44, is leſs than half of the Denominator 100, and therefore leſs than half a Farthing.

Example.

The Numerator	62 of the Fraction
Multiply by	4 Farthings
The Product is	248, which divide

by the Denominator 100,

And the Quotient is 2 Farthings, and ſomething more. } 248 q. / ~~1~~ ~~0~~ ~~0~~ (2 $\frac{48}{100}$

Thus the Rate of 1 Foot of Brickwork after the Rate of 5 *l.* 5 *s.* *per* Rod, is found to be Fourpence half-penny half Farthing; which was required.

The ſame Rule holds in Tyling, or Carpenters Work, only changing the Number 272. 25. (the Feet in a Rod) and inſtead of it uſe 100 (the Number of Feet in a Square.)

The End of the FOURTH BOOK.

MEASURING

OF

Superficial PLAINS,

BY

Vulgar ARITHMETICK,

without reducing the Integers into the least Denomination.

TOGETHER WITH

Directions for Measuring

OF

LAND.

The FIFTH BOOK.

By *VEN. MANDEY.*

LONDON:

Printed in the YEAR M.DCC.XXVII.

MEASURING OF *Superficial* PLAINS.

BOOK V.

CHAP. I.

By measuring of Plains, is meant the finding the Area or Content of Superficial Figures, such as are Quadrates, Triangles, Circles, &c. and as a Line is made by the Motion of a Point, so likewise a Superficie is made by the Motion of a Line.

IN measuring of Plains, the first Figure to begin with (as being most easy) is a Quadrate or Square, which is a Figure inclosed with four equal Sides, making four right Angles.

The next is an Oblong or long Square, which is a Figure having four right Angles, and four Sides, but not equal or of one Length. Two of which Sides being opposite, are of one Length and Parallel; and the other

other two Sides being of another Length, are opposite and parallel.

The measuring of these two kinds of Plains, you have been already shewn in the preceding Tract of Measuring (to wit, by multiplying the Length by the Breadth.)

PROP. I. *Fig.* I.

To measure (or find the Content of) a Rhombus, being a Figure like a Quarry of Glass, having four equal Sides; and two Pair of equal Angles.

SUppose the Rhombus ABCD in *Fig.* I. one Side whereof is 5 Feet, and 4 Inches; take this for your Length.

Then for the Breadth take the nearest Distance between any two of the Sides, as suppose the Prick Line EF being 5 Feet, multiply these two, and the Product is 26 Feet, 8 Inches, which is the Area or Content of the Rhombus ABCD.

Otherwise thus,

Draw the Diagonal Line AC, which divides the Rhombus into two Triangles; next draw the Perpendicular Line DB.

Then take the Diagonal Line AC for your Length, and the half Length of the Perpendicular for your Breadth, and multiply them, and the Product is the Content.

See the Examples cast up both Ways.

	Feet.	In.
The Length of one Side AB (the other three being each the ſame Length) is	5	4
The neareſt Diſtance (or EF) is	5	0
	25	
	1	8
Being caſt up the firſt Way, the Content is	26	8

The Second Way.

	F.	In.	P.
The Length of the Diagonal Line AC is	9	2	0
The half Length of the Perpendicular is	2	10	11
	18	4	
	7	6	
		1	8
		8	3
			1
The ſame Content as before	26	8	0

The multiplying of the Parts of Inches into the Feet and Inches, you are taught in the foregoing Tract of Meaſuring of Glaziers Work.

PROP.

PROP. II. *Fig.* II.

To measure (or find the Content of) a Trapezium, or four-sided Figure, two sides whereof are parallel.

SUppose the Trapezium ABCD in *Fig.* II. whose longest Side AB is 9 Feet, 4 Inches $\frac{1}{2}$; the Side parallel to it is DC, being 7 Feet, 2 Inches and $\frac{10}{12}$ of an Inch; the Breadth being taken the nearest Way, is the prick Line whose Length is 6 Feet, and $\frac{1}{4}$ of an Inch.

Add the two parallel Sides together, whereof take half the Sum for your Length; then take the nearest Distance between those two Sides that are nearest together for your Breadth, and multiply them, the Product whereof is the Content.

Example.

	F.	In.	P.	
The longest Side AB, is	9	4	6	
The shortest parallel Side DC, is	7	2	10	
Those two being added, is	16	7	4	
The mean Length being one half, is	8	3	8	
This being mult. by the nearest Distance	6	0	3	
	48			Sec.
	1	6		
		4		
		2		9
				2
The Content required is	50	0		11

The

The 11 beyond the Place of Parts of Inches is $\frac{11}{12}$ Parts of $\frac{1}{12}$ Part of an Inch, or $\frac{11}{144}$ Parts of an Inch, which is not conſiderable, except the Superficies that you meaſure be ſome rich or coſtly thing.

Note, That I account, or ſuppoſe the Inch to be divided into 12 Parts, for two Reaſons: The firſt is, The more Parts it is divided into, the nearer to the Truth you may take the Length and Breadth of any Superficies or Solid, and likewiſe proceed the nearer to the true Content.

The ſecond Reaſon is, Becauſe the Parts of Inches thus divided, have the ſame Proportion to the Inches, that the Inches have to the Feet, and by that means theſe Parts may be multiplied into the Feet and Inches; which according as the Inch is divided on our common Rulers into 8 Parts, thoſe Parts cannot be multiplied into the Feet and Inches, but the Length, and likewiſe the Breadth of any Superficies muſt be reduced into the leaſt Denomination, and then multiplied each by other, which cauſeth a great deal of Multiplication, and alſo of Diviſion, before you arrive at the Surperficial Area or Content, as you may ſee in the preceding Tract at the beginning of Meaſuring of Glaziers Work, from Page 216, to Page 219, where I have proved this Way of multiplying the Parts of Inches into the Feet and Inches to be true, both by Decimal Arithmetick, and alſo by reducing the Length and Breadth into the leaſt Denomination (to wit, Quarters of Inches.)

And indeed (as Mr. *Oughtred* ſaith in his Circles of Proportion) the dividing of the Inches on the Rulers, into Halfs, and Quarters, and Half-quarters, is moſt inartificial; and I ſuppoſe was done before Decimal Arithmetick was brought to light. Which if the Foot

were

were divided into 10 Parts (which may as well be called Inches, as now it is divided into 12 Parts) and each of thoſe Parts into 10 Parts, and the Length and Breadth of Plains or Solids taken therewith, and ſet down Decimally, and caſt up by Decimal Arithmetick; the Science of Meaſuring would be much eaſier than now it is, and ſave a great deal of Labour.

Note, That a four-ſided Superficies which hath none of the Sides parallel, likewiſe every plain Figure of more Sides than four, being propoſed to be meaſured, muſt with Diagonal Lines be divided into Triangles.

And Note, That every ſuch Figure containeth ſo many Triangles as it hath Sides, abating two out of the Number (to wit, if the Figure to be divided into Triangles have ſix Sides, it will contain four Triangles, if ſeven Sides, five Triangles); then theſe Triangles are to be meaſured ſeverally, and their Contents being added, gives the Content of the irregular Figure.

PROP. III. *Fig.* III.

To meaſure any Triangle.

MUltiply the longeſt Side (being uſually called the Baſe) by half the Perpendicular (which is a Line let fall Perpendicular from the Angle, oppoſite to the Baſe) and the Product is the Content of the Triangle.

Suppoſe a Triangle A B C in *Fig.* III. whoſe Baſe AC is 7 Feet, the Side CB 6 Feet, and the Side BA 5 Feet, and the Perpendicular, which is the prick

Line 3 Feet, 2 Inches, the half whereof is 2 Feet, 1 Inch; which being multiplied by the Baſe Line 7 Feet, makes 14 Feet, 7 Inches, for the Content of that Triangle.

Example.

	F.	I.
The Length of the Baſe is	7	0
Half the Length of the Perpendicular is	2	1
The Content of the Triangle is	14	7

Otherwiſe thus:

Multiply the whole Perpendicular by half the Baſe.

Example.

	F.	I.
The whole Perpendicular is	4	2
Being multiplied by half the Baſe	3	6
	12	6
	2	1
The Content (as before) is	14	7

PROP. IV. *Fig.* IV.

The Side of an Equilateral Triangle being given, to find the Perpendicular Arithmetically.

LET the Side of the Triangle given be AC in *Fig.* IV. whoſe Length is 8 Feet.

First ſquare the Side, then ſquare half the Baſe, and

and ſubtract that from the Square of the Side, and the ſquare Root of the Remainder, is the Length of the Perpendicular.

Example.

	F.	I.
The given Side is A C	8	0
Which to ſquare, mnltiply by	8	0
The Square of the Side A C is	64	0
The whole Baſe is 8, the half whereof is	4	0
Which to ſquare, multiply by	4	0
The Square of half the Baſe is	16	0
Which ſubtract from	64	0
The Square of the given Side	16	0
and the Remainder is	48	0

The Square Root whereof is 6 Feet, 11 Inches, and $\frac{2}{12}$ Parts of an Inch *Fere*, being the Length of the Perpendicular required.

It is ſuppoſed that the Reader hereof can extract Square and Cube Roots; if he cannot, he may learn it in Mr. *Wingate*'s Arithmetick, in which Book they are plainly and briefly demonſtrated; or he may work by Logarithms.

PROP.

PROP. V. *Fig.* IV.

The Perpendicular and Side of an Equilateral Triangle being given, to find the Content of that Triangle.

IN the former Diagram, *Fig.* IV. the Perpendicular 6 Feet, 11 Inches $\frac{2}{12}$ *Fere*, and the Side 8 Feet being given, the Content is required.

Multiply the whole of either, by half the other, and the Product is the Content required.

Example.

	F.	I.	P.	
The whole Baſe or Side is	8	0	0	
The half of the Perpendicular is	3	5	7	*Fere*
	24			
	3	4		
		4	8	
The Content of the Triangle is	27	8	8	*Fere*

If you multiply the whole Perpendicular 6 Feet, 11 Inches, by half the Side 4 Feet, it produceth the ſame as before (to wit) 27 Feet, 8 Inches, $\frac{8}{12}$ of an Inch almoſt, for *Fere* ſignifies almoſt or near.

To find the Content of the ſame Equilateral Triangle, without knowing the Length of the Perpendicular.

Make it as 30 to the Square of one Side, ſo 13 to a fourth Number, which will be the *Area* requir'd.

Example.

64	22	F.
13	832(27 $\frac{2}{30}$ the ſame as before.	
192	330	
64		
832		

PROP.

PROP. VI. *Fig.* V.

The Perpendicular and Base of a right angled Triangle being given, the Content of that Triangle is required.

IN *Fig.* V. the Perpendicular FG, being 12 Feet, and the Base CE 25 Feet, is given, and the Content required.

Multiply the whole of either by half the other, as 12 Feet by 12 Feet, 6 Inches, or 25 Feet by 6, the Product is 150 Feet, the Superficial Content required.

Or multiply the whole by the whole, the Product is 300, whereof take half, it is 150 as before.

Or thus, without the Perpendicular.

Multiply the two Sides 15 by 20, the Product is 300, whereof take half for your Demand; or multiply half of one Side by the whole of the other Side, the Product is 150, as before.

These Rules hold good in those right-angled Triangles, whose longest Side (or Base) being squared, the Product is as much as the Square of both the other Sides, as in *Fig.* V. the Square of CE is 625, the Square of CF is 225, and the Square of FE is 400; which two last Squares being added, make 625, the Square of the Base; so likewise, in a right-angled Triangle, whose Sides are 6 Feet, 8 Feet, and 10 Feet, it holds the same Proportion; for the Square of 10, which is 100, is as much as the Square of 6, which is 36, and the Square of 8, which is 64, being added together.

PROP.

PROP. VII. *Fig.* VI.

The Perpendicular and Base of an Isosceles Triangle given, to find the Area or Superficial Content.

IN *Fig.* VI. add all the three Sides together, whereof take half, then take the Difference of each Side from that half, and multipiy those three Differences one by another, the Product whereof multiply by the half Sum of the three Sides, and the Square Root of that Product is the Superficial Content.

Example.

The 3 Sides being added, make 56, the half whereof is 28, then the Difference of the Side A B (being 20) from 28 is 8; the like Difference is between the Side B C and 28, namely 8; and the Difference between the Side AC (being 16) and 28, is 12. Now these 3 Differences being multiplied one by another, namely, 8 by 8 is 64, and 64 by 12, makes 768, which being multiplied by 28, the half Sum of the 3 Sides, the Product is 21504, whereof the Square Root is 146 Feet, 7 Inches, 8 Parts, and $\frac{9}{12}$ of a Part, or 146 Feet, 7 Inches, and $\frac{72}{144}$ of an Inch.

Which Rule is general for all right-lined Triangles whatsoever.

PROP.

PROP. VIII. *Fig.* VII.

To find the Perpendicular of any Triangle Arithmetically, the Sides being given.

IN *Fig.* VII. let the Sides given be ED 6 Feet, DF 8 Feet, and FE 10 Feet, it is required to find the Perpendicular?

Square the 3 Sides ſeverally, then add the Square of the Baſe (or longeſt Side) to the Square of one of the other two Sides, as to the Square of ED, from whence ſubtract the Square of the remaining Side, and from the Remainder take half, which divide by the Baſe, the Quotient thereof being ſquared, deduct or ſubtract it from the Square of the ſhorteſt Side, and the Square Root of the Remainder is the Length of the Perpendicular.

Example.

The Square of ED is 36, the Square of DF is 64, and the Square of FE is 100; then the Square of the Baſe 100 being added to the Square of ED 36, makes 136, from whence the Square of DF being ſubtracted, which is 64, the Remainder is 72, whereof the half is 36, which being divided by the Baſe 10, produceth 3 Feet $\frac{3}{5}$; or 3 Feet, 7 Inches, 2 Parts, and $\frac{4}{5}$ of a Part, for the leſſer Segment of the Baſe EG; the Square of which Segment is 12 Feet $\frac{24}{25}$, or 12 Feet, 11 Inches and 6 Parts, and being ſubtracted from the Square of ED 36, the Remainder is 23 Feet $\frac{1}{25}$, or 23 Feet and $\frac{5}{12}$ of an Inch +, whoſe Square Root is 4 Feet, 9 Inches, and

and 7 Parts +, is the Length of the Perpendicular DG required. But this being very troubleſome, I will give you a Table in the *Appendix*, which will ſhew the Length of ſeveral Perpendiculars.

PROP. IX. *Fig.* VIII.

The Side of a Pentagon being given, to find the Area or Content.

IN *Fig.* VIII. let the given Side be 10, it is required to find the Area of that Pentagon.

Deſcribe an Iſoſceles Triangle on any Side of the Pentagon, whoſe Top is the Centre of the Pentagon, as ADB, whoſe Perpendicular being found is 6 Feet, 10 Inches, and 5 Parts + of an Inch, divided into 12 Parts, which being multiplied in half the Perimetry 25, produces 171 Feet, 8 Inches, and 5 Parts, for the Area required.

This Rule is general in all kind of regular Polygons, of how many Sides ſoever, as well for their Superficial Content, as finding their Perpendicular.

But becauſe it is troubleſome finding the Centre Geometrically, I will here ſhew how to find it by Arithmetick.

The Side of a Pentagon being given, to find the Radius of a Circle inſcribed within that Pentagon.

The Rule is this, As 182 to 125, ſo is the Side of the Pentagon, to the Semidiameter of a Circle inſcribed in it.

Example.

The Side given is 10 Feet, what is the neareſt Diſtance from the Centre of the Pentagon to one of the Sides?

Multiply

Multiply and Divide according to the Rule of Three, and you will find 6 Feet, 10 Inches, and 5 Parts +.

PROP. X. *Fig.* IX.

The Diameter of a Circle being given, to find the Circumference thereof Arithmetically.

THE Proportion of the Diameter to the Circumference was first found out by *Archimedes*, who brought the Length of the Perimeter of a Circle within the Limits of Numbers, very little differing from the Truth, demonstrating the same to be less than three Diameters and a seventh Part, but greater than three Diameters and $\frac{10}{71}$ Parts of the Diameter, so that supposing the Radius to consist of 10000000 equal Parts, the Arch of a Quadrant will be between 15714285 and 15704225 of the same Parts. But since *Ludovicus Van Cullen* hath come yet nearer to the Truth, and pronounced from true Principles that the Arch of a Quadrant (putting as before 10000000 for the Radius) differs not one whole Unite from the Numb. 15707963. But for our purpose we shall make choice either of the Proportion in whole Numbers by *Archimedes*, which is as 7 to 22, or of the Proportion which *Metius* hath given, which is as 113 to 355, so is the Diameter to the Periphery or Circumference.

Let the Diameter given be 14 Feet, in *Fig.* IX. it is required to find the Circumference thereof.

Multiply the Diameter given (being 14) by 22, the Product is 308, which divided by 7, bringeth 44 Feet for the Circumference required; or multiply the Diameter by 3 $\frac{1}{7}$ the Product is 44, as before; but this makes the Circumference a small Matter too great, as

as you may ſee if you multiply 14 by 355, the Product is 4970, which being divided by 113, the Quotient is 43 and $\frac{111}{113}$ the Remainder, which is 111 Parts of an Unite (whether it be a Foot, or an Inch, or any other Meaſure) divided into 113 Parts.

If the Circumference be given, and the Diameter required, it appeareth by this Rule, that the Circumference 44 being multipily'd by 7, and the Product divided by 22, brings 14 the Diameter.

PROP. XI. *Fig.* IX.

The Diameter and Circumference of a Circle being given, to find the Area or Superficial Content thereof Arithmetically divers ways.

LET the Diameter of the Circle be 14, as in *Fig.* IX. and the Circumfererence thereof 44. It is required to find the Superficial Content.

Multiply half the Diameter by half the Circumference, it ſhews the Content of any Circle.

Thus the half Circumference is 22, which being multiplied by the Semidiameter 7, the Product is 154, the Superficial Content required.

Or multiply the whole Circumference 44, by the Semidiameter 7, the Product will be 308, whereof take half, which is 154, as before.

Or multiply the Square of the Diameter 196 by 11, the Product will be 2156; which divide by 14, brings 154, as before.

PROP.

PROP. XII. *Fig.* IX.

Having only the Diameter of a Circle given, to find the Content.

SAY, as 7 is to 22, ſo is the Square of the Semidiameter to the Content of the Circle.

Let the Diameter given be 14, as in *Fig.* IX.

Then ſay by the Rule of Three, if 7 give 22, what will 49 give? (which 49 is the Square of the Semidiameter;) multiply the third Number 49, by the ſecond Number 22, the Product is 1078, which divide by the firſt Number 7, and you have 154 for the Content, as before.

PROP. XIII. *Fig.* IX.

Having the Circumference given, to find the Content.

THE Rule is at 88 (which is 4 times 22) is to 7, ſo is the Square of the Circumference to the Content.

Example.

Let the Circumference be 44, the Square thereof is 1936; then ſay by the Rule of Three,

If 88 give 7, what will 1936 give?

Multiply 1936 by 7

The Product is 13552

Which divide by the first Number 88, and the Quotient is 154 Feet.

```
  43
  573
13552(154 Feet, the Content of the Circle.
 8888
  88
```

PROP. XIV. *Fig.* IX.

The Content of a Circle being given, to find the Diameter.

THE Rule is as 22 to (4 times 7, which is) 28, so is the Content to the Square of the Diameter.

Example.

Let the Content given be 154, and it is required to find the Diameter.

If 22 give 28, what will 154 give? Multiply and Divide, and you will find 196, the Square Root whereof is 14, the Diameter required.

PROP. XV. *Fig.* IX.

The Content of a Circle being given, to find the Circumference.

THE Rule is, as 7 is to (4 times 22, which is) 88, so is the Content to the Square of the Circumference.

Example.

Example.

Let the Content given be 154, and the Circumference required to be found.

Then ſay, by the Rule of Three,

If 7 give 88, what will 154 give? Multiply and divide, and you will find 1936, the Root whereof being 44, is the Circumference required.

PROP. XVI. *Fig.* IX.

The Content of a Circle being given, to find the Side of a Square, the Content of which Square ſhall be equal to the Content of the Circle.

EXtract the Square Root of the Content given, and that Root is the Side of the Square required.

Example.

Let the Content given be 154, the Square Root whereof is 12 Feet, 4 Inches $\frac{11}{12}$ Parts of an Inch, which is the Side of a Square, whoſe Content is equal to the Content of the Circle given.

PROP. XVII.

The Diameter of any Circle being given, to find the Side of a Square, the Area of which Square shall be equal to the Area of the Circle of the given Diameter.

THIS Propoſition may be ſolved ſeveral Ways, as firſt according to *Archimedes* his Proportion in whole Numbers, thus:

As 7 is to 22, ſo is the Square of the Semidiameter to the Content required.

Let the Diameter given be 7 Feet, the Semidiameter will be 3 Feet, 6 Inches, the Square whereof is 12 Feet, 3 Inches; which being multiplied by 22 Feet, the Product is 269 Feet, and 6 Inches, which divided by 7, makes 38 Feet, 6 Inches for the Content of the Circle, which Content is a ſmall Matter more than the true Content.

Secondly, According to *Metius* his Proportion, thus:

As 113 to 355, ſo is the Square of the Semidiameter to the Content required.

Example.

The Square of the Semidiameter aforeſaid is 12 Feet, 3 Inches, which being multiplied by 355 Feet, produces 4348 Feet, 9 Inches; which Product being divided by

by 113 Feet, makes 38 Feet, 5 Inches, 9 Parts, and $\frac{9}{12}$ of a Part, which Content is leſs than the Content produced from *Archimedes*'s Proportion, by more than $\frac{2}{144}$ Parts of a Superficial Foot, and the greater the Diameter is, the more the Difference will be.

Or thus, As 452 (which is 4 times 113) is to 355: So is the Square of the Diameter to the Content required.

Thirdly, It may be ſolved by Decimal Arithmetick, thus; as 1 Foot to Feet 3.1416, ſo is the Square of the Semidiameter to the Content required.

Example.

Feet 3.1416 being multiplied by Feet 12.25, produceth Feet 38.484600, which ſhould be divided by the firſt Number 1; but becauſe 1 doth neither increaſe any Number being multiplied by it, nor diminiſh any Number being divided by it, therefore we find the Content to be Feet 38.484600, which being reduced to Feet, Inches, and Parts, is 38 Feet, 5 Inches, 9 Parts, 9 Seconds, and 4 Thirds.

Fourthly, It may be ſolved Decimally thus:

As Feet 114.5915492 is to 360 Feet, ſo is the Square of the Semidiameter to the Content required.

Example.

360 Feet being multiplied by Feet 12.25, produceth Feet 4410.00 which being divided by Feet 114.5915492, produceth Feet 38.48451, which being reduced, is 38 Feet, 5 Inches, 9 Parts, 9 Seconds, and 2 Thirds; which is a ſmall Matter leſs than the former Content.

But peradventure the Reader hereof may not underſtand Decimal Arithmetick, I will therefore exhibit one Way more by Vulgar Arithmetick, for the ſolving of this Propoſition; which is this:

Multiply the Diameter given, by 10 Feet, 7 Inches, 7 Parts, 5 Seconds; and divide the Product by 12 Feet, it produces the Side of a Square, which being multiplied by itſelf, is the Content of the Circle required.

Which I look upon to be as quick a Way as by Decimals, eſpecially if the Diameter given conſiſts of Feet, Inches, Halfs, Quarters, *&c.* and the Solution be required in the ſame, and cometh within a Trifle as near the Truth.

And here I ſhall a little inſiſt upon the multiplying the Fractional Parts of Inches into one another, for two Reaſons.

Firſt, As being a Way very uſeful, and ſaves a great deal of Labour (to wit) of redicing the Integers into the leaſt Denomination (or Fractional Parts) it being the uſual Way ſo to do, among thoſe who underſtand not Decimal Arithmetick.

Secondly, Becauſe hitherto I have had occaſion only to multiply Twelfths of Inches into Feet and Inches; and having occaſion now to multiply other Twelfths into thoſe Twelfths, and other Twelfths into theſe Twelfths, *&c.* which I name thus,

Feet, *Inches*, *Parts*, *Seconds*, *Thirds*, *Fourths*, &c.

And here Note, that as a Foot contains 12 Inches Line meaſure, ſo an Inch is ſuppoſed (or accompted) to contain 12 Parts Line meaſure (although upon the Rulers the Inches are divided but into 8 Parts, or Half-Quarters)

Quarters) and each Part to contain 12 Seconds, and each Second to contain 12 Thirds, and each Third to contain 12 Fourths, *&c.* And ſo the Proportion might be continued, if there were occaſion, to Fifths, Sixths, *&c.*

But according to ſuperficial Meaſure, a Foot contains 144 Inches, and an Inch 144 Parts, and a Part 144 Seconds, *&c.*

Alſo obſerve, that each Place of Twelfths is diſtinguiſhed by having a Letter put over it; thus, over the Feet write F, over Inches I, over Parts P, over Seconds S, over Thirds T, over Fourths Fo, *&c.* as beneath.

And for the better underſtanding how to multiply the Fractions or Twelfths into one another, obſerve the Rules in the following Table.

	F.	I.	P.	S.	T.	F.
1. *Note*, That Inches mult. into F. every 12 of the Product is and any Number leſs than 12 is Inches	1					
2. In. mult. into (or by) In. every 12 of the Prod. is and any Number of the Prod. leſs than 12 are Parts	0	1				
3. Parts mult. into Feet, every 12 of the Prod. is and any Number under 12 are Parts.	0	1				
4. Parts multiplied into Inches, every 12 of the Prod. is and any Number of the Prod. under 12 are Seconds	0	0	1			
5. Parts multiplied by Parts, every 12 of the Prod. is and any Number leſs than 12 are Thirds	0	0	0	1		
6. Seconds multiplied into Fe. every 12 of the Prod. is and any Number leſs than 12 are Seconds	0	0	1			
7. Sec. mult. into Inches, every 12 of the Prod. is and any Number leſs than 12 are Thirds.	0	0	0	1		
8. Sec. mult. into Parts, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	
9. Sec. multiplied into Sec. every 12 of the Prod. is and any Number leſs than 12 are Fifths.	0	0	0	0	0	1
10. Thirds multiplied into Feet, every 12 of the Prod. is and any Number leſs than 12 are Thirds.	0	0	0	1		
11. Thirds multiplied into Inches, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	
12. Thirds multiplied into Parts, every 12 of the Prod. is and any Number leſs than 12 are Fifths.	0	0	0	0	0	1
13. Fourths multiplied into Feet, every 12 of the Prod. is and any Number leſs than 12 are Fourths.	0	0	0	0	1	

Farther in Fractional Parts you need not proceed.

Here *Note*, if the greateſt Denomination of the Dimenſion to be caſt up be Inches, and the Content be required in ſuperficial Inches; then Parts multiplied into Inches, every 12 of the Product is one Inch; and Seconds multiplied into Parts, every 12 is one Part, *&c.*

Explanation of the Table in 3 or 4 Examples.

Firſt, Suppoſe you have 6 Parts to be multiplied into 7 Feet;

Say 6 times 7 is 42; then ask yourſelf how many times 12 is 42? and the Anſwer is 3 times, and 6 remaining; therefore according to the 3d Rule of the preceding Table, ſet 3 in the Place of Inches, noted with the Letter I, and the 6 that remains, being leſs than 12, you muſt write in the Place of Parts, noted above the Dimenſion with the Letter P.

Secondly, Suppoſe 8 Thirds to be multiplied into 8 Inches, ſay 8 times 8 is 64, which contains 5 twelves, and 4 remaining; therefore, according to the 11th Rule, ſet 5 in the Place of Thirds, noted by T; and the 4 that remains, ſet in the Place or Column of Fourths, noted with Fo.

Thirdly, Suppoſe you have 10 Seconds to be multiplied by 11 Parts, ſay 10 times 11 is 110, which contains 9 times 12, and 2 remaining; then, according to the 8th Rule of the foregoing Table, ſet down 9 in the Place of Thirds, noted on the Top with T, and the 2 that remains ſet in the Place of Fourths, noted by Fo, *&c.*

Theſe

Theſe being premiſed, I come now to ſolve the foregoing *Prop.* (to wit) the Diameter of a Circle being given, whoſe Length is 7 Feet, it is required to find a Square, whoſe Content ſhall be equal to that Circle.

The Proportion is, as 12 Feet is to 10 Feet 7 Inches, 7 Parts, 5 Seconds; ſo is the Diameter to the Side of a Square, or the Square of the Diameter to the Content required.

The Rule thus; Multiply the Diameter by 10 Feet, 7 Inches, 7 Parts, 5 Seconds, and divide the Product by 12 Feet. The Quotient, together with the Remains, if there happen to be any, is the Side of a Square, which being multiplied in itſelf, is equal to the Content required.

Example.

	F.	I.	P.	S.	T.	Fo.
The Length of the Diam. given is	7	0	0	0	0	0
Which multiply by	10	7	7	5		
The Prod. of 10 F. by 7 F. ss	70					
The Prod. of 7 Inches by 7 F. is	4	1				
The Prod. of 7 Parts by 7 F. is	0	4	1			
The Prod. of 5 Sec. by 7 F. is	0	0	2	11		
The Total Product is	74	5	3	11		

Which Product, according to the foregoing Rule, muſt be divided by 12. Therefore divide the 74 Feet, by 12 Feet, thus,

Inches

$\cancel{1}$ (2 Inches.
$\cancel{7}\,\cancel{4}$(6 Feet.
$\cancel{1}\,\cancel{2}$

And the Quotient is 6 Feet, and 2 remains, which is 2 Inches, ſo the Side of the Square equal to the Circle, is 6 Feet, 2 Inches, 5 Parts, 3 Seconds, 11 Thirds; for inſtead of dividing the 5 Inches, 3 Parts, 11 Seconds (that belonged to the 74 Feet in the Total Product) by 12, only do thus; Remove them one Place forwarder towards the Right Hand, it is the ſame as if you had divided them by 12; ſo then the 5 Inches will ſtand in the Place of Parts, and be 5 Parts, and the 3 Parts will be 3 Seconds, and the 11 Seconds will be 11 Thirds; which obſerve as a general Rule in any Number to be divided after this manner.

When you have divided the Feet by 12, if there happen to be no Inches remaining, then ſet down your Feet under the Place of Feet, and put a Cypher in the Place of Inches, and then ſet your other Fractions next; as in this laſt Example, if there had been no Remainder beſides the Quotient, then you muſt have put 0 in the Place of Inches, and then the 5 Inches which belonged to the 74 Feet will be changed into 5 Parts, as if they had been divided by 12. And likewiſe the 3 Parts will be turned into 3 Seconds, &c.

So

	F.	I.	P.	S.	T.	Fo.
So that 74 Feet, 5 Inch. 3 Parts, 11 Seconds, being divided by 12, is	6	2	5	3	11	
Which is the Side of a Square; which multiply by	6	2	5	3	11	
The Prod. of 6 Feet by 6 Feet, is	36					
The Prod. of 2 Inch. by 6 Feet, is	1					
The ſame again is	1					
The Prod. of 2 Inch. by 2 Inch. is	0		4			
The Prod. of 5 Parts by 6 Feet, is		2	6			
The ſame again is		2	6			
The Prod. of 5 Parts by 2 Inch. is	0	0	0	10		
The ſame again is				10		
The Prod. of 5 Parts by 5 Parts, is				2	1	
The Prod. of 3 Sec. by 6 Feet, is			1	6		
The ſame again is			1	6		
The Prod. of 3 Sec. by 2 Inch. is					6	
The ſame again is					6	
The Prod. of 3 Sec. by 5 Parts, is					1	3
The ſame again is					1	3
The Prod. of 11 Thirds by 6 F. is				5	6	
The ſame again is				5	6	
The Prod. of 11 Thirds by 2 In. is					1	10
The ſame again is					1	10
Farther you need not proceed, the Value of the Products being inconſiderate.						
The Total Product is	38	5	9	10	7	2

Which is the Content required, and agrees with the Decimal way within $\frac{1}{1728}$ Parts of an Inch.

Although I have ſet the Product of each Multiplication one under another, for Demonſtration; yet never-

nevertheleſs when you are perfect in caſting up, you may ſet the Products of 4 or 5 Multiplications in one level Line.

Let a ſecond Example be,

A Diameter given, whoſe Length is 42 Feet, and the Content of that Circle is required.

According to *Archimedes* his Proportion in whole Numbers, working as is ſhewn before, the Content will be 1386 Feet.

And according to *Metius*, in whole Numbers the Content will be found to be 1385 Feet and $\frac{50}{113}$ of a Foot, which reduced, is 1385 Feet, 5 Inches, 3 Parts, 8 Seconds.

Let us try what the Content will be our Way.

	F.	I.	P.	S.	T.	F.
The Diameter is	42	0	0	0	0	0
Which multiply by	10	7	7	5		
The Prod. of 10 F. by 42 F. is	420					
The Prod. of 7 Inch. by 42 Feet, is	24	6				
The Prod. of 7 Parts by 42 Feet, is	0	24	6			
The Prod. of 5 Sec. by 42 Feet, is			17	6		
The Total Product is	446	7	11	6		

Which 446 Feet, divide by 12.

1
18(2 Inches.
446(37 Feet.
122
1

The

The Quotient is 37 Feet and 2 Inches, to which add the 7 Parts 11 Seconds 6 Thirds, by changing their Places one Degree forwarder toward the Right Hand, (inſtead of dividing them by 12) and it will be

	F.	I	P.	S.	T.	Fo.
For the Side of the Square	37	2	7	11	6	0
Which to ſquare, multiply by	37	2	7	11	6	0
The Prod. of 7 Feet by 37 is	259					
The Prod. of 30 F. by 37 F. is	111					
The Prod. of 2 Inch. by 37 Feet is	6	2				
The ſame again is	6	2				
The Prod. of 2 Inch. by 2 Inch. is			4			
The Prod. of 7 Parts by 37 Feet, is		21	7			
The ſame again is		21	7			
The Prod. of 7 Parts by 2 Inches, is			1	2		
The ſame again is			1	2		
The Prod. of 7 Parts by 7 Parts, is				4	1	
The Prod. of 11 Sec. by 37 Feet, is			33	11		
The ſame again is			33	11		
The Prod. of 11 Sec. by 2 Inch. is				1	10	
The ſame again is				1	10	
The Prod. of 11 Sec. by 7 Parts, is					6	5
The ſame again is					6	5
The Prod. of 11 Sec. by 11 Sec. is						10
The Prod. of 6 Thir. by 37 Feet, is				18	6	
The ſame again is				18	6	
The Prod. of 6 Thir. by 2 Inches, is					1	
The ſame again is					1	
The Prod. of 6 Thir. by 7 Parts, is						3
The ſame again is						3
The Content or Total Prod. is	1385	5	8	0	1	2

Which

Which Content is four Parts more than the Content, according to *Metius*'s Proportion, but near four Parts less than the Content according to *Archimedes*'s Proportion.

And by the way, note, That a Circle of 14 Feet Diameter, contains 4 times as much as a Circle that is 7 F. Diameter. The Rule is thus, the Diameter 7 is contained in the Diameter 14 two times; now the Square of 2 is 4, therefore a Circle whose Diameter is 14, contains a Circle whose Diameter is 7 four times.

Likewise the Diameter in the last Example being 42, contains the Diameter 7 six times; for the Square of 6 is 36, therefore a Circle whose Diameter is 42, contains 36 Circles of 7 Feet Diameter apiece.

For if you multiply 38 Feet, 5 Inches, 9 Parts, 10 Seconds, + (being the Content of a Circle whose Diameter is 7) by 36, you will find it produce the Content of a Circle whose Diameter is 42.

Example.

	F.	I.	P.	S.	T.
A Circle whose Dia. is 7 F. the Con. is	38	5	9	10	6
Which multiply by	36				
	228	0	0	0	0
	114				
	15	27	30	18	0
A Cir. whose Dia. is 42 F. contains	1385	5	7	6	0

Let the Third Example be

A Diameter whose Length is 38 Feet, 6 Inches, and an half and half a quarter, and it is required to find the Content of that Circle.

The half Inch is 6 Parts, but the half quarter is $\frac{1}{8}$ Part of an Inch, which must be reduced to Twelfths, thus;

thus; Multiply the Numerator of the Fraction (to be reduced) by 12, and divide the Product by the Denominator of the fame, and the Quotient is the Numerator of a Fraction that hath 12 for his Denominator; and if any Number remain befides the Quotient, multiply that Remainder by 12, and divide the Product by 8, fo long until you have no Remainder befides the Quotient.

Example.

$\frac{1}{8}$ of an Inch is to be reduced; multiply the Numerator 1 by your new Denominator 12, it is ftill 12; divide it by the Denominator 8, the Quotient is 1, and 4 remains; the 1 in the Quotient is 1 Part, which add to the $\frac{1}{2}$ Inch or 6 Parts, and then it makes 7 Parts: Then multiply the Remainder 4 by 12, the Product is 48; which divide by the Denominator 8, and the Quotient is 6 and no Remainder, which is 6 Seconds. So $\frac{1}{8}$ of an Inch being reduced to 12ths, is 1 Par. 6 Sec.

	F.	I.	P.	S.	T.
Now I fet down the Diameter thus	38	6	7	6	0
Which I multiply by	10	7	7	5	
	380				
	5	3	6		
	22	2	4	1	
		5	10		
		22	2	4	
			3	6	
			5		
			15	10	
				3	6
				2	6
The Product is	409	11	11	3	0

Which

Which 409 divide by 12.

	F.	I.	P.	S.	T.	
14(1 In. The Side of the Sq. is	34	1	11	11	3	
409(34 F. Which multiply by	34	1	11	11	3	
122						
1	136	31	2			
	1022	10	1			
	2	10		11	10	
		31	2	11	10	
				10		
			31	2	11	
			31	2	11	10
				8	6	
				8	6	
The Content required is	1167	3	11	8	6	10

This laſt *Example* could not be ſolved ſo near the Truth by vulgar *Arithmetick* any other way, without reducing the 38 Feet, 6 Inches, and half Inch into half quarters of Inches, which is a long way about.

PROP. XVIII.

The Superficial Content of a Circle being given; to find the Diameter.

LET the Content given (being found according to *Archimedes*'s Proportion) be 346 Feet 6 Inches; and the Diameter required.

The Proportion is as 11 is to 14, ſo is the Content given, to the Square of the Diameter required.

The

The Rule thus, Multiply the Content given by 14, the Product whereof divide by 11, and the Square Root of the Quotient is the Length of the Diameter.

Example.

Multiply the Content given, being 346 Feet 6 Inches, by 14, the Product is 4851, which being divided by 11, the Quotient is 441 for the Square of the Diameter, the Roof whereof is 21, the Length of the Diameter requir'd.

Or thus, and more exact, As 355 to 452, so is the Content given, to the Square of the Diameteter.

Example.

Multiply the Content given, being 346 Feet 6 Inches, by 452, the Product is 156618, which being divided by 355 produces 441 Feet and $\frac{63}{355}$ of a Foot, which, when reduced, is 441 Feet 2 Inches 1 Part, 6 Seconds, the Square Root whereof is 21 Feet 1 Inch 5 Parts 6 Seconds, the Diameter required.

To those who understand Decimals, the Proportion for solving the 18th Proposition is this,

As 1.00000 is to 1.27324, so is the Content given to the Square of the Diameter.

Therefore if you multiply 1.27324 by the given Content, the Square Root of that Product will be the Diameter required.

PROP. XIX.

The Diameter of a Circle being given; to find the Side of a Square which may be inscrib'd within that Circle.

THE Rule is this: Square the Diameter, then take one half of the Product, then find the Root of that Product, it will be the Side of the Square desired.

Example.

Let the Diameter given be 28 Feet 3 Inches, the Square whereof is 798 Feet 9 Parts, the Half whereof is 399 Feet 4 Parts and 6 Seconds; the Square Root whereof is 19 Feet 11 Inches 8 Parts 5 Seconds +, for the Length of the Side of the Square required.

Decimally thus, As 1.000000 is to .707106, so is the Diameter to the Side required.

Therefore if you multiply the said .707106 by the Diameter given, the Product will be the Side of the inscrib'd Square requir'd; for, as you read before, if you should divide the Product by 1.000000, it will still be the same.

PROP. XX. *Fig. X.*

The Diameter and Arch-Line of a Semi-circle being given; to find the Area thereof.

LET ABC in *Fig. X.* be a Semi-circle given, whose Diameter is AC, and the Arch-Line ABC, it is required to find the Area of that Semi-circle.

Multi-

Multiply half the Arch-Line 11 by the Semidiameter 7, the Product will be 77 for the Area required.

PROP. XXI. *Fig. X.*

The Semidiameter and Arch-Line of a Sector of a Circle being given; to find the Area.

IN *Fig.* X. let BCD be the Sector of a Circle, whose Semidiameter is DC, or DB, and the Arch-Line BC, and it is required to find the Area.

Multiply the Semidiameter 7, by half the Arch-Line BC 5 Feet 6 Inches, the Product is 38 Feet 6 Inches for the Area required.

PROP. XXII. *Fig. X.*

Any Segment or part of a Circle being given, to find the Content.

IN *Fig. X.* let AF BE be the Segment of a Circle, the Content whereof is required.

First find out the Centre of the Arch-Line AFB, (by the 21ſt Prop. of the 20th Page of Geometry) then draw the Lines DA and DB, and caſt up the whole Figure AFBD, as in the laſt Prop. which will be 38 Feet 6 Inches, then find the ſuperficial Content of the Triangle ABD, which is 24 Feet 5 Inches 9 Parts, and deduct the ſame out of the whole Content 38 Feet 6 Inches; the Remains is 14 Feet and 3 Parts, for the Content of the given Segment, which was required.

By this Rule (obſerved with Diſcretion) may all manner of Segments or Parts of a Circle, whether greater or leſs than a Semi-circle, be eaſily meaſured.

PROP. XXIII. *Fig. X.*

To find the Diameter of a Circle, by having one Part of the Diameter given, alſo having the Length of the Chord croſſing the Diameter in the given Part.

IN *Fig. X.* let FE be the Part of the Diameter given, alſo let AB be the given Chord which cuts off the given Part of the Diameter; it is required to find the whole Diameter Arithmetically.

FE the Part of the Diameter given, is 2 Feet and $\frac{7}{12}$ of an Inch, AB the Chord interſecting it is 9 Feet 10 Inches $\frac{10}{12}$ of an Inch.

The Rule is thus: Square one half of the Chord, and divide the Product by the Part of the Diameter given, which Quotient being added to the ſaid Part, is the Length of the Diameter required.

Example.

The whole Chord AB is 9 Feet 10 Inches 10 Parts, the Half whereof is 4 Feet 11 Inches 5 Parts, the Square of which is 24 Feet 6 Inches 2 Parts and 4 3ds, which being divided by 2 Feet 7 Parts the Part of the Diameter given) the Quotient is 11 Feet 11 Inches 8 Parts, to which 2 Feet 7 Parts being added, make 14 Feet for the Length of the Diameter required.

PROP.

PROP. XXIV. *Fig.* X.

Any Segment of a Circle being given, whose Chord-Line doth not exceed the Chord of the Quadrant of the same Circle; to find the Content without finding the Diameter, and without describing any more of the Circumference.

ALthough, by the precedent Prop. XXII. any Segment of a Circle may be measured; yet because the finding of the Center, and measuring of the Sector, and then measuring the Triangle and subtracting it from the Sector, requires a great deal of Trouble and Time; I will here shew how to do it with less Trouble, and very near the Truth, in small Segments.

Let the Segment given be the same (whose Content was found by 22 Prop.) in *Fig.* X. whose Chord-Line A B is 9 Feet 10 Inches and 9 Parts, and the Part of the Diameter F E (cut off by the Chord Line, is 2 Feet 7 Parts and 6 Seconds; the Content of which Segment A F B E is required.

Note, If the versed Sine, *viz.* the Line F E be not given, you must divide the Chord A B in the Middle, and from that middle Point, as at E, raise a Perpendicular to the Arch-Line.

Then take the whole Length of the Chord A B, and two Thirds of the Length of the Line F E, to which two Thirds add 7 Parts, then multiply those two Lengths, and the Product gives you the Content.

Example.

The verſed Sine FE is 2 Feet 7 Parts 6 Seconds, two third Parts whereof is 1 Foot 4 Inches 5 Parts, to which 7 Parts being added, makes it 1 Foot 5 Inches; which multiply'd by the Chord AB, (namely) 9 Feet 10 Inches 9 Parts, produces 14 Feet and 2 Parts for the Content, which is 1 Part leſs than the Content found by the 22 Prop.

Another Example.

In *Fig. XI.* the Semi-diameter is 14 Feet, and the Content of the Quadrant ABCD will be found by the former Rules, according to *Metius*'s Proportion, to he 153 Feet 11 Inches 1 Part 6 Seconds.

Now it is required to find the Content of the Segment ABCE.

By ſubtracting the Content of the Triangle ACD, which is 98 Feet 4 Parts and 5 Seconds, from the Content of the Quadrant which is as above, there remains 55 Feet 10 Inches 9 Parts for the Content of the Segment ABCE.

Let us ſee what the other Way will produce.

The Length or Part of the Diameter BE is 4 Feet 1 Inch 2 Parts, two third Parts whereof is 2 Feet 8 Inches 9 Parts 4 Seconds, to which add 7 Parts, and it makes 2 Feet 9 Inches 4 Parts 4 Seconds; which being multiplied by the Chord-Line AC, whoſe Length is 19 Feet 9 Inches 7 Parts, the Product is 54 Feet 11 Inches 6 Parts for the Content of the Segment.

A

A Third Example.

Let the Content of the two little Segments, A B, B C, in *Fig. XI.* be required.

The Line F G is 1 Foot, two third Parts whereof is 8 Inches, to which 7 Parts being added, makes it 8 Inches 7 Parts, which being multiplied by 10 Feet 10 Inches, the Length of the Chord-Line A B produces 7 Feet 8 Inches 11 Parts, for the Content of the Segment A F B G, the other Segment B C being equal to this, add 7 Feet 8 Inches 11 Parts to the Content of the firſt, and the Product is 15 Feet, 5 Inches 10 Parts for the Content of both the little Segments.

The Triangle ABCE, whoſe Content is 40 Feet, 6 Inches 5 Parts, being deducted from the Content of the Segment ABCE, which is 55 Feet 10 Inches 9 Parts, there remains 15 Feet 4 Inches 4 Parts, for the Content of the two little Segments; which is half an Inch leſs than was found before.

PROP. XXV. *Fig. XII.*

An irregular Plat or Figure being given; to find the Area or Content thereof.

LET a b c d e f g h i in *Fig. XII.* be an irregular Plat, whoſe ſuperficial Content is required.

Reduce the ſame into as many Trapezia's as it will contain, at firſt the Trapezium a b c d; Secondly, a d e f; Thirdly, a f g, i, and there remains the Triangle, i g h. In which three Trapezia's, draw the Diagonals b d, d f, and f i, which

ſhall be as common Baſes to each Triangle on either Side; on which Baſes let fall the Perpendiculars from the ſeveral Angles at a, c, e, and g; then in every Trapezium take the Length of the Baſe by itſelf, and the Length of the two Perpendiculars thereon falling, joined together, in one Number by itſelf; then multiply one half of the one in the whole of the other, and the Product is the Area of that Trapezium, which reſerve by itſelf, and working in like ſort with the reſt; and laſtly, the Triangle i, g, h: Collect all their Products together, which ſhall ſhew the Content required.

PROP. XXVI.

To find the Superficial Content of any Oval.

TAke both the Diameters, and multiply them one into another; then extract the Square Root of that Product, and that Root will be the Diameter of a Circle, whoſe Content will be equal to the Content of the Oval. For the Area of a Circle is to the Area of an Ellipſis, as the Square of the Diameter of that Circle is to the rectangled Figure of the tranſverſe and conjugate Diameters of the Ellipſis by the 6th of *Archimedes*'s Conoides and Spheroides.

Example.

Let *Fig. XIII.* be an Oval, whoſe longeſt Diameter is 10 Feet, and the ſhorteſt Diameter is 6 Feet, and the Content required.

Multiply

Multiply the Length 10 by the Breadth 6, the Product is 60, the ſquare Root whereof is 7 Feet 8 Inches 11 Parts 5 Seconds, which is the Length of the Diameter, the Content of whoſe Circle is equal to the Content of the Oval; then multiply the Diameter by 10 Feet 7 Inches 7 Parts 5 Seconds (as you were ſhewn at the 5th Example of Prop. XVII. of this) and the Product is 82 Feet 4 Inches 6 Parts 3 Seconds, which divide by 12, it produces 6 Feet 10 Inches 4 Parts 6 Seconds 3 Thirds, which being multiplied by it ſelf (*viz.* ſquared) the Product is 47 Feet 1 Inch 5 Parts 11 Seconds, the Content of the Oval required.

See the whole Work.	F.	I.	P.	S.	T.
The Length of the Oval	10	0	0	0	0
The Breadth of the Oval	6	0	0	0	0
Being multiplied, the Product is	60	0	0	0	0
The Square Root whereof is	7	8	11	5	
Which being the M. Diam. mult. by	10	7	7	5	
	70	4			
	6	8	8	5	5
	4	1	6	6	
		9	2	3	4
		4	1	2	11
			4	8	2
			4	2	4
			2	11	
The Product is	82	4	6	3	2

Which

Which Product divide by 12.

(1 Inch. And it is
2 (0 Which multiply by
82 (6 Feet
12

F.	I.	P.	S.	T.
6	10	4	6	3
6	10	4	6	3
36	8	4	1	4
5	2	3	4	2
5	2	3	4	2
		3	5	3
		3	5	2
			1	6
			1	6
				2
47	1	5	11	3

The Content of the Oval is 47 | 1 | 5 | 11 | 3

In Decimals thus: Multiply the Length of the Oval by the Breadth, and divide the Product by 1.27324 Foot, it gives the Content.

PROP. XXVII. *Fig. XIV.*

To find the Superficial Content of a Cylinder, the Diameter being given.

NOte, That a Cylinder is a ſolid Body, which may well be repreſented by a Roll either of Timber or Stone, ſuch as are uſed in Gardens for the rolling of Walks.

The Proportion is, *As* 7 *to* 22, or *As* 113 to 355; *So* is the Diameter and Length of the Side multiplied one by the other, *To* the ſuperficial Content of the Outſide of the Cylinder, beſides the two Baſes or Ends.

The

The Rule thus: Multiply the Diameter by the Length or Side; then multiply that Product by 355, the Product whereof divide by 113, and it gives you the Content of the round Superficies.

Let *Fig. XIV.* be a Cylinder, whoſe Diameter is 4 Feet, and the Side or Length 10 Feet, and the Content required.

Firſt, Multiply the Diameter 4 by the Length 10, the Product is 40; which multiply by 355, and the Product is 14200; which divide by 113, and the Quotient 125 Feet, and 75 remaining, which is $\frac{75}{113}$ of a Foot; and being reduced, is 125 F. 7 I. 11 P. 6 S. for the Content required.

And if you had wrought after the Proportion of 7 to 22, the Content would have been 125 F. 8 I. 7 P. *fere*, which is a ſmall matter more than the former.

If you have the Circumference and Length of a Cylinder given, and the Content required;

Multiply the Circumference by the Length, and the Product is the ſuperficial Content.

Example.

Let the Circumference and Length of the Cylinder, *Fig. XIV.* be given, and the Content required.

Multiply the Circumference 12 F. 6 I. 9 P. 7. S. by the Length 10 F. and the Product is 125 F. 7 I. 11 P. for the Content required.

If you are minded to add the ſuperficial Content of the two Baſes or Ends of the Cylinder, you will find the Content of them as you are ſhewn before in meaſuring of Circles.

PROP.

PROP. XXVIII. *Fig. XV.*

To find the Superficial Content of a Cone.

A Cone is a Body which hath a Circle for its Base, from whence it diminisheth equally (like a round Spire of a Steeple) till it end in a Point.

Let *Fig. XV.* be a Cone, the Circumference of whose Base is 24 F. and the Length of the Side 10 F. and the Content required.

If you require the Content of the whole Cone, that is to say, the Content of the Base, as well as the Content of the Outside; then multiply the whole Side by half the Compass or Periphery of the Base; and to the Product add the Content of the Plain of the Base.

Example.

The whole Circumference of the Base being 24 F. the half thereof is 12 F. which being multiplied by the Length 10 F. produces 120 F. for the Content of the Outside, beside the Base: Then by the 13th Prop. of this, find the Content of the Circle whose Circumference is 24, and add it to the former Content, and the Product is the superficial Content of the Cone, together with its Base.

Example.

	F.	I.	P.	S.
The Cont. of the Cone without the Base is	120	0	0	0
The Content of the Base is	45	9	9	9
The Content of the Cone, together with its Base, is	165	9	9	9

PROP.

PROP. XXIX. *Fig. XVI.*

To find the Superficial Content of a Pyramid.

As a Cone hath a round Baſe, and diminiſheth equally till it end in a Point; ſo a Pyramid hath an angular Baſe of 3, 4, 5, 6, or any Number of Sides, and diminiſheth to a Point at the Top.

Let *Fig. XVI.* be a Pyramid, whoſe Baſe is a Square, whoſe Side is 5 Feet, and the Length from the Baſe to the Top 10 Feet, and the ſuperficial Content required.

Add the four Sides of the Baſe together, of which Product take half; then multiply that half by the Side or Length.

Example.

One Side being 5 Feet, the four Sides added make 20 Feet, the Half whereof is 10 Feet; which multiplied by 10 Feet the Length, produces 100 Feet for the ſuperficial Content, beſides the Baſe: If you are minded to add the Content of the Baſe to it, which is 5 times 5 Feet, or 25 Feet, then the ſuperficial Content of the Pyramid, together with the Baſe, will be 125 Feet.

Likewiſe, if the Baſe be Triangular, you muſt add the three Sides of the Baſe together, and take half of the Product, which multiplied by the Length, gives you the Content: Then if you would add the Content of the Baſe to the Content of the Outſide, meaſure it as you are ſhewn before in meaſuring of Triangles.

Alſo

Alſo if the Baſe be a Pentagon, take half the Product of the five Sides of the Baſe (being added together) and multiply it by the Length, and the Product gives you the Content: If you are minded to add the Baſe, find the Content, as you are ſhewn at Prop. IX. of this.

PROP. XXX.

To find the Superficial Content of a Globe or Sphere.

MUltiply the Diameter or Axis by the Circumference, and the Product will be the ſuperficial Content of the round Body or Globe.

Example.

Let the Diameter be 14 Feet, the Circumference will be 44 *fere*, which being multiplied one by the other, the Product is 616 Feet for the Content of the Superficies of the Globe.

Or find the Content of the Circle that hath the ſame Diameter as the Globe, and multiply that Content by 4, and the Product is the ſuperficial Content required.

Example.

A Circle whoſe Diameter is 14, the Content is 154, which multiplied by 4, the Product is 616, as before.

PROP. XXXI. *Fig. XVII.*

To find the Superficial Content of a Fragment, or Part of a Globe.

THE Superficie of any Fragment, or Portion of a Globe, is to the Superficie of the remaining Portion, as the Portion of the Axis or Diameter cut off, is to the remaining Portion of the same.

Example.

Let there be in *Fig. XVII.* a Portion of a Globe given ABC, whose Part (or Segment) of the Axis (or Diameter) is 5 Feet, and the whole Diameter 14 Feet, and the superficial Content of the Circular Part is required.

Say, by the Rule of Three, If 14 give 616, what will 5 give? Multiply 616 by 5, the Product is 3080; which divide by 14, and you have 220 Feet for the superficial Content of the Segment, or Part of the Globe.

I shall now conclude the Measuring of Plains, *with some Directions whereby to measure* Land.

Wherein Note, That 5 $\frac{1}{2}$ Yards, or 16 $\frac{1}{2}$ Feet in Length, is called (according to Statute) a *Pole* or *Perch*; and one Perch in Breadth, and 40 in Length, is called a *Rood*; and 4 Perches in Breadth, and 40 in Length, is one *Acre*: So that an Acre contains 160 square Perches, half an Acre 80 Perches, and a quarter of an Acre, commonly called a Rood, 40 Perches.

In

In meaſuring of Land, it is uſual to take the Lengths and Breadths with a Chain, the Length of which Chain ſome make to contain 4 Poles, (or Perches) as Mr. *Gunter* and others; and ſome make it to contain 2 Poles, as Mr. *Rathborn*, and others.

The Chain which I would adviſe is thus:

To contain 2 Perches (or 33 Feet) in Length, each of which Perches, divide into 12 equal Parts, or (as I may call them) Links, and each Link into 12 equal Parts, with little Wyers or Notches croſs-wiſe.

Thus the Length of a Perch will be divided into 144 Parts, and the whole Chain, conſiſting of 2 Perches, will contain 288 Parts; in the Middle of which Chain it will be convenient to have a pretty large Ring, that ſo you may diſtinguiſh where one Perch ends, and the other begins: Alſo midway between that large Ring and one end of the Chain hang a Curtain-Ring; and likewiſe hang another Curtain-Ring midway between the other end and the large Ring: So the whole Chain will be divided into four equal Parts or half Perches by the three Rings.

The Chain being thus divided, you have every whole Pole equal to 12 Links, or 144 Parts; every $\frac{3}{4}$ of a Pole equal to 9 Links, or 108 Parts; every half Pole equal to 6 Links, or 72 Parts: And laſtly, every quarter of a Pole equal to 3 Links, or 36 Parts.

The Chain being divided after this manner, you have only three Denominations to ſet down, namely, *Chains*, *Links*, *Parts*, if you will meaſure ſo near the Truth; but in moſt Menſurations you need proceed no farther than to Chains and Links, and then you have but two Denominations to ſet down.

Alſo the Manner of caſting up the Dimenſions, is the ſame as is ſhewn all along before, only altering the

the two first Denominations, to wit, whereas you did set over the Dimensions F. I. P. *&c.* signifying Feet, Inches, Parts; so in measuring of Land, you must set over the Dimensions C. L. P. signifying Chains, Links, Parts: For as a Foot contains 12 Inches, and an Inch 12 Parts, Line-Measure (or in Length) so likewise a Pole or Chain contains 12 Links, and one Link 12 Parts.

You must remember to double the Number of your Chains when you set them down, because a Chain contains 2 Poles or Perches.

Suppose in *Fig.* V. CFEB to be a Piece of Land lying in form of a long Square, and the Length FE being measured with the Chain, to contain 8 Chains, 6 Links, and 6 Parts; because the double of 8 is 16, I set it down thus . . . . . Also the Breadth EB set thus, containing

	C.	L.	P	
	16	6	6	
	14	9	3	
	64	4	6	
	167	7	1	6
Which I multiply as if they were Feet, Inches, and Parts . . .	12	4	4	6
				1
And the Product in square Perches, is	244	4	0	1

Which 244 Chains or Perches you must divide by 160 to bring them into Acres, as you read above, that an Acre contains 160 square Poles or Perches.

(84
244(1
160

So you have 1 the Quotient, which is 1 Acre, and the 84 that remains being Perches, divide by 40, the Number of Perches in a Rood, and you have the Quotient 2 Roods and 4 Perches remaining.

(4
84(2
40

Thus the Content of that Piece of Land

is found to be 1 Acre, 2 Roods, 4 Perches, and the 4 Links which remain in the Multiplication is one third Part of a Perch.

Again,

Suppose *Fig.* VII. to be a Field inclosed with three Sides, like a Triangle EDF, and you require to know the Content thereof;

Let the Side ED be 120 Chains or Perches, the Side DF 160 Chains or Perches, and the Side FE 200 Perches, and the Perpendicular DG 95 Perches.

As you were shewn in measuring of Triangles, take half the Base or Side EF, which is 100 Perches, and multiply it by the Perpendicular DG 95 Perches.

Example.

	C.	L.	P.
The Length of half the Base EF, is	100	0	0
Which multiplied by the Perpendicular	95	0	0
	500		
	900		
The Product is, in square Perches	9500	0	0

Which being divided by 160, produces 59 Acres, 1 Rood and an half; (for if you divide 60 by 40, it is one and a half) which is the Content of that Triangular Piece of Land.

(6
150(0
9500(59
1600
16

Example.

Example.

A ſquare Piece of Land being in Breadth 120 Chains, 9 Links, 6 Parts; and in Length 125 Chains, 6 Links, and 2 Parts, and the Content is required.

	C.	L.	P.
The double of 120 is 240, ſet thus	240	9	6
The double of 125 is 250, ſet it thus	250	6	2
	12000	4	6
	480	125	1
	120		3
	187	6	
		40	
The Product in Perches is	60321	7	10

Which divide by 160, and the Quotient is 377 Acres, and 1 remaining, which is 1 Perch; to which add the 7 Links that are in the Product of the Multiplication, and the whole Content will be 377 Acres, 1 Perch, and 7 Links, which is half a Perch, and 1 Link.

The 10 Parts being but of a ſmall Value, are ſeldom ſet down.

11
1232(1
60321(377
16000
166
1

Foraſmuch as in ſome Countries they allow 18 *Feet to the Pole or Perch, and in other Countries more, as* 20 *or* 24; *and all allow* 160 *of their Perches to an Acre: It will be convenient here to ſhew how to reduce one kind of Meaſure into another.*

 Example.

Example.

Suppoſe it were required to reduce 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure (that is, 16 Feet and an half to the Pole) into cuſtomary Meaſure of 18 Feet to the Pole.

Firſt, reduce your given Quantity, 4 Acres, 3 Roods, and 4 Perches, into the leaſt Denomination, to wit, Perches; by multiplying the 4 Acres, by 160 (the Number of Perches in one Acre) and the Product is 640 Perches in the 4 Acres: Then multiply the 3 Roods by 40 (the Number of Perches in one Rood) and the Product is 120 Perches in the 3 Roods. Laſtly, add the 640 Perches, and the 120 Perches, alſo the 4 Perches together, and the total Product is 764 Perches, being contained in the given Number, *viz.* 4 Acres, 3 Roods, and 4 Perches.

Secondly, ſquare the two Poles or Perches, that is to ſay, multiply each in itſelf: As firſt, multiply the Pole of 18 Feet by 18 Feet, and the Product is 324 Feet: Then multiply the Pole of 16 Feet and an half, by 16 Feet and an half, and the Product is 272 Feet and 3 Inches.

Thirdly, ſay by the Rule of Three: *As* 324 (which is the Square of the 18 Feet Pole) *is* to 272 Feet and 3 Inches (which is the Square of the 16 Feet and an half Pole) *ſo* is 764 Perches (the Number given) *to* a fourth Number required.

Firſt, multiply 764 by 272 Feet and 3 Inches, and the Product is 207999, which divide by the firſt Number in the Queſtion, namely, 324, and the Quotient is 641 Perches, $\frac{315}{324}$ of a Perch, which being abbreviated, is $\frac{35}{36}$ of a Perch.

Fourthly,

Fourthly, to reduce the Perches into Acres, divide them by 160; ſo 641 Perches being divided by 160, you find 4 Acres and 1 Perch remaining.

Thus you ſee, that 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure, of 16 Feet and a half to the Pole, being reduc'd, is 4 Acres, 1 Perch, and $\frac{35}{6}$ of a Perch cuſtomary Meaſure of 18 Feet to the Pole.

An Example of the whole Work.

The Quantity given is 4 Acres, 3 Roods, 4 Perches of Statute Meaſure, to be reduced to Cuſtomary Meaſure of 18 Feet to the Pole or Perch.

Firſt reduce the Quantity given into the leaſt Denomination, namely, Perches.

Firſt reduce the	4 Acres.
The Perches in 1 Acre is	160; being multiplied,
The Perches in 4 Acres is	640
Secondly, Reduce the	3 Roods.
The Perches in 1 Rood is	40; being multiplied,
The Perches in 3 Roods is	120

The third Number 4 being the leaſt Denomination, namely, Perches, needs no reducing.

Now add the Perches together,	640
	120
	4
And you find the Quan. given contains	764 Perches.

The Quantity given being brought into Perches, the enſuing Work will be to find the Number of Superficial Feet that each Pole contains.

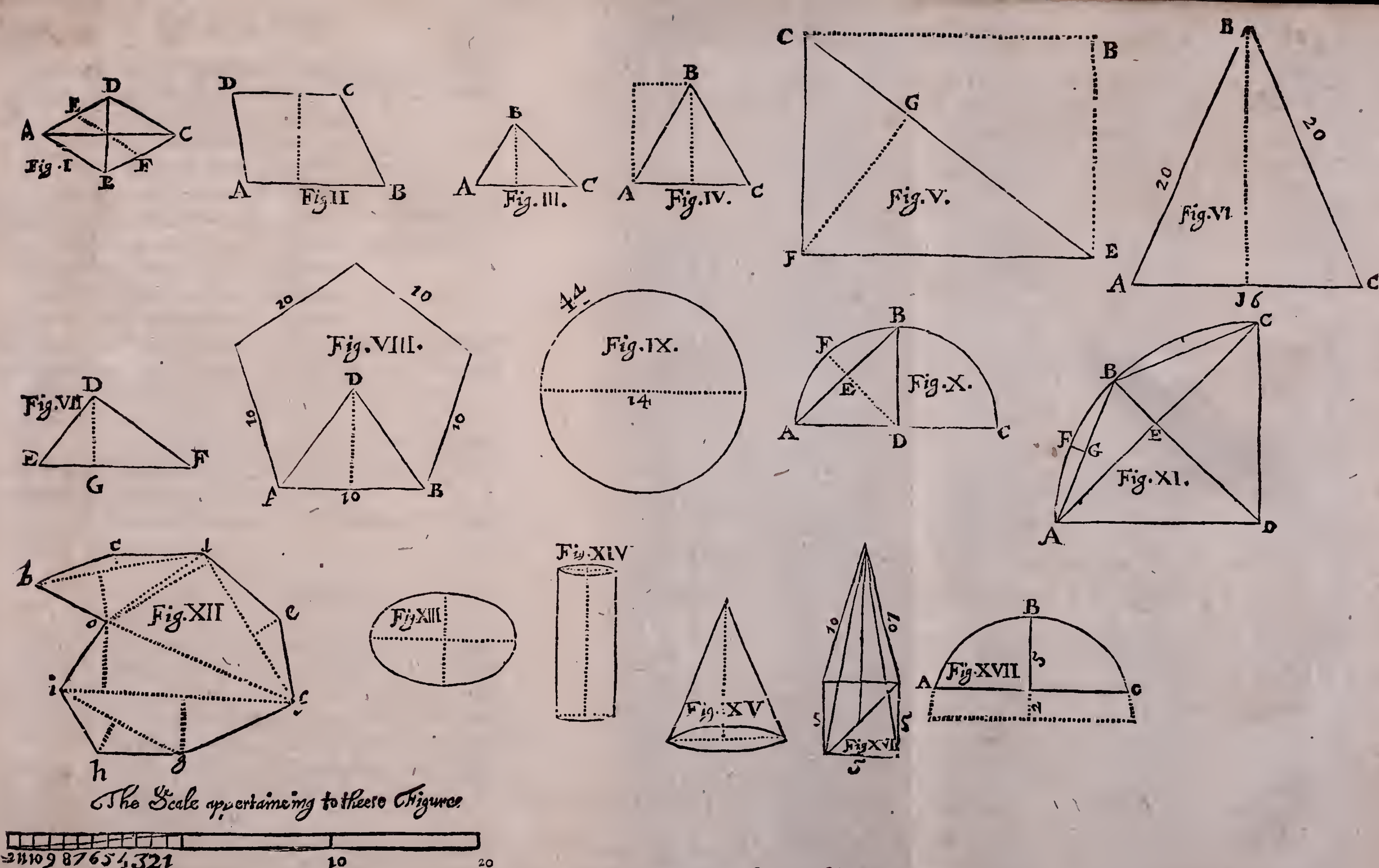
And firſt the Pole of	18 Feet.
Being ſquared, or multiplied by	18
	144
	18
Contains, or the Product is	324 Feet.

	F.	I.
Secondly, The Pole of	16	6
Being ſquared, or multiplied by	16	6
	96	
	168	
	8	3
Contains, or the Product is	272	3

Thirdly, by the Rule of Three ſay, *As* 324 Feet *is to* 272 Feet 3 Inches, *ſo* is 764 Perches *to* a Fourth Number required.

	F.	I.
Multiply the Second Number	272	3
By the Third	764	
	1088	
	1632	
	1904	
	191	
The Product is	207999	

Which



Place this at the End of the Fifth Book.

Which divide by the firſt Number 324, and the Quotient is 641 Perches and $\frac{315}{324}$ Parts of a Perch, which abbreviated is $\frac{35}{36}$ Parts of a Perch, which is almoſt a Perch.

```
      (3
     6(1
  1353(5
207999(641
 32444
   322
     3
```

Fourthly, Having now reduced the Quantity of given Perches (namely, 764 of Statute Meaſure) to 641 Perches, and $\frac{35}{36}$ of a Perch of Cuſtomary Meaſure of 18 Feet to the Pole or Perch; the next Work is to bring theſe Perches into Acres: Which to do, divide them by 160, and the Quotient (as you ſee in the Margin) is 4 Acres and 1 Perch; to which add the $\frac{35}{36}$ of a Perch, and then you have 4 Acres 1 Perch and $\frac{35}{36}$ of a Perch of Cuſtomary Meaſure (of 18 Feet to the Pole) contained in 4 Acres, 3 Roods, and 4 Perches of Statute Meaſure, of 16 Feet and an half to the Pole.

```
   (1 Perch
641(4 Acres.
160
```

A Second Example.

Suppoſe, on the contrary, that 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch of cuſtomary Meaſure of 18 Feet to the Pole, were required to be reduced into Statute Meaſure of 16 Feet and an half to the Pole.

As in the laſt Example, Firſt, reduce the given Quantity into the leaſt Denomination, to wit, Feet; for in this Example, the leaſt Denomination is Parts of a Perch, which are Feet.

Therefore firſt multiply the 4 Acres by 160, and the Product is 640, to which add the 1 Perch given, and it is 641 Perches.

 Secondly,

Secondly, to bring theſe Perches into Feet, you muſt multiply them by the Number of Feet contained in a Perch; which as you found before in the laſt Example, by multiplying 18 by 18, the Product was 324: So then you muſt multiply 641 Perches by 324 Feet, and the Product will be 207684 Feet: To which you muſt add the Fraction $\frac{35}{36}$ whoſe Value you muſt find thus:

Firſt multiply the Numerator 35 by the Number of Feet in a Perch, to wit, 324, and the Product is 11340, which divided by the Denominator 36, and the Quotient is 315 Feet, which is the Value of the Fraction $\frac{35}{36}$, and muſt be added to the 207684 Feet and then the whole Number of Feet will be 207999. Thus you have the Quantity given reduced into the leaſt Denomination, namely, Feet.

And becauſe the Statute Pole 16 Feet and an half is a Fraction, and being ſquared, it produces a Fraction, and muſt be the firſt Number in the Queſtion, according to the Rule of Three, and by conſequence, the Diviſor of the Diviſion, which cannot be in Vulgar Arithmetick, to divide by an Integer, mix'd with a Fraction, altho' it may be done Decimally: We muſt therefore reduce the Feet into Inches, by multiplying 207999 by 12, the Product is 2495988 Inches; likewiſe multiply 272 Feet by 12, and the Product is 3264, to which add the 3 Inches that belong to the 272 Feet, and it is 3267 Inches.

Then divide 2495988 by 3267, the Number of Inches in a Statute Perch, and the Quotient is 764 Perches of Statute Meaſure, which divide by 160, and you have 4 Acres in the Quotient, and 124 Perches remaining; which 124 divide by 40, the Perches in a Rood, and you have 3 Roods and 4 Perches.

So

So that 4 Acres, 1 Perch, and $\frac{35}{36}$ of a Perch of Customary Measure, being reduced to Statute Measure, contains 4 Acres, 3 Roods, and 4 Perches, which was required to be done.

After the same Method may any Customary Quantity whatsoever be reduced.

I shall not need to mention any more Examples of Land that is circular, or irregular Plats, having already shewn how to find their Contents.

Note, When you measure the Length or Breadth of a Field with your Chain, you must measure in a strait Line as near as you can, otherwise you will make the Length longer than it is.

I shall conclude the measuring of superficial Plains with this Advice:

If you intend to learn any thing that is herein contained, you must not only read, but likewise put Pen to Paper, and cast up the Examples, and also other Examples of your own proposing, and with a little Use, mix'd with Diligence, you will soon attain to your Desire; and you will find this Way of multiplying Fractions into Integers very pleasant and brief, in respect of the other Way in Vulgar Arithmetick, whereby the Integers must be reduced into the least Denomination, which is 4 times more Labour than this Way.

The End of the FIFTH BOOK.

MEASURING

OF

SOLIDS.

The SIXTH BOOK.

By *VEN. MANDEY.*

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OF

MEASURING

OF

SOLIDS.

BY Meaſuring of Solids, is meant the Meaſuring of Stone, Timber, digging of Earth, and all ſolid Magnitude whatſoever.

And as from the Motion of a Line is made a Superficie; ſo likewiſe from the Motion of a Superficie, is made a Solid or Mathematical Body.

And as a Superficie conſiſts of Length and Breadth, ſo a Solid or Mathematical Body conſiſts of Length, Breadth, and Depth (or Thickneſs.)

There is no farther Progreſs; for which Way ſoever a ſolid Body is moved, a ſolid Magnitude will be thereby deſcribed.

In meaſuring of Solids, I ſhall begin with the Cube, it being moſt eaſy to meaſure.

A Cube,

A Cube is a Rectangular or Square Solid, that hath an equal Length, Breadth, and Depth, and is comprehended under ſix equal Squares, and may well be repreſented by a Dye, which is itſelf a little Cube.

And as in meaſuring of Plains, you only multiply the Breadth by the Length, for the gaining of the Content;

So in meaſuring of Solids, you muſt firſt multiply the Breadth by the Length, for the gaining the Superficial Content; and then multiply that ſuperficial Content by the Depth or Thickneſs of the ſolid Body, and the Product is the ſolid Content.

Of Timber or Stone being ſquare and equal-ſided.

PROP. I. *Fig.* I.

Fig. I. *being a Cube* 12 *Inches in Length, and* 12 *Inches in Breadth, and* 12 *Inches in Depth; what is the ſolid Content thereof?*

FIRST multiply 12 Inches the Length, by 12 Inches the Breadth, and the Product is 144 Inches for the ſuperficial Content; which multiplied by 12 Inches the Depth, produces 1728 Inches for the ſolid Content of that Cube.

Example.

†

Example.

	Inches.
The Length	12
The Breadth	12
	24
	12
The ſuperficial Content is	144
Which multiply by the Depth	12
	288
	144
And the ſolid Content required, is	1728

Where by the Way, you may take Notice, That as a ſuperficial Foot contains 144 ſuperficial Inches, ſo likewiſe a cubical or ſolid Foot contains 1728 cubical Inches, being produced as above.

So that by a Foot of Timber or Stone, or any other ſolid Material, in whatſoever kind of Solid it be found, is underſtood a Cube containing 1728 cubical Inches, and conſequently half a Foot Solid contains 864 cubick Inches, and a quarter of a Foot contains 432 cubick Inches.

PROP. II.

If the Side of a Cube of Timber or Stone be 4 Feet, 5 Inches, what is the Content of that Cube?

FIRST find the Content of the Side, by multiplying 4 Feet, 5 Inches by itſelf, that is, by 4 Feet, 5 Inches,

ches, it produces 19 Feet, 6 Inches, 1 Part for the ſuperficial Content of the Side ; which Product multiplied by the ſaid 4 Feet, 5 Inches, produces 86 Feet, 1 Inch, 10 Parts, and 5 Seconds, for the Content required.

Example.

	F.	I.	P.	S.	T.
The Side given is	4	5	0	0	0
Which multiply by itſelf	4	5	0	0	0
	16	2	1		
	1	8			
	1	8			
The Superficial Content of the Side is	19	6	1		
Which multiply by the Depth or Side	4	5			
	76	2	6	5	
	2				
	7	11	4		
The ſolid Content required, is	86	1	10	5	

PROP.

PROP. III. *Fig.* II.

Suppose in Fig. II. ABCD *to be a strait Piece of Timber, terminated at both Ends by two equal long Squares (which may be called the Bases) the Length of which Piece* BD *is* 3 *Feet,* 4 *Inches, and the broader Side of the Base* AB 1 *Foot,* 8 *Inches, and the narrower Side* BC 1 *Foot,* 1 *Inch, and the Content of this Piece be required;*

FIRST find the superficial Content of the Base or End, by multiplying the broader Side 1 Foot, 8 Inches, by the narrower Side 1 Foot, 1 Inch, the Product is 1 Foot, 9 Inches, 8 Parts for the superficial Content of the End; which 1 Foot, 9 Inches, 8 Parts, being multiplied by the Length 3 Feet, 4 Inches, produces 6 Feet, 2 Parts, 8 Seconds, for the solid Content of that Piece of Timber.

Example.

Example.

	F.	I.	P.	S.	T.
The broader Side of the Base AB, is	1	8	0	0	0
The narrower Side BC, is	1	1	0	0	0
Being multiplied.	1	8			
		1	8		
The supeficial Content of the Base is	1	9	8		
Which multiply by the Length AE	3	4			
	34				
	23				
	3	2	8		
	2				
The solid Content required, is	6	0	2	8	

PROP. IV.

Suppose a Piece of Timber or Stone, whose Length is 3 Feet 4 Inches, be terminated at both Ends by two equal Quadrats (or Squares) the Side whereof is 1 Foot 7 Inches; what is the Content of such Piece?

FIRST find the Content of one End or Base, by multiplying 1 Foot 7 Inches, by 1 Foot 7 Inches, it produces 2 Feet 6 Inches, and 1 Part for the Content; then multiply the Content of the End by the Length 3 Feet 4 Inches, and it produces 8 Feet, 4 Inches, 3 Parts, and 4 Seconds, for the solid Content.

An Advertiſement.

The uſual Way among many Men to meaſure Timber, whoſe Sides are unequal (although it be a falſe Way) is this: They uſually add the Breadth and Depth together, and take the half for the Side of a mean Square; and after they have found the Content of that mean Square, as they call it (falſe Square as I term it) they multiply it into (or by) the Length, and the Product they conclude is the Content of the Piece.

Let us compare this falſe Way with the true Way.

Suppoſe the Piece of Timber in the preceding *Prop.* III. whoſe Sides are unequal, to wit, one Side of the Baſe (or End) AB, is 1 Foot 8 Inches, and the other Side of the ſame Baſe (or End) BC, is 1 Foot 1 Inch.

Example of the falſe Way.

Firſt they add the two Sides together, namely, 1 Foot 8 Inches, and 1 Foot 1 Inch, which make 2 Feet 9 Inches, the half whereof is 1 Foot 4 Inches and 6 Parts, which they multiply by itſelf, namely, 1 Foot 4 Inches and 6 Parts, and the Product is 1 Foot 10 Inches 8 Parts and 3 Seconds, which they alſo multiply by the Length 3 Feet 4 Inches, and the Product is 6 Feet, 3 Inches, 7 Parts, and 6 Seconds, for the Content, which is more than the true Content by above a quarter of a Foot, and the more unequal the two Sides are, the more falſe this Way of Meaſuring gives the Content.

Of

Of Timber or Stone, consisting of 3, 5, or more Sides.

PROP. V. *Fig.* III.

Admit Fig. III. *to be a strait Piece of Timber or Stone, whose Length* A B *is* 30 *Feet, to be terminated at both Ends with two equilateral Triangles, the Side whereof* G F *is* 12 *Feet; the Content whereof is required.*

Take this as a general Rule for regular Polygons.

FIRST measure all the Sides round about, or girt them with a Line; then take half that Length for one Sum or Breadth; then find the Center H of the Base, which you may do by finding the Middle of the Base Line, as also of one Side; and from the Angles opposite to the Base Line and that Side, draw two Lines to the Middle of them, and where those Lines intersect, there is the Center of the Triangle, as in the Figure at H; then take the nearest Distance from the Center to one of the Sides for the other Sum or Depth; then multiply that Breadth and Depth one by the other, and the Product is the superficial Content of the End or Base of the Piece, which Content multiplied by the Length of the Piece, produces the solid Content.

Example.

The Girt of the 3 Sides is 36 Feet, the half whereof is 18 Feet, which multiply by the nearest Space from the Center to one Side, which is 3 Feet, 5 Inches, 6 Parts, 10 Seconds, and the Product is 62 Feet, 4 Inches, and 3 Parts, for the Content of the Base or End of the Piece, which multiplied by the Length of the Piece 30 Feet, produces 1870 Feet, 7 Inches, and 6 Parts for the Content of the Piece.

The Operation.

	F.	I.	P.	S.	T.
The 3 Sides being added, is 36 Feet, the half whereof is	18	0	0	0	0
Which multiply by the nearest Space from the Center, to wit,	3	5	6	10	0
	54	9	15		
	7	6			
The superficial Content of the Base is	62	4	3		
Which multiply by the Length	30				
	1860	7	6		
	10				
The solid Content of the whole Piece is	1870	7	6		

Otherwise

Otherwise thus:

Find the Perpendicular of the Triangle by the 4th Prop. of meaſuring of Plains; then multiply the Perpendicular by half the Baſe, the Product whereof being multiplied into the Length, gives the ſolid Content.

Example.

	F.	I	P.	S.
The Perpendicular is	10	4	8	6
Which multiply by half the Baſe	6			
	60	4	3	
	2			
The Content of the End is	62	4	3	
Which multiply by the Length	30			
	1860	7	6	
	10			
The Content (as before) is	1870	7	6	

PROP. VI. *Fig.* IV.

Admit Fig. IV. *were a strait Piece of Timber, terminated at both Ends with two equal Pentagons, whose Side* AB *is* 12 *Inches, and the nearest Distance from the Center* C, *to the Middle of the Side at* D *is* 8 *Inches, and the Length* EF, 15 *Feet, and the Content required.*

Example.

	F.	I.	P.
THE 5 Sides added together is 5 Feet, the half whereof is	2	6	0
Which multiply by CD	0	8	
	1	4	
		4	
The Content of the Base is	1	8	
Which multiply by the Length	15		
	15		
	10		
The solid Content is	25		

Of

Of squared Timber or Stone, being bigger at one End than at the other, in which Form some Timber Trees grow; and being fell'd, are so hewn, and brought to a Square.

PROP. VII. *Fig.* V.

Admit Fig. V. *to be a strait Piece of Timber or Stone, terminated at both Ends with two unequal Squares; the Side of the greater Square (or End)* AB, *is* 1 *Foot* 8 *Inches, and the Side of the lesser Square (or Upper-end)* EF, *is* 1 *Foot* 3 *Inches, and the Length* AE 20 *Feet; what is the Content?*

The Rule is this:

1. FIND the Content of the lower End (or greater Square) ABC. 2. Find the Content of the upper End (or lesser Square) EGDF. 3. Multiply one Content by the other. 4. From that Product extract the Square Root. 5. Add this Square Root, and the whole Content of both the Bases into one Sum. 6. Multiply this Sum by one Third Part of the Altitude (or Length) of the Piece, and the Product will be the solid Content required.

Which Rule is demonſtrated by Mr. *Oughtred*, for meaſuring a Segment of a Pyramid, in *Ch.* 19. *Prob.* 21. of his *Clavis Mathematicæ.*

Example.

Firſt, Find the ſuperficial Content of the lower End ABC, by multiplying 1 Foot 8 Inches, by 1 Foot 8 Inches, it produces 2 Feet, 9 Inches, 4 Parts, for the ſuperficial Content of the lower End.

Secondly, Find the Content of the upper End (or leſſer Square) E G D F, by multiplying 1 Foot 3 Inches, by 1 Foot 3 Inches, the Product whereof is 1 Foot, 6 Inches, and 9 Parts, for the ſuperficial Coutent of the leſſer Square (or upper End.)

Thirdly, Multiply one Content by the other, to wit, 1 Foot, 6 Inches, and 9 Parts, by 2 Feet, 9 Inches, and 4 Parts, and the Product is 4 Feet, 4 Inches, and 1 Part.

Fourthly, Extract the Square Root of 4 Feet, 4 Inches, and 1 Part, which Root is 2 Feet and 1 Inch.

Fifthly, Add this Square Root and the Content of both the Baſes together, all three in one Sum, and the Product is 6 Feet, 5 Inches, and 1 Part.

Sixthly, Multiply the laſt Sum by a Third Part of the Length, namely, 6 Feet 8 Inches, and the Product will be 42 Feet, 9 Inches, 10 Parts, and 8 Seconds, for the ſolid Content required.

The

The Operation.

	F.	I.	P.	S
The Side AB is	1	8	0	0
Which square or multiply by itself	1	8	0	0
	1	8		
		8		
		5	4	
The superficial Cont. of the lower End is	2	9	4	
The Side of the lesser Square, or upper End, is	1	3		
Which square or multiply by itself	1	3		
	1	3	9	
		3		
The superficial Cont. of the upper End is	1	6	9	
Which multiply by the Cont. of the lower	2	9	4	
The Product of one End multiplied by the other is	4	4	1	
The Square Root whereof is	2	1		
To which add the Cont. of the lower End	2	9	4	
Also add the Content of the upper End	1	6	9	
And the Product is	6	5	1	
Which multiply by 1 Third of the Leng.	6	8		
	36	3	4	
	2	6	6	
	4			8
The solid Content required is	42	9	10	8

An

An Advertisement.

The usual Way which a great many Men use in measuring such a Piece of Timber or Stone, is this, which is a false Way, and finds a Content less than the true Content.

They take the Sides of the Square about the Middle of the Piece, and this Square of the Middle of the Piece they suppose to be a mean Square between the greater End and the lesser End; then they multiply the Content of this Square by the Length of the Piece.

Let us compare this false Way with the true Way, and see what Difference there will be between the Content of the Piece found by the foregoing 7th *Prop.* and the Content of the same Piece found according to the usual (but false) Way.

Example.

The Side of the Square taken in the Middle of the Piece is 1 Foot, 5 Inches, and 6 Parts, which multiplied by itself produces 2 Feet, 1 Inch, 6 Parts, and 3 Seconds, for the Content of the mean Square, as they account it; which Content they multiply by the Length of the Piece 20 Feet, and it produces 42 Feet, 6 Inches, and 5 Parts, for the solid Content which is above a quarter of a solid Foot less than the true Content found before; and the more the Tree (or Solid whatsoever) diminisheth, the more the Content, measured this false Way, will want of the true Content.

Example.

Example.

Suppoſe a Piece of Timber, being the ſame Length 20 Feet, were 1 Foot 8 Inches Square, at the lower (or greater) End, and 1 Foot Square at the upper (or leſſer) End, and the Content required.

Firſt, We will find it the true Way.

	F.	I.	P.	S.
The Content of the greater End (whoſe Side is 1 F. 8 I.) is	2	9	4	0
The Content of the leſſer End (whoſe Side is 1 F.) is	1	0	0	0
Which being multiplied produces	2	9	4	
The Square Root whereof is	1	8		
To which add the Content of the greater End	2	9	4	
Alſo add the Content of the leſſer End	1	0	0	
The Product is	5	5	4	
Which mult. by $\frac{1}{3}$ Part of the Length	6	8		
	30	3	4	
	2	6	2	8
	3	4		
		2		
The true ſolid Content is	36	3	6	8

Now

Now let us ſee what the uſual (but falſe) Way will produce.

	F.	I.	P.
The Side of the Square taken in the Middle is	1	4	0
Which multiply by itſelf, namely,	1	4	0
	1	4	
		4	
		1	4
The Content of the mean Square is	1	9	4
Which multiply by the Length	20		
	20		
	15	6	8
The Content (the falſe Way) is	35	6	8

Which Content is leſs than the true Content by almoſt ¾ of a ſolid Foot, which in meaſuring of a great Quantity of Taper-grown Timber would be conſiderable.

Of Pyramids.

A *Pyramid* is a Solid, comprehended under plain Surfaces, and from a Triangular, Quadrangular, or any Multangular Baſe, diminiſheth equally leſs and leſs, till it finiſh or end in a Point at Top; ſuch are the Spires of ſome Steeples.

Note, If a Pyramid be cut into two Segments or Parts by a Plain parallel to the Baſe, one of thoſe Segments or Parts will be a Pyramid (to wit, that which diminiſheth to a Point) and the other Segment or Part will have two unequal Baſes, ſuch as is *Fig.* V. The manner of finding its Content is ſhewn at *Prop.* VII.

PROP.

PROP. VIII. *Fig.* VI.

Admit Fig. VI. *were a Pyramid, whose Base is an Equilateral Triangle, and one Side of the Base* AB *is* 1 *Foot, and the Perpendicular of the Base* CD 10 *Inches,* 4 *Parts,* 6 *Seconds, and the Length of the Pyramid* RS 8 *Feet; what is the Content of this Pyramid?*

The Rule for finding the Content, is this:

MUltiply the Content of the Base by one Third Part of the Height of the Pyramid, and the Product is the solid Content. (By Height is to be understood a Perpendicular Line, that falleth from the Top of the Pyramid, to the Middle or Center of the Base: You are not to understand by Height, the Length of the Slope-line on the Outside RS, for that is longer than the Altitude or Height.) And before we proceed any farther, it will be convenient to shew how to find the Height of any Pyramid.

How to find the Height of any Pyramid.

First, Square the Semidiameter of the Base.

Secondly, Square the Length of the Pyramid, which is the Slope-line RS.

Thirdly, Subtract the lesser Square out of the greater.

Fourthly, Find the Square Root of the Remainder, and that is the Height required.

Example.

Example.

First, The Semidiameter of the Base EF is 3 Inches, 5 Parts, 7 Seconds, which being squared, is 12 Inches (or 1 one of those Inches whereof 12 make a superficial Foot.)

Secondly, The Length of the Pyramid RS is 8 Feet, which being squared is 64 Feet.

Thirdly, The lesser Square, 1 Inch being taken out of the greater Square 64 Feet, there remains 63 Feet and 11 Inches.

Fourthly, The Root of 63 Feet and 11 Inches being extracted, is 7 Feet, 11 Inches, 11 Parts; which is the Height of the Pyramid required.

And this Rule is general for the finding the Height of any Pyramid, whether Triangular, Quadrangular, or Multangular, always remembring to find the Center of the Base, that so you may have the Semidiameter thereof; which Centers are found by drawing Lines from the Angles, to the Middle of the Sides opposite to them.

The Operation.

	I.	P.	S.	T.	
The Semidiameter of the Base being EF, is	3	5	7	0	
You must square, that is, mult. by	3	5	7		
		9	2	1	
		1	3	2	11
		1	3	2	11
			1	9	
			1	9	
The Square of the Semidiameter is	1	0	0	0	10

The

	F.	I.	P.	S.
The Length of the Pyramid RS is	8	0	0	0
Which ſquare or multiply by itſelf	8			
And the Length being ſquared, is	64	0	0	0
Out of which ſubtract the Square of the Semidiameter		1		
And there remains	63	11	0	0
The Square Root whereof (being the Height) is	7	11	11	11 +

Thus having found the Height of the Pyramid, we will proceed in the next Place, to find the Content of the Baſe.

	F.	I.	P.	S.	
The Perpendicular C D is	0	10	4	6	
Which multiply by half the Baſe AC		6			
The Content of the Baſe is	0	5	2	3 +	
Which multiply by $\frac{1}{3}$ of the Height	2	7	11	11	
		10	4	1	10
		2	11	6	2
			1	2	2
			4	7	
				1	9
				4	7
The ſolid Content of the Pyramid, is	1	1	9	11	6

PROP.

PROP. IX. *Fig.* VII.

Admit Fig. VII. *were a Pyramid, whoſe Length* AE *is* 15 *Feet, and the Baſe of it* ABCD *a Square, whoſe Side* AB, *or* CD, *is* 2 *Feet* 6 *Inches; what is the Content?*

FIRST find the Altitude or Height, which is the prick Line EF, by the Rule in the foregoing *Prop.* VIII. then multiply one Third Part of the Heighth by the Content of the Baſe ABCD, and the Product is the Content required.

The Operation.

	F.	I.	P.	S.	T.
The Side of the Square of the Baſe AB, or CD, is	2	6	0	0	0
Which multiply by itſelf	2	6			
	4	3			
	1				
	1				
The Content of the Baſe is	6	3	0	0	0
Which multiply by $\frac{1}{3}$ of the Height	4	11	9	6	4
	24	2	9	3	
	1	6	2	1	6
	5	4	6	2	1
			3		
The ſolid Content of the Pyram. is	31	1	8	6	7

How,

How to find the Third Part of any Number.

Suppose you were to find the one Third Part of the Height of the Pyramid aforesaid.

	F.	I.	P.	S.	T.
Set down your Number thus	14	11	4	7	0
Then say, The $\frac{1}{3}$ Part of 14 Feet is	4	8			
The $\frac{1}{3}$ Part of 11 Inches, is		3	8		
The $\frac{1}{3}$ Part of 4 Parts, is			1	4	
The $\frac{1}{3}$ Part of 7 Seconds, is				2	4
Being added, the Product is	4	11	9	6	4

Which is the Third Part required.

After the same manner you may find the Third Part of any other Number; you may likewise find the Fourth Part or Fifth Part of any Number after the same manner.

PROP. X. *Fig.* VIII.

Admit Fig. VIII. *were a Pyramid, whose Length* AH *is* 15 *Feet, and the Base thereof a Pentagon, whose Side* AB *is* 2 *Feet,* 6 *Inches, and the Content required.*

FIRST find the Content of the Base ABCDE (by the Rule delivered at the 9th Prop. of *Book* 5. *Of Measuring of Plains)* which I will here repeat.

The Rule is this:

As 182 *is to* 125: *So* is the Side of the Pentagon given, *to* the Semidiameter of a Circle inscribed within that Pentagon.

Then say, If 182 give 125, what will (the Side of the Pentagon) 2 Feet 6 Inches give?

	F.	I.
Multiply the Second Number	125	0
By the Third Number	2	6
	250	
	62	6
And the Product is	312	6

Which divide by the first Number 182, and the Quotient is 1 Foot, and 130 remains, which multiply by 12, and the Product is 1560, which divide by 182, and the Quotient is 8 Inches, and 104 remains; which multiply by 12, and the Product is 1248, which divide by 182; and the Quotient is 6 Parts, and 156 remaining; which multiply by 12, and the Product is 1872, which divide by 182, and the Quotient is 10 Seconds, and 52 remains; which multiply by 12, and the Product is 624, which divide by 182, and the Quotient is 3 Thirds, as you may see in the Margin: There is yet a

(130
~~312~~ (1 Foot.
~~182~~

130
12

260
130

1560

(104
~~1560~~ (8 Inches.
~~182~~

Remainder,

Remainder, but the Value of it is not $\frac{1}{1000}$ Part of an Inch; and in many Cafes you will not need to proceed any farther than to Parts of Inches.

$$\begin{array}{r} 104 \\ 12 \\ \hline 208 \\ 104 \\ \hline 1248 \\ \hline \end{array}$$

In the next Place, there is the 6 Inches that belong to the 312 Feet, to be divided by 182; but becaufe the Divifor is greater than the Dividend, multiply the 6 Inches by 144, and the Product is 864 Seconds; then divide the 864 by 182, and the Quotient is 4 Seconds, which you muft add to the 10 Seconds, and they make 1 Part and 2 Seconds.

(156
1248 (6 Parts.
182

$$\begin{array}{r} 156 \\ 12 \\ \hline 312 \\ 156 \\ \hline 1872 \\ \hline \end{array}$$

So is there found the Length of the Semidiameter of the Circle infcribed within the Pentagon, (whofe Side was given) namely, 1 Foot, 8 Inches, 7 Parts, 2 Seconds, and 3 Thirds, which is the neareft Diftance from the Center of the Pentagon, to either of the Sides.

ø(5
1872 (10 Sec.
1822
18

The next Work will be to find the Content of the Bafe of the Pyramid.

	F.	I.	P.	S.	T.
First, add the 5 Sides togther, and they make 12 Feet and 6 Inches the half whereof is	6	3	0	0	0
Which multiply by the Semidiameter found, which is	1	8	7	2	3
	6	3	1	9	6
	4	2	6	1	6
		3	1		
And the Content of the Base ABCDE, is	10	8	8	11	0
Which multiply by a Third Part of the Height of the Pyramid	5				
	50		4	7	
	3	4			
		3	4		
And the Solid Content of the Pyramid, is	53	7	8	7	

I shall not instance any more Examples of Pyramids; for if the Base consist of 6 or more Sides, the Content of the Base may be found by the Rules delivered in *Book* 5. and then by multiplying the Content of the Base into one Third Part of the Height of the Pyramid, the Product is the solid Content.

Of

Of round Solids, ſuch as are Cylinders, Cones, Spheres, Columns, &c.

PROP. XI. *Fig.* IX.

Admit Fig. IX. *were a Cylinder (by Cylinder is meant ſuch a like Solid as is uſed in Gardens for the Rolling of Walks) whoſe Length* AB *is* 6 *Feet, and the Diameter* BC 1 *Foot and* 4 *Inches, and the Content required.*

The Rule for Meaſuring ſuch a Solid, is this:

MUltiply the Content of the Baſe BC, or AD, by the Length AB, and the Product is the ſolid Content.

Firſt let us find the Content of the Baſe, the Rule is this: *As* 452 (that is four times 113) *is to* 355; *So* is the Square of the Diameter *to* the Content required.

	F.	I.	P.	S.
The Diameter is 1 F. 4 I. the Square of it is	1	9	4	0
Which multiply by	355			
	355	118	4	
	266	3		
And the Product is	631	1	4	

 Which

(179
631 (1 Foot.
452

179
12
358
179
2148

(340
2148 (4 Inches.
452

340
12
680
340
4080

(12
4080 (9 Parts.
452

Which divide by the first Number 452, and the Quotient is 1 Foot, as you see in the Margin, and 179 remains; then continuing multiplying the Remainders by 12, and dividing the Products by 452, till you come to the Denomination of Thirds, as you see in the Margin, at last your Quotient is 9 Parts, and 12 remaining, which 12 is inconsiderate in Value; but for the 1 Inch and 4 Parts that belong to the 631 Feet; and for the 12 that remain above your last Division, you may add 6 Thirds; for if you should continue multiplying and dividing the Remains, you would produce 6 Thirds.

	F.	I.	P.	S.	T.
Then the Content of the Base is found to be	1	4	9	0	6
Which must be multiplied by the Length AB	6				
	6	4	6	3	
	2				
And the Product is	8	4	6	3	

Which is the solid Content of the Cylinder.

Of

Of a Cone.

A *Cone* is a Solid, which hath a Circle for its Baſe, from whence it grows equally leſs and leſs (like a Sugar-Loaf) till it finiſh or end in a Point at Top.

PROP. XII. *Fig.* X.

Admit Fig. X. *were a Cone, whoſe Length* AC *is* 15 *Feet, and the Diameter of the Baſe* AB 1 *Foot* 4 *Inches; what is the Content thereof?*

THE Rule for finding the ſolid Content of a Cone, is the ſame as for the finding the ſolid Content of a Pyramid, to wit, by multiplying the Content of the Baſe by one Third Part of the Height.

Firſt find the Height of the Cone (according to the Rule preſcribed in *Prop.* 8. of this Book) for the Height of a Cone is found by the ſame Rule that the Height of a Pyramid is found.

	F.	I.	P.
1. The Semidiam. of the Baſe AB, is	0	8	0
Which ſquare (or multiply by itſelf)	0	8	0
The Square of the Semidiameter is	0	5	4

	F.	I.	P.	S.	T.
2. The Length of the Cone AC is	15				
Which square (or multiply by itself)	15				
	75				
	15				
The Product is	225				
3. Out of the Square of the Length AC, subtract the Square of half the Diam. AB, to wit,		5	4		
And the Remainder is	224	6	8		
The Square Root whereof is	14	11	9	11 *fere*	
Which is the Height of the Cone; of which Height we must take one Third Part, which is	4	11	11	3	8

Having found the Height, and taken a Third Part thereof, the next Work will be to find the Content of the Base.

Then say, by the preceding Rule in *Prop.* XI. *As* 452 *is* to 355: *So* is the Square of the Diameter *to* the Content of the Base.

	F.	I.	P.
The Square of the Diameter is	1	9	4
Which multiply by	355		
	355	118	4
	266	3	
And the Product is	631	1	4

Which

	F.	I.	P.	S.	T.
Which divide by 452, and you will find the Cont. of the Base to be (as in *Prop.* XI.)	1	4	9	0	6
Which multip. by one Third Part the Height of the Cone	4	11	11	3	8
	4	11	11	3	5
	1	4	8	3	
		3	8	8	3
		3	3	8	2
				1	8
				2	2
The ſolid Content of the Cone is	6	11	8	2	8

Note, If a Cone be cut into two Segments (or Parts) by a Plain parallel to the Baſe; one of thoſe Parts will be a Cone, and the other Part will have two unequal Circles for the Baſes: As in *Fig.* XI. the upper Part is a Cone, and the lower Part is a Segment of a Cone, which hath two unequal Circles for the Baſes.

PROP. XIII. *Fig.* IX.

A Segment of a Cone may well be repreſented by a Tree that grows taper or diminiſhing.

ADmit the Segment in *Fig.* XI. were a round Taper Piece of Timber, whoſe Length AB is 15 Feet, and the Diameter of the lower Baſe AC 1 Foot 4 Inches; and the Diameter of the upper End, or Baſe DB, is 1 Foot; what is the Content of this Piece?

The

The Length A B being 15 Feet, and the Diameter A C being 1 Foot 4 Inches; the Length, and likewiſe the Diameter is the ſame as was the Cone in the laſt Propoſition.

For finding the Content of this Piece of Timber (or Segment of a Cone) you muſt have reſpect to the Rule delivered in the preceding 7th Prop. of this Book, for the ſame Rule that ſerveth for ſquare Timber, whoſe Baſes or Ends are unequal, ſerveth likewiſe for round Timber, or Bodies whoſe Baſes or Circles at the Ends are unequal.

Example.

The Content of the lower Baſe AC, is, by the laſt foregoing Propoſition found to be 1 F. 4 I. 9 P. 0 S. 6 T.

Now we muſt find the Content of the upper Baſe, or leſſer End BD, whoſe Diameter is 1 Foot.

And becauſe the Square of the Diameter is but 1 Foot, the Content of the Baſe cannot be ſo much as 1 Foot; therefore we muſt ſtate the Queſtion in Inches, ſaying thus:

If 452 give 355; what will the Diameter, whoſe Square is 12 Inches, give?

For, if you remember, the Rule is, *As* 452 *is to* 355: *So* is the Square of the Diameter, *to* the Content of the Circle of the Baſe (or End of the Piece.)

Therefore

	Inches.
Therefore multiply the ſecond Number	355
By the third Number in the Queſtion, to wit,	12
	710
	355
And the Product is	4260

Which divide by the firſt Number 452, and the Quotient is 9 Inches, and $\frac{192}{452}$ of an Inch; which, when reduced, as is ſhewn before, by multiplying the Remainders by 12, and dividing the Products by 452, you will find 9 Inches, 5 Parts 1 Second, and 2 Thirds, for the Content of the upper End or Baſe of the Piece.

(192
4260 (9 Inches.
452

Having now the Content of both the Baſes, the next Work will be to multiply them one by the other.

	F.	I.	P.	S.	T.	Fo.
The Content of the leſſer Baſe BD, is	0	9	5	1	2	0
Which multiply by the Content of the greater Baſe AC	1	4	9	0	6	0
		9	6	9	2	9
		3	1	8	4	
			5	3	9	8
				1	4	6
						1
						2
The Product of the two Baſes multiplied, is	1	1	1	10	9	2

The Square Root whereof is 3 I. 7 P. 6 S.

Which

Which Square Root, and the Content of both the Bases, must be added together.

	F.	I.	P.	S.	T.
The Square Root is	0	3	7	6	0
The Content of the greater Base is	1	4	9	0	6
The Content of the lesser Base is	0	9	5	1	2
And the Product of the Addition is	2	5	9	7	8
	5				
Which should be multiplied by $\frac{1}{3}$ Part of the Height, namely, 4 F. 11 I. 11 P. 3 S. 8 T. but because $\frac{1}{3}$ Part of the Height, and $\frac{1}{3}$ Part of the Length differ but by 8 Seconds, you may multiply it by $\frac{1}{3}$ Part of the Length, namely, 5 Fe.	10				
	2	1			
		3	9		
			2	11	
				3	4
and the Product is	12	5	0	2	4

Which is the solid Content of the Segment of the Cone.

Of a Globe or Sphere.

A Globe or Sphere is a perfect round Body, contained under one Circular Plain, the middle Point whereof is called the Center, from whence all strait Lines drawn to the Outside are of equal Length, and are called Semidiameters, or rather Semiaxes.

PROP.

PROP. XIV. *Fig.* XII.

Let Fig. XII. *be a Sphere, whose Axis* AB *is* 8 *Feet; what is the solid Content thereof?*

The Rule is this:

A*S* 21 *is to* 11 (or, *As* 42 *to* 22): *So* is the Cube of the Diameter or Axis *to* the solid Content required.

	F.
The Axis is 8, and the Square of it	64
Which multiplied by the Axis	8
The Cube of the Axis is	512

Then say by the Rule of Three: If 21 give 11, what will (the Cube of the Diameter) 512 give?

Multiply 512 by 11, and the Product is 5632 (as you see in the Margin) which divide by the first Number in the Question, to wit, 21, and the Quotient is 268 Feet, and $\frac{4}{21}$ of a Foot, which being reduced, is 2 Inc. 3 Parts, and 5 Seconds.

So that the solid Content of the Sphere, whose Axis is 8 Feet, is 268 Feet, 2 Inches, 3 Parts, and 5 Seconds.

```
 512
  11
----
 512
512
----
5632
----

 1 1
1 4 7 (4
5 6 3 2 (268 Feet.
2 1 1 1
  2 2
```

PROP.

PROP. XV.

A Sphere is given, whose solid Content is 268 Feet, 2 Inches, 3 Parts, and 5 Seconds; what is the Length of the Axis?

The Rule is this:

AS 22 *is* to 42: *So* is the solid Content given, to wit, 268 Feet, 2 Inches, 3 Parts, 5 Seconds, *to* the Cube of the Axis, whose Length is required.

State the Question thus: If 22 give 42, what will 268 F. 2 I. 3 P. 5 S. give?

F.	I.	P.	S.
268	2	3	5
42			
536			
10727	10	6	
		17	6
11263	11	11	6

1 2 4
1 1 2 6 4 (512
2 2 2 2
2 2

Multiply the third Number by the second (as in the Margin) and the Product is 11264 Feet *fere*; which divide by the first Number in the Question 22, and the Quotient is 512 (which is the Cube of the Axis) the Cube Root whereof is 8 Feet, which is the Length of the Axis required.

The briefest Way to extract Square and Cube Roots, is by a Table of Logarithms.

PROP.

PROP. XVI. *Fig.* XII.

Admit in Fig. XII. CED *were a Segment (or Portion) of a Sphere, whose Segment of the Axis* EF *is* 2 *Feet (the whole Axis* AB *being* 8 *Feet) and the Chord (or Subtenſe) of the Segment* CD, *is* 6 *Feet,* 11 *Inches, and* 2 *Parts; what is the ſolid Content of this Portion of the Sphere?*

The Rule is this:

First, increaſe the Altitude of the other Segment (not given) by half the Axis.

Then ſay by the Rule of Three: *As* the Altitude of the other Segment not given, *is* to the Altitude (or Height) of the given Segment (or Portion) *So* is the Altitude of the other Segment increaſed by half the Axis, *to* a fourth Proportional.

Secondly, Square half the Chord (or Subtenſe) of the given Segment, and multiply the Square of that half Chord by the fourth Proportional found by the firſt Work (or Rule.)

Then ſay by the Rule of Three: *As* 21 *is* to 22: *So* is the Product of the Square of half the Chord of the Segment given, multiplied by the fourth Proportional (found as above) *to* the ſolid Content of the Segment given.

Example.

Example.

The Segment not given is CDG, whoſe Altitude FG is 6 Feet, which muſt be increaſed by half the Axis, namely, 4 Feet, and then it is 10 Feet.

Then ſay, *As* (the Altitude of the other Segment not given, which is) 6 Feet, *is* to (the Altitude of the given Segment) 2 Feet: *So* (is the Altitude of the other Segment not given, being increaſed by half the Axis, to wit, 6 Feet more 4 Feet, that is) 10 Feet, *to* a fourth Proportional required.

Let us find this fourth Proportional.

Firſt, Multiply 10 F. by 2 F. and the Product is 20 F. which divide by 6 F. and the Quotient is 3 F. $\frac{3}{6}$ Parts of a Foot, that is, 3 F. 4 I. which is the fourth Proportional.

In the next Place, Square half the given Chord CF; the whole Chord CD is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. *fere*, the Square whereof is 12 F. 0 I. 1 P. 2 S. + which muſt be multiplied by the fourth Proportional found, namely, 3 F. 4 I. and the Product is 40 F. 0 I. 3 P. 10 S. 8 T.

Then ſay by the Rule of Three: *As* 21 *is* to 22: *So* is the Square of half the given Chord, multiplied by the fourth Proportional found (as before) 40 F. 0 I. 3 P. 10 S. 8 T. *to* the Content required.

Then multiply the third Number by the ſecond, and the Product is 880 F. 7 I. 1 P. 6 S. 8 T. which divide by the firſt Number 21, and the Quotient is 41 F. and $\frac{19}{21}$ of a Foot, which (when reduced) is

41

41 F. 11 I. *fere*, which is the ſolid Content of the Segment required.

For note, That a Sphere is equal to two Cones, having the Height and the Diameter of their Baſe the ſame with the Axis of the Sphere: Or, which is all one, a Sphere is two Third Parts of a Cylinder, having the Height and the Diameter of the Baſe the ſame with the Axis of the Sphere.

But in caſe you have only the Segment of a Sphere given, and not the Axis of the whole Sphere, then you muſt find the Axis.

PROP. XVII. *Fig.* XII.

The Segment or Portion of a Sphere being given, and the Axis required;

The Rule is this:

SQuare one half of the Chord of the Segment, and the Product divide by the Altitude of the given Segment; then to the Quotient add the Altitude of the given Segment, and the Product is the Length of the Axis required.

Example.

Admit in Fig. XII. *the Segment given to be* CED, *and the Axis* AB *required to be found.*

The given Chord CD is 6 F. 11 I. 2 P. the half whereof is 3 F. 5 I. 7 P. which being ſquared, is 12 F. which divide (by the Altitude EF) 2 F. and

the Quotient is 6 F. To which add the Altitude of the Segment (EF) 2 Feet, and the Product is 8 F. which is the Length of the Axis of the whole Sphere, whereof C E D is a Segment.

PROP. XVIII.

The Axis of a Sphere being given; to find the ſuperficial Content:

One Rule is this:

AS 7 *is* to 22: *So* is the Square of the Diameter, *to* the ſuperficial Content required.

Example.

Let the Diameter or Axis given be 8 Feet, which being ſquared is 64 Feet.

Then ſay by the Rule of Three: If 7 give 22, what will 64 give?

Multiply 64 by 22, and the Product is 1408; which divide by 7, and the Quotient is 201 Feet and $\frac{1}{7}$ of a Foot; which reduced, is 201 F. 1 I. 8 P. 7 S. *fere*, which is the ſuperficial Content of the Sphere.

On the contrary, If the ſuperficial Content of a Sphere be given, and the Axis required,

The Rule is this:

As 22 *to* 7: *So* is the ſuperficial Content given, *to* the Square of the Axis.

Example.

Example.

The Content given is 201 F. 1 I. 8 P. 7 S. what is the Axis?

Multiply the Content given by 7, and the Product is 1408, which divide by 22, and the Quotient is 64, which is the Square of the Diameter; the Square Root whereof is 8, the Axis or Diameter required.

PROP. XIX. *Fig.* XIII.

A Portion or Segment of a Sphere being given; to find the Content of the Convex Superficies.

ADmit in *Fig.* XIII. BAC to be a Portion of a Sphere, whoſe Altitude is AE 4 Feet, and whoſe Baſe or Chord is BC 8 Feet, and the Content of the Convex Superficies is required.

Draw the ſtrait Line AB from the Pole A, to the Baſe in B; then, I ſay, the Content of a Circle, whoſe Radius (or Semidiameter) is AB is equal to the Content of the Convex Superficies of the Portion BAC.

Let us try.

The Semidiameter AB is 5 F. 7 I. 10 P. 6 S. now the Content of a Circle deſcribed from that Semidiameter (being found by *Prop.* XVII. of *Meaſuring of Plains*) will be 100 F. 6 I. 6 P. which is the Content of the convex Superficies of the Portion BAC, which was required.

Which may be proved by the laſt foregoing Propoſition; for the Content Superficial of a Sphere, whoſe Diameter is 8 F. was found to be 201 F. 1 I. 8 P. +: Now this Portion BAC is half the Sphere of the ſame Axis; therefore double the Content of it, which was found to be 100 F. 6 I. 6 P. and the Product is 201 F. and 1 I. for the Content of the whole; which is the ſame that was found by the preceding Propoſition, within (or want) 8 P. which want of the 8 Parts is cauſed by the Length of the Line AB, it being a ſmall Matter more than 5 F. 7 I. 10 P. 6 S.

The Truth of this laſt Propoſition, that is to ſay, That the Circle deſcribed by the Radius AD, or AB, is equal to the convex Superficies of the Portion of a Sphere BAC, is demonſtrable from the Compariſon of Motion, thus:

Let the Plain AEBD be underſtood to make a Revolution about the Axis AE; and it is manifeſt, that by the ſtrait Line AD, a Circle may be deſcribed; and by the Arch AB, the Superficies of a Portion of a Sphere; and laſtly, by the Subtenſe AB, the Superficies of a right Cone will be deſcribed. Now ſeeing both the ſtrait Line AB, and the Arch AB, make one and the ſame Revolution, and both of them have the ſame extreme Points A and B; the Cauſe why the Spherical Superficies which is made by the Arch, is greater than the conical Superficies which is made by the Subtenſe, is, That AB the Arch is greater than AB the Subtenſe; and the Cauſe why it is greater, conſiſts in this: That although they be both drawn from A to B, yet the Subtenſe is drawn ſtrait, but the Arch angularly, namely, according to that Angle which the Arch makes with the Subtenſe, which Angle is equal to the Angle DAB (for an Angle of Contingence

tingence adds nothing to an Angle of a Segment.) Wherefore the Magnitude of the Angle DAB is the Cause why the Superficies of the Portion described by the Arch AB is greater than the Superficies of the right Cone described by the Subtense AB.

Again, The Cause why the Circle described by the Tangent AD, is greater than the Superficies of the right Cone described by the Subtense AB (notwithstanding that the Tangent and Subtense are equal, and both moved round in the same time) is this: That AD stands at right Angles to the Axis, but AB obliquely; which Obliquity consists in the same Angle DAB. Seeing therefore the Quantity of the Angle DAB is that which makes the Excess both of the Superficies of the Portion, and of the Circle made by the Radius AD, above the Superficies of the right Cone described by the Subtense AB; it follows, That both the Superficies of the Portion, and that of the Circle do equally exceed the Superficies of the Cone. Wherefore the Circle made by AD or AB, and the Spherical Superficies made by the Arch AB, are equal to one another, which was to be proved.

Of the Proportions between a Cube, a Prism, and a Pyramid, a Cylinder, Sphere, and Cone, whose Altitudes and Bases are as follow.

	F.	I.	P.	S.	T.
A *Cube*, whose Side or Base is 1 F. 4 I. the Content is	2	4	5	4	0

	F.	I.	P.	S.	T.	Fo
A *Prism* (having its Base and Altitude severally equal to the Base of the aforesaid Cube, or of any other Rectangular Parallelepipedon is the half of it, and the Content is	1	2	2	8	0	0
A *Square Pyramid*, having its Height and Base severally equal to the Base of the aforesaid Cube, is $\frac{1}{3}$ Part of it, and its Content is	0	9	5	9	4	0
A *Triangular Pyramid*, whose Height is as aforesaid, and each Side of the Base as afore (that is, 1 F. 4 I.) the Solid Content is	0	4	1	3	1	4
A *Cylinder*, having the same Height and Diameter with the Cube aforesaid, the Solid Content of such a Cylinder is	1	10	4	2	1	0
A *Sphere*, whose Axis is as aforesaid, its Solid Content is	1	2	10	9	4	8
A *Cone*, whose Altitude and Diameter of its Base are severally equal to the Side of the Cube aforesaid, its Solid Content is	0	7	5	4	8	4

Hence it appears that a *Cube* is double the *Prism*, and three times as much as the *Square Pyramid* of equal Base and Altitude.

The

The *Triangular Pyramid* is something more than $\frac{1}{2}$ of the Cube.

The *Cylinder*, whose Diameter and Height is severally equal to the Height of the Cube, is in proportion to it as 11 to 14; or the Cylinder contains 11 of those Parts whereof the Cube contains 14.

The *Globe* or *Sphere*, whose Axis is equal to the Height of a Cube, contains 11 such Parts, whereof the Cube contains 21; or the Sphere is in proportion to the Cube, as 11 to 21.

The *Cone*, whose Diameter and Altitude are severally equal to the Altitude and Diameter of the Cylinder, is in proportion to the Cylinder as 1 to 3.

Whence it appears, that the Sphere is $\frac{2}{3}$ of the Cylinder, and the Cone $\frac{1}{2}$ of the Sphere.

Of GAUGING.

GAuging is comprehended in the Measuring of *Solids*, and there is only this Difference between Gauging (or measuring of Vessels) and measuring of other Solids: The Content of the latter is given in Feet, Inches, Parts, *&c.* but the Content of the former is given in Gallons, Quarts, Pints, *&c.*

But before you can give the Content in Gallons, *&c.* you must first find the solid Content in Cubick Inches, Parts, *&c.* and having first found the Content in Inches and Parts, you must afterwards reduce them into Gallons, Quarts, and Pints, *&c.*

There are several Methods shewn for the finding the Content of these irregular Solids; but the Method shewn by Mr. *Oughtred*, is generally esteemed for one of the best; and therefore I shall make use of his Method.

Most Liquid Vessels, such as are Pipes, Hogsheads, Barrels, Kilderkins, Firkins, &c. are made in form of a Spheroides, having the two Ends equally cut off; and accordingly may be measured thus:

Measure the two Diameters of the Vessel in Inches, the one at the Bung-hole, the other at the Head, and also the length within; and by the Diameters found, find out the Areas, or Contents of the Circles: Then add together two third Parts of the Content of the greater Circle, and one third Part of the Content of the lesser Circle. Lastly, Multiply the Product of these two Sums added together, by the Length of the Vessel; so shall you have the Content of the Vessel in Cubick Inches.

Of which 231 make a Wine Gallon; and 272, and 3 Parts, make an Ale or Beer Gallon, according to Mr. *Oughtred*, who would have a Gallon to consist of a Number of Cubick Inches, the Square Root whereof is Palms $5\frac{1}{2}$.

That 231 Cubick Inches make a Wine Gallon, is the Opinion of most Men; but the Quantity of the Ale or Beer Gallon, is not as yet fully agreed on: Formerly the Ale Gallon hath been accounted to contain 288 Cubick Inches, and $\frac{3}{4}$ or 9 Parts; but since the Excise, it is computed to contain but 282 Cubick Inches.

PROP.

PROP. XX.

Admit a Vessel to have the Diameter at the Bung 32 Inches, and at the Head 18 Inches, and the Length 40 Inches, what is the Content of this Vessel in Gallons, and Parts of a Gallon?

THese Dimensions being given in Inches, and the Content required in Inches, you must have respect to the Note at *Page* 288. Book 5.

We must, first of all, find the Content of the two Circles belonging to the two Diameters given, by the second Rule delivered at the 17th Proposition of the Fifth Book.

	Inches.
The Square of the Diameter 32, is	1024
Which according to the Rule, multiply by	355
	5120
	5120
	3072
And the Product is	363520

Which divide by 452 (according as the aforesaid Rule directs) and the Quotient is (as in the Margin) 804 Inches, and $\frac{112}{452}$ Parts of an Inch; which reduced, is 804 I. 2 P. 11 S. 8 T. for the

(1
49(12 I.
363520 (804
45222
455
4

Content

Content of the greater Circle, to wit, the Circle at the Bung, whose Diameter was 32 Inches.

	Inches.
In the next Place, the Square of the Diameter 18 Inches, is	324
Which multiply by	355
	1620
	1620
	972
And the Product is	115020

Which divide by 452 (as in the Margin) and the Quotient is 254 Inches, and $\frac{212}{452}$ of an Inch; which reduced, is 254 I. 5 P. 7 S. 6 T. and is the Content of the lesser Circle at the Head, whose Diameter was 18 Inches.

(2
20(1
2462(2
115020 (254Inches.
45222
455
4

The next Work is, to take the two third Parts of the Content of the greater Circle, and one third Part of the Content of the lesser Circle, and add them together.

The

	I.	P.	S.	T.	
The Content of the greater Circle, is	804	2	11	8	
The Content of the lesser Circle, is	254	5	7	6	
Two third Parts of the greater Circle, is	536	1	11	9	+
And one third Part of the lesser Circle, is	84	9	10	6	
Which being added, the Product is	620	11	10	3	
Which multiply by the Length	40				
	24800	33	4		
	36	8	10		
The Content of the Vessel in Cubick Inches, is	24839	6	2		

The next Work will be, to find how many Gallons is contained in 24839 Cubick Inches.

And if you would know the Content in Wine Measure, divide the Number of Inches by 231, and the Quotient gives you the Content in Wine Gallons.

But if you would know the Content in Ale Measure, then divide the Number of Cubick Inches by 282, and the Quotient gives you the Content: Or if you would know the Content in Ale Measure, according as the Gallon was accounted (before the Excise) to contain 288 Cubick Inches, and $\frac{3}{4}$ (or 9 Parts.) After you have reduced the Inches into Wine Measure, then multiply the Content in Gallons (and Parts, if there be any) by 4, and divide the Product by

by

†

by 5, and the Quotient gives you the Content required; for 231 being multiplied by 5, produces 1155 I. which being divided by 4, the Quotient is 288 I. and $\frac{3}{4}$ (or 9 P.) for 231 bears the ſame Proportion to 288 Inches and 9 Parts, that 4 does to 5.

Example.

Suppoſe you would know how many Gallons of Wine Meaſure is contained in 24839 Cubick Inches; divide the Number given by 231 (as in the Margin) and the Quotient is 107 Gallons, and $\frac{122}{231}$ Parts of a Gallon; which reduced, is 107 Gallons and an half, and about $\frac{1}{4}$ of a Pint.

```
  (1
 17(22
24839 (107 Gallons.
23111
 233
  2
```

Then if you would know how many Ale Gallons it contains, accounting 288 I. 9 P. to the Gallon, ſay,

As 5 *is* to 4; *ſo* is the Content in Wine Gallons, *to* the Content in Ale Gallons.

Multiply 107 Gallons and an half, by 4, and the Product is 430, which divide by 5, and the Quotient is 86 Gallons, and near a Quarter of a Pint.

```
107 ½
  4
———
428
  2
———
430
———
 3
430 (86 Gal. ¼ Pint.
 55
```

Or if you would know how many Ale Gallons (of 288 I. 9 P. to the Gallon) there is in 24839 I. 6 P. without reducing it firſt to Wine Meaſure, do thus:

Multiply

Multiply the Content given in Inches, by 12 (to bring them into Parts) and the Product is 298068, to which add the 6 Parts that are in the Content besides the Inches, and the Product is 298074 P. (as in the Margin.)

I.	P.
24839	6
12	0
49678	
24839	
298068	
	6
298074	

Then reduce the Inches of the Gallon into Parts, by multiplying 288 by 12, and the Product is 3456; to which add the 9 Parts, and the Product is 3465 Parts; lastly, divide 298074 by 3465, and the Quotient is 86 Gallons, and $\frac{84}{3465}$ of a Gallon; which 84 Parts being reduced is 7 Inches, which is not a quarter of a Pint, for a quarter of a Pint contains 9 Inches.

I.	P.
288	9
12	0
576	
288	
3456	
	9
3465	

By the same Method you may find the Content (of any Number of Inches and Parts given) in Ale Measure, allowing (with M. *Oughtred*) 272 Cubick Inches and 3 Parts to the Gallon.

```
          (8
     2087 (4
   298074 (86 Gallons.
    34655
     346
```

Now because the finding the two third Parts of the Content of the Circle at the Bung, and the one third Part of the Content of the Circle at the Head, is something tedious; I have therefore found out proportional Numbers to shorten the Work, which are as follow.

As

As 1728 I. *is* to 904 I. 9 P. 4 S. 5 T. 2 Fo. *so* is the Square of the Diameter at the Bung, *to* two third Parts of the Content of the Circle thereof.

Again: *As* 1728 I. *is* to 452 I. 4 P. 8 S. 2 T. 7 F. *so* is the Square of the Diameter at the Head, *to* one third Part of the Content of the Circle thereof.

Having these proportional Numbers, the Operation will be after this manner.

First, Square each Diameter.

Secondly, Multiply the Square of the Diameter at the Bung by 904 I. 9 P. 4 S. 5 T. 2 Fo.

Thirdly, Multiply the Square of the Diameter at the Head by 452 I. 4 P. 8 S. 2 T. 7 Fo.

Fourthly, Add these two Products together.

Fifthly, Divide the Product of the Addition by 1728.

Sixthly, Multiply the Quotient by the Length of the Vessel, and the Product is the Content in Cubick Inches.

Example.

	I.	P.	S.	T.	Fo.
I. The Diameter at the Bung is 32 Inches, the Square whereof is	1024	0	0	0	0
II. Which multiply by	904	9	4	5	2
	4096	341	4		
	92160		426	8	
	768			170	8
The Product is	926495	6	0	10	8

III. The

	I.	P.	S.	T.	Fo.
III. The Diameter at the Head is 18 Inches, the Square whereof is	324	0	0	0	0
Which multiply by	452	4	8	2	7
	648	216	54		
	1620			189	
	129608				
	1				
The Product is	146574	5	9	9	0
IV. To which add the former Product	926495	6	0	10	8
And the Sum is	1073069	11	10	7	8

V. Divide this laſt Sum by 1728, and the Quotient is 620 I. and $\frac{1709}{1728}$ Parts of an Inch, which being reduced, is 620 I. 11 P. 10 S. 5 T.

(17
362(0 I.
1073069 (620
172888
1722
17

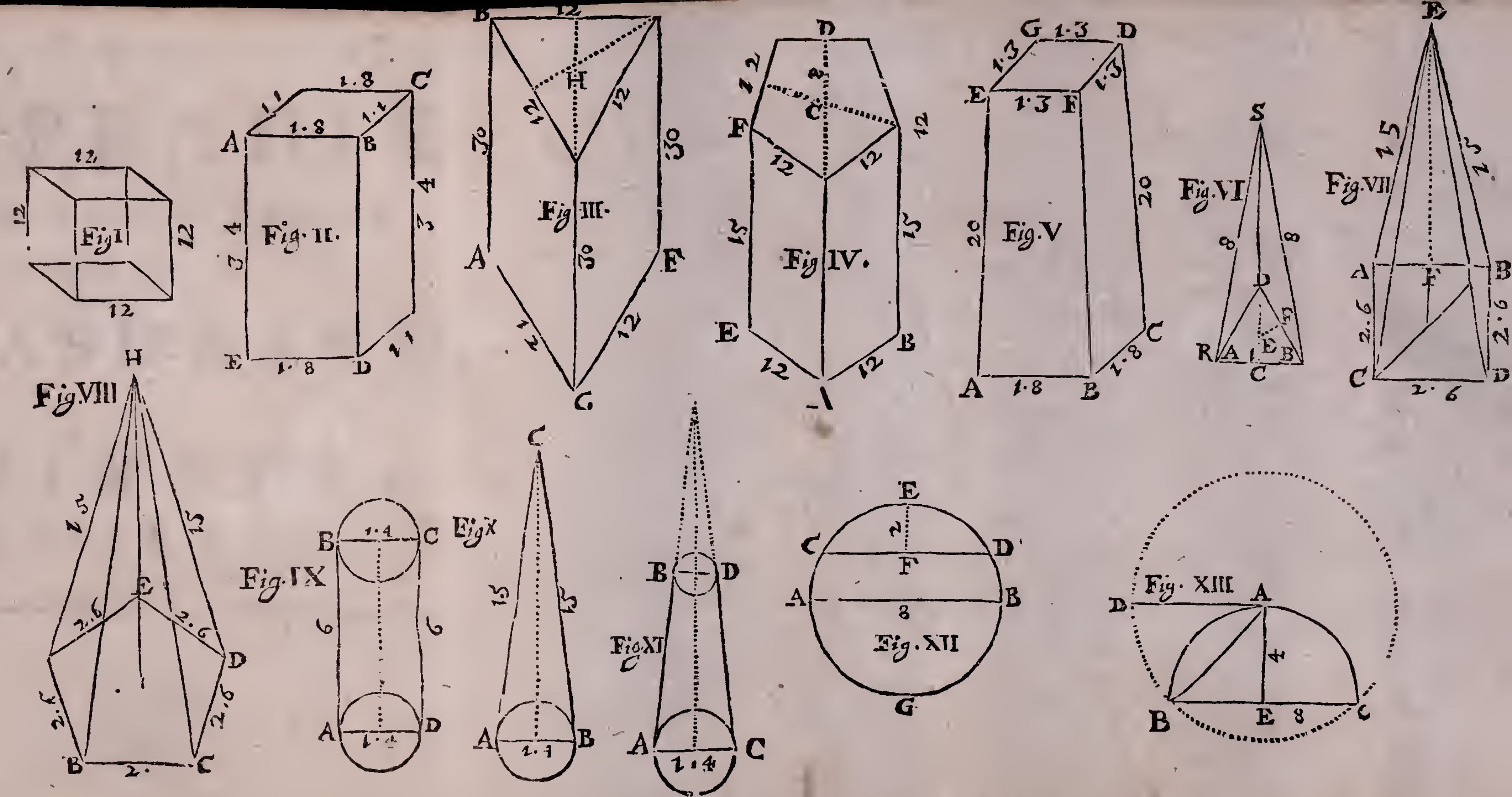
VI. Multiply this laſt Sum by the Length of the Veſſel 40 I. and the Product is (as in the Margin) 24839 I. 6 P. 8 S. 8 T. which is the ſolid Content in Cubick Inches and Parts, and the ſame that was found the firſt Way.

I.	P.	S.	T.
620	11	10	5
40			
24800	33	4	
36	8	16	8
24839	6	8	8

For

For measuring or gauging of *Brewers* Tuns or Vessels, whether they be square or round, or any other Form, work thus:

First find the Content in Cubick Inches, by the preceding Rules of measuring such Bodies; then divide that Content by 282 (the Number of Inches agreed upon for a Gallon to contain since the Excise) and the Quotient is the Content in Gallons. Lastly, to bring the Gallons into Barrels, divide the Number of them by 36 (the Gallons in one Barrel) and the Quotient gives you the Content of the Vessel or Tun, in Barrels.



Place this at the End of the Sixth Book.

THE

THE APPENDIX.

Treating in the FIRST PLACE

OF THE

MEASURING

OF

CHIMNEYS

REFORMED.

By *VEN. MANDEY.*

LONDON:

Printed in the YEAR M.DCC.XXVII.

THE
APPENDIX.

Treating in the first place of the Measuring of Chimneys Reformed.

THE Brick-work of Chimneys being a great part of it not to be seen, hath occasioned much Error in the Measuring thereof; and, it may be, not one Measurer in ten is sensible how the Funnels are wrought, and in what manner they are carried up: I thought it might be a Service to the Publick to correct this Error, that so neither the Gentleman, nor he that pays for the Work, nor the Workman that doth it, may receive any Injury. The usual way of girting round the Jaumbs and Breast of the Chimney, for the Length to be multiplied by the Height of the Story, is erroneous, and more in one Story than in another, as will appear hereafter. Some Years ago, it was my Hap to measure the Bricklayers Work of an House, wherein Mr. *Leonard Sowersby*, the late notorious Measurer, was concerned against me. We had oftentimes measured together before that time, and also since in several

 Places:

Places: But here we happen'd to disagree about measuring the Chimneys very considerably, which gave me occasion to delineate the Chimneys on Paper, and to shew him wherein he was mistaken: The Chimneys were a double Stack placed Diagonal ways against the Party Wall, as you may see in Fig. 1. But before we proceed any farther, it will be convenient that you understand some Symbols which I shall have occasion to use in demonstrating the Mensuration of the following Particulars in this Appendix, only to abbreviate the Work.

Symbols.	*Signification.*
=	Equal to.
+	More.
—	Less.
×	Multiplied by
dd	Deduct.
△	Triangle.
□	Square.
▭	Oblong.
Z	The Sum.
X	The Difference.
1 B	One Brick-work.
$1\frac{1}{2}$B	One Brick and half work.
f. i.	Feet. Inches.

Proceed we now to the Measure of these two Chimneys in the Shop or Parlour Story; for in the Cellar there were Piers wrought, and Arches turn'd, if my Memory fail not.

And first, Mr. *Sowersby*'s way was thus: He girt them round from the Wall, which Length was 16 *f.* 2 *i.* and multiplied that by the Height of the Story, which

which was 10 *f.* 8 *i.* at 1 Brick in Thickneſs, and would allow no more.

feet	*in.*
16	2
10	8
160	0
1	8
10	8
	1
172	5

of 1 Brick-work, which being reduced, is 114 *f.* 11 *i.* of one Brick and $\frac{1}{2}$.

I told him, that he had not allowed the true Meaſure of the Work by a great deal, and that I did conceive the trueſt and readieſt way, was to meaſure them as a Solid, and deduct the Vacancies; which accordingly I did in the following manner: The two Chimneys made the half of a Quadrate, each ſide being 8 *f.* 1 *i.* which Quadrate was divided in two in the middle by the Party Wall, as you may perceive by Fig. 1. Therefore I multiply 8 *f.* 1 *i.* by 8 *f.* 1 *i.* it makes 65 *f.* 4 *i.* half whereof is 32 *f.* 8 *i.* which being multiplied by the Height of the Story, *viz.* by 10 *f.* 8 *i.* is = 348 *f.* 5 *i.* the ſolid Content: In the next place we muſt deduct the Vacancies; the Chimneys were both alike in Magnitude, and each Chimney was 4 *f.* wide between Jaumb and Jaumb, and 1 *f.* 8 *i.* deep, from Breaſt to Back, and the Mantle-trees lay 4 *f.* high from the Floor. The neareſt way, as I conceive, is to deduct the whole as a Vacancy, and afterwards

to add the Breaſts, Wings, and Wieths: The Vacancy of one Chimney is 4 *f.* × 1 *f.* 8 *i.* = 6 *f.* 8 *i.* × 10 *f.* 8 *i.* = 71 *f.* 1 *i.* which being doubled for both Chimneys is 142 *f.* 2 *i.* which deducted from the Solidity, leaves Cubick feet, 206 *f.* 3 *i.* which muſt be reduced to 1 Brick and half in thickneſs, by ſaying, As 14 to 12, ſo 206 *f.* to 176 *f.* 6 *i.* of 1 Brick and half Work. Now becauſe we have deducted all the Breadth and Depth of the Chimneys, by the Height of the Story, and there being the Breaſts, and Wings, and Wieths, included in this Deduction, we muſt add the Quantity of them to the 176 *f.* 6 *i.*

The mean Length of one Wing, and both the four Inches of the Breaſt, and the croſs Wieth, is 8 *f.* 5 *i.* which multiplied by 6 *f.* 8 *i.* the Height of the Mantle-tree to the Top of the Story, is = 56 *f.* 1 *i.* of ½ Brick-work, which reduced is 18 *f.* 8 *i.* of one Brick and half work in one Chimney, the double of which for both Chimneys, is 37 *f.* 4 *i.* which added to 176 *f.* 6 *i.* makes 213 *f.* 10 *i.* of one Brick and half work, the true Content of theſe two Chimneys in the Parlour Story: Then we muſt add the Pargetting of theſe two Funnels in this Story, (for ſince we meaſure no more Brick-work, than is really done in the Chimneys, we ought in Juſtice to meaſure, and allow for the Pargetting; by reaſon the Chimneys are more troubleſome in working than ſtrait Walls; and yet when they are meaſured by the Rod, the Bricklayer hath no greater Price by the Rod for them, than he hath for working the ſtrait Walls: The mean Girt of the four Sides of one Funnel, is 5 *f.* 8 *i.* which multiplied by 6 *f.* 8 *i.* = 37 *f.* 9 *i.* which doubled for both Funnels, is 75 *f.*

6 *i.*

6 *i.* which divided by 9 is 8 Yards + 3 *f.* + 6 *i.* of Pargetting, at 4 *d. per* Yard, is 2 *s.* 9 *d.*

	feet.	in.	
So that the true Measure of these two Chimneys, is	213	10	of 1 ½ Brick-work.
And Mr. *Sowersby* made the Content to be but	114	11	
The Difference	98	11	

A very considerable Quantity of Work, for the Workmen to lose in one Story, besides the Pargetting of the Funnels, and so proportionally in every Story.

EXAMPLE II.

Suppose the Dining-Room Story to be 11 *f. high, and the Chimneys the same Girt as in the Parlour; See Fig.* 1. *and the same Bigness within.*

First, Mr. *Sowersby*'s Way was,

feet.	in.	
16	2	
11	0	
16		
161	10	
177	10	of 1 Brick in Thickness, being = to
118	6	of 1 Brick ½ Work.

Now my way,

		feet.	in.
The Solidity		8	1
		8	1
		64	8
			8
		65	4
	½ =	32	8
	which ×	11	0
		32	
		327	4
The Solid Content		359	4

The Solid Content of the two Chimneys in the Dining-Room Story, is 359 *f.* 4 *i.* of Cubick Feet; now we muſt deduct the two Funnels of the Parlour Story, and the two Funnels of this Story: One Funnel of the Parlour Story, is 1 *f.* 2 *i.* × 1 *f.* 2 *i.* = 1 *f.* 4 *i.* × 11 *f.* the Height of the Story is = 14 *f.* 8 *i.* the Double whereof, for both the Parlour Funnels, is 29 *f.* 4 *i.* Then ſuppoſing the Chimneys in the Dining-room Story to be the ſame Breadth between the Jaumbs, and the ſame Depth as in the Parlour Story, we will deduct the Breadth by the Depth, the whole Height of the Story, and add the Wings, and Breaſts, and Wieths afterwards: Well then 4 *f.* × 1 *f.* 8 *i.* = 6 *f.* 8 *i.* × 11 *f.* the Height of the Story, is = 73 *f.* 4 *i.* for one Chimney, the double whereof

for

for both Chimneys, is 146 *f.* 8 *i.* which added to 29 *f.* 4 *i.* the other Deduction makes 176 *f.* which muſt be deducted from the Solidity, and there remains Cubick Feet 183 and 4 *i.* Say then, if 14 give 12, what will 183 give? Multiply and divide according to the Rule of Proportion, and you will have 156 *f.* 10 *i.* of one Brick and half Work, to which we muſt add the Wings, and Breaſts, and Wieths: The mean Length of both Wings, and of the half Brick-work of the inſide and outſide Breaſt, and croſs Wieth, is 10 *f.* 5 *i.* for one Chimney, the double whereof, is 20 *f.* 10 *i.* for both Chimneys, which multiplied by 7 *f.* the Height of them, from the Mantle-tree to the Top of the Story, produces 145 *f.* 10 *i.* of half Brick-work, in the Breaſts, &c. of both Chimneys, which reduced, is 48 *f.* 7 *i.* of one Brick and half Work, to be added to 156 *f.* 10 *i.* which makes 205 *f.* 5 *i.* of 1 Brick and half Work in the Dining-Room Story, being 86 *f.* 11 *i.* more than Mr. *Sowersby*'s way: next we muſt meaſure the Pargetting of the four Funnels: The four Sides of one of the Parlour Funnels, is = 4 *f.* 8 *i.* (for we ſuppoſe the Funnels to be 14 *i.* □) which multiplied by 11 *f.* the Height of the Story, is = 51 *f.* 4 *i.* the Double whereof is 102 *f.* 8 *i.* for both Funnels: And for the Pargetting of the other two Funnels, from the Mantle-tree, to the Top of the Story, the mean Length of the 4 Sides, is 5 *f.* 8 *i.* × 7 *f.* the Height from the Mantle-tree to the Top of the Story, is = 39 *f.* 8 *i.* for one, the Double whereof, is 79 *f.* 4 *i.* for both Funnels, which added to 102 *f.* 8 *i.* the Pargetting of the Parlour Funnels makes 182, which divided by 9, produces 20 Yards and 2 *f.* which at 4 *d.* *per* Yard, comes to 6 *s.* + 9 *d.*

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EXAMPLE III.

Of Measuring Chimneys that ſtand at right Angles, adjoining to a Party or other Wall, as Fig. 2.

Suppoſe Fig. 2. to be the firſt Story: The uſual way is to girt the Chimney about, for the Length, and multiply that Length by the Height of the Story, allowing the Work to be of the ſame Thickneſs as the Jaumbs, which in ſome Caſes it comes pretty near the Truth, and in others it makes leſs than the true Quantity by a great deal: The Jaumbs of this Chimney being 3 *f.* from the Wall, and the Chimney 4 *f.* within, and the Jaumbs 9 *i.* in Thickneſs, the Girt is 11 *f.* 6 *i.* × by 10 *f.* the Height of the Story, is = 115 *f.* of one Brick-work, which reduced is = 76 *f.* 8 *i.* of one half Brick-work. Let's try it by meaſuring each Particular; the two Jaumbs being added together, make 6 *f.* × 10 *f.* = 60 *f.* the Back, is 4 *f.* × 10 *f.* = 40 *f.* of two Brick-work, which being doubled is = 80 *f.* of one Brick-work: The outſide Breaſt is 4 *f.* × 6 *f.* (that is from the Vacancy under the Mantle-tree, to the Top of the Story, ſuppoſing the Mantle-tree to lie 4 *f.* high from the Floor,) is = 24 *f.* of half Brick-work, = 12 *f.* of one Brick-work: Then the inſide Breaſt, and Wing, and Wieth, the mean Length, is 3 *f.* 9 *i.* × 6 *f.* = 22 *f.* 6 *i.* of half Brick-work, = 11 *f.* 3 *i.* of one Brick-work. All theſe being added together, makes 163 *f.* 3 *i.* of 1 Brick-work, being = 108 *f.* 10 *i.* of one half Brick-work, out of which we muſt deduct the falling back for the Funnel, being 1 *f.* 2 *i.* × 7 *f.* = 8 *f.* 2 *i.* of one half Brick-work, and there remains

100 *f.*

100 *f.* 8 *i.* one and a half Brick-work, being 24 *f.* more than the uſual way. But if the Jaumbs had been one Brick and a half in Thickneſs, the uſual way of Meaſuring would have wanted but 5 *f.* 8 *i.* of the true Meaſure or Quantity; the Pargetting muſt be added, which is about 4 Yards, at 4 *d. per* Yard, comes to 1 *s.* 4 *d.*

EXAMPLE IV.

To meaſure Fig. 2. *in the ſecond Story.*

Suppoſing it to be 11 *f.* high to the Top of the Floor, and the Girt the ſame, as in the firſt Story: 11 *f.* 6 *i.* × 11 *f.* = 126 *f.* of one Brick-work = 84 *f.* 4 *i.* of one and a half Brick-work, according to the uſual way of meaſuring: We will try it the true way, the two Jaumbs = 6 *f.* × 11 *f.* = 66 *f.* of one Brick-work. The Back 4 *f.* × 11 *f.* = 44 *f.* of two Brick-work = 88 *f.* of one Brick-work; the outer Breaſt, 4 *f.* × 7 *f.* = 28 *f.* of half Brick-work = 14 *f.* of one Brick-work; the inner Breaſt, and Wing, and Wieth, is 3 *f.* 9 *i.* the mean Length, which × 7 *f.* the Height from the Mantle-tree to the Top of the Story, is = 26 *f.* 3 *i.* of half Brick-work = 13 *f.* 1 *i.* of one Brick-work, all being added, makes 181 *f.* 1 *i.* of one Brick-work, which reduced, makes 120 *f.* 8 *i.* of one and a half Brick-work, out of which we muſt deduct the Funnel of the firſt Story, and the Funnel of this Story: The Funnel of the firſt Story, is 1 *f.* 2 *i.* × 11 *f.* = 12 *f.* 10 *i.* of one and a half Brick-work, (for we ſuppoſe the Funnels to be 14 *i.* □ within) the Funnel of this Story, is 1 *f.* 2 *i.* × 7 *f.* = 8 *f.* 2 *i.* of one and a half Brick-work, which added to the

the 12 *f.* 10 *i.* makes 21 *f.* to be deducted from 120 *f.* 8 *i.* so there remains 99 *f.* 8 *i.* of one and a half Brick-work, being 15 *f.* 4 *i.* more than the usual way. Then also the Pargetting of these two Funnels is to be measur'd; the four Sides of the first Story Funnel being added together, is $=$ 4 *f.* 8 *i.* $\times$ 11 *f.* $=$ 51 *f.* 4 *i.* the Sides of the Funnel in this Story being added, the mean Length is 5 *f.* 4 *i.* $\times$ 7 *f.* $=$ 37 *f.* 4 *i.* both being added, produce 88 *f.* 8 *i.* which is nine Yards; and 7 *f.* of Pargetting, at 4 *d.* *per* Yard, comes to 3 *s.* 3 *d.*

EXAMPLE V.

To measure Fig. 2. *in the third Story.*

Supposing the Story to be 9 *f.* 6 *i.* to the Top of the Floor, and the Girt the same as in the two other Stories, 11 *f.* 6 *i.* $\times$ 9 *f.* 6 *i.* $=$ 109 *f.* 3 *i.* of one Brick-work, $=$ 72 *f.* 10 *i.* of one and a half Brick-work according to the usual way. Now for the true way, the two Jaumbs added make 6 *f.* $\times$ 9 *f.* 6 *i.* $=$ 57 *f.* of one Brick-work. The Back 4 *f.* $\times$ 9 *f.* 6 *i.* $=$ 38 *f.* of half Brick-work $=$ 19 *f.* of one Brick-work; the two Wieths of the Funnels, is 2 *f.* 4 *i.* $\times$ 9 *f.* 6 *i.* $=$ 22 *f.* 2 *i.* of half Brick-work $=$ 11 *f.* 1 *i.* of one Brick-work; the outside Breast 4 *f.* $\times$ 5 *f.* 6 *i.* $=$ 22 *f.* of half Brick-work, 11 of one Brick; the two Wings and inside Breast, and Wieth, the mean Length, is 5 *f.* $\times$ 5 *f.* 6 *i.* $=$ 27 *f.* 6 *i.* of half Brick-work $=$ 13 *f.* 9 *i.* of one Brick-work; all these being added, make 111 *f.* 10 *i.* of one Brick-work $=$ 74 *f.* 7 *i.* of one and a half Brick-work, which exceeds the usual way, but 1 *f.* 9 *i.* besides the Pargetting: For which, the four Sides of one Funnel, is 4 *f.* 8 *i.* $\times$ 9 *f.* 6 *i.* $=$ 44 *f.*

4 *i.*

4 *i.* the other Funnel being the ſame, is = 44 *f.* 4 *i.* and the Funnel in this Story, the mean Length of the four Sides, is 8 *f.* 2 *i.* × 5 *f.* 6 *i.* = 44 *f.* 11 *i.* all three being added, make 133 *f.* 7 *i.* = 14 Yards and 7 *f.* at 4 *d. per* Yard, comes to 4 *s.* 11 *d.* which added to the Price of the Exceſs of the Brick-work, is 5 *s.* 6 *d.* being too much for a Workman to loſe in one Story.

EXAMPLE VI.

Of Meaſuring Angle Chimneys, or Chimneys ſtanding in an Angle.

We will ſuppoſe Fig. 3. to be the Chimney in the Cellar, or Ground Story, and to be 9 *f.* high from the Foundation to the Top of the Floor. The uſual way of Meaſuring this Chimney, is to take the Length of the Foreſide, or Breaſt, and to multiply it by the Height of the Story, and to allow the Work to be half Brick thinner than the Breadth of the Jaumbs; nay ſome will allow the Work to be but one Brick thick, let the Jaumbs be of never ſo much in Thickneſs: Well then the Length of the Foreſide of Fig. 3. is 7 *f.* × 9 *f.* = 63 *f.* of one and a half Brick-work, becauſe the Jaumbs are two Bricks thick: We will try it the true way; ſuppoſing the Walls A and B that are at right Angles to be old Walls, or if new, to be meaſured before by themſelves: But before we proceed to the true way of Meaſuring theſe kind of Chimneys, it will be convenient to inſert a Rule, whereby we may know by Meaſuring the Foreſide, what the Length of the two ſide Walls A and B are, which are included by the Foreſide, which is from the 47 Propoſition of the firſt Book of *Euclid*; and 'tis thus, Square the Length of the Foreſide, and

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and take $\frac{1}{4}$ part of the Product for the superficial Content: This Rule is of good use for the deducting of any Angle Chimney out of Cieling, or Paving, or Flooring, &c. Therefore note it well, for it is of great use. Now to come to the Business, the Length of the Foreside is 7 *f.* which squared, or × 7 *f.* = 49 *f.* a fourth Part of it is 12 *f.* 3 *i.* for the superficial Content, which multiplied by the Height of the Story, is 9 *f.* produces Cubick Feet 110 *f.* 3 *i.* then we will deduct the Vacancy of the whole Length, Breadth, and Height, including the Breasts, and Wings, and add them afterwards; well then 4 *f.* × 1 *f.* 6 *i.* = 6 *f.* × 9 *f.* = 54 *f.* which deducted from 110 *f.* 3 *i.* leaves Cubick Feet 56 *f.* 3 *i.* which must be reduced to one and a half Brick-work, by making it as 14 to 12, so 56 *f.* 3 *i.* to 48 *f.* 3 *i.* of one and a half Brick-work. In the next place, we must deduct the Funnel from the Mantle-tree upwards, which must stand in the Triangle, let it be 1 *f.* 2 *i.* × 1 *f.* = 1 *f.* 2 *i.* × 5 *f.* = 5 *f.* 10 *i.* of one and a half Brick-work, which deduct from 48 *f.* 3 *i.* and there remains 42 *f.* 5 *i.* of one and a half Brick-work: In the next place we must add the Breasts, and Wings, and Wieths: The outside Breast, is 4 *f.* × 9 *f.* = 36 *f.* of half Brick-work = 12 *f.* of one and a half Brick-work: The inside Breast, and Wing, and Wieth, is 5 *f.* × 5. *f.* (the Height from the Tarsels to the Top of the Floor) makes 25 *f.* of half Brick-work = 8 *f.* 4 *i.* of one and a half Brick-work; these two being added, make 20 *f.* 4 *i.* of one and a half Brick-work, which added to the 48 *f.* 3 *i.* we had before, makes 68 *f.* 7 *i.* of one Brick and a half Work, for the true Content of this Chimney, being 5 *f.* 7 *i.* more than the usual way makes it, the Pargetting is likewise to be measured, and allowed for.

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EXAMPLE VII.

Suppoſing the ſame Chimney in Fig. 3. to be 4 *f.* wide, and the Jaumbs but 14 *i.* thick on the Foreſide in the ſecond Story, which Story we will for Example ſuppoſe to be 10 *f.* high; the Length of the Foreſide will be 4 *f.* + 1 *f.* 2 *i.* + 1 *f.* 2 *i.* = 6 *f.* 4 *i.* which multiplied by 10 *f.* makes 63 *f.* 4 *i.* of one Brick-work, if you allow the Work to be half a Brick thinner than the Jaumbs, as moſt Meaſurers generally do, which reduced, makes 42 *f.* 2 *i.* of one and a half Brick-work. But to find the true Quantity of the Chimney in this Story, we work thus; the Length of the Foreſide is 6 *f.* 4 *i.* which, when ſquared, makes 40 *f.* 1 *i.* 4 *p.* a fourth Part of it is 10 *f.* (rejecting the 1 *i.* and 4 *p.*) for the Area of the Baſe, which multiplied by 10 *f.* the Height of the Story produces 100 Cubick Feet: Then we muſt deduct the Vacancy of the Funnel in the firſt Story, being 1 *f.* 2 *i.* × 1 *f.* × 10 *f.* = 11 *f.* 8 *i.* Alſo we muſt deduct the Vacancy in this Story, being 4 *f.* × 1 *f.* 6 *i.* = 6 *f.* × 10 *f.* = 60 *f.* which added to the other Deduction 11 *f.* 8 *i.* makes 77 *f.* 8 *i.* which taken from 100 *f.* leaves Cubick Feet 22 *f.* 4 *i.* which muſt be reduced to one and a half Brick-work, by making it as 14 to 12, ſo 22 *f.* 4 *i.* to 19 *f.* of one and a half Brick-work, to which we muſt add the Breaſts, and Wieths, and Wings, the mean Length of which, is 9 *f.* × 6 *f.* = 54 *f.* of half Brick-work = 18 *f.* of one and a half Brick-work, which added to the 19 *f.* of one and a half Brick-work, makes 37 *f.* of one and a half Brick-work; to which the Pargetting muſt be added of both Funnels.

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EXAMPLE VIII. Fig. 4.

Suppoſing Fig. 4. to be a double Stack of Chimneys in the Ground Story ſtanding ſingly by themſelves, and the Height of the Story, to be 10 *f.* the Chimneys to be each 4 *f.* wide, and 18 *i.* deep, and the Jaumbs one Brick thick: To meaſure theſe Chimneys the uſual Way, is to girt them about, or to take the Length of one Jaumb, and the Foreſide, and double them for the Length, and multiply it by the Height of the Story; the Length of one Jaumb is 5 *f.* and the Length of the Foreſide is 5 *f.* 6 *i.* which being added, makes 10 *f.* 6 *i.* the Double whereof is 21 *f.* for the Girt, or Length, which multiplied by 10 *f.* the Height of the Story is = 210 *f.* of one Brick-work, which reduced, is = 140 *f.* of one and a half Brick-work, according to the uſual way: Let's try it the true way; the two Jaumbs added, are 10 *f.* which × 10 *f.* = 100 *f.* of one Brick-work = 66 *f.* 8 *i.* of one and a half Brick-work: The Back 4 *f.* × 10 *f.* = 40 *f.* of two and a half Brick-work = 66 *f.* 8 *i.* of one and a half Brick-work: One outſide Breaſt being 4 *f.* in Length, the other being the ſame, and added makes 8 *f.* of half Brick-work; the mean Length of the two inſide Breaſts, and Wings, and Wieths, is 6 *f.* 9 *i.* which add to the 8 *f.* and it makes 14 *f.* 9 *i.* which × by 6 *f.* the Height from the Tarſels to the Top of the Story, makes 88 *f.* 6 *i.* of half Brick-work, which when reduced, produces 59 *f.* of one and a half Brick-work, which three Contents being added, makes 192 *f.* 4 *i.* of one and a half Brick-work, out of which we muſt deduct the Funnel, from the Tarſels to the Top of the Story, which is 1 *f.* × 6 *f.* = 6 *f.* of one and a half Brick-work, becauſe the Funnel, is 14 *i.* by 12 *i.* Well then deduct 6 *f.* from

from 192 *f.* 4 *i.* and there remains 186 *f.* 4 *i.* of one and a half Brick-work, for the true Content of these two Chimneys in these Story; to which must be added the Pargetting of the Funnel, which Content found the true Way, is 46 *f.* 4 *i.* of one and a half Brick-work more than the usual Way, besides the Allowance to be added for the Pargetting.

An Advertisement.

Because the Pargetting of the Funnels is a Work differing both in Quality and Price from the Brick-work, it will be convenient to find out a Method, that it may be reduced to its Equivalency in Brick-work, which may be done thus; one Yard of Pargetting comes to 4 *d.* and one Foot of one and a half Brick-work, at 5 *l.* *per* Rod, comes to 4 *d.* 1 *q.* $\frac{64}{100}$, therefore we may allow one Yard of Pargetting against one Foot of Brickwork, without any great difference, it being but somewhat more than a Farthing, and the Pargetting of a Funnel 1 *f.* in Height is about half a Yard; therefore, when we make Deductions for the Funnels, if we allow but one half of the Deduction, the other half will recompence for the Pargetting, and so we need not trouble ourselves to measure the Pargetting any more hereafter in what follows, but use this Method of taking but one half of the Deduction.

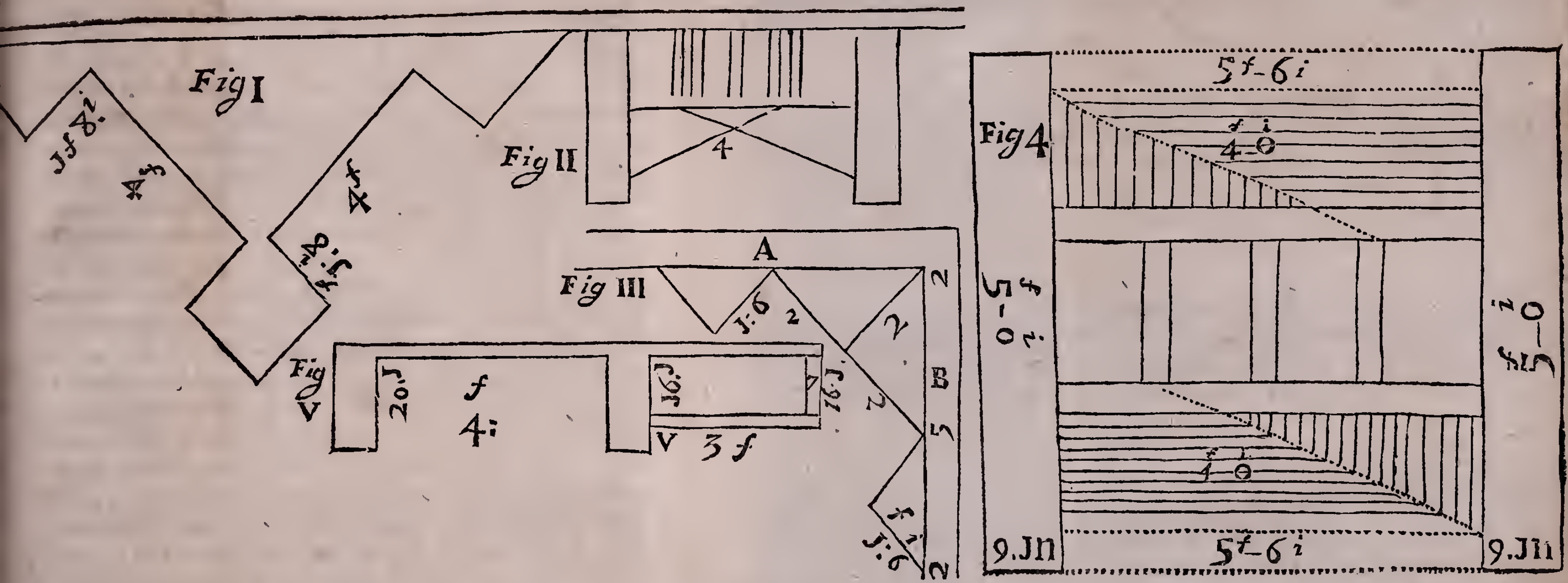
EXAMPLE IX. Fig. XIV.

Let's try the same Stack of Chimneys in the second Story, which we suppose to be 12 *f.* high: And first by the usual way, we will imagine the Girt to be the same as in the Ground Story, *viz.* 21 *f.* × 12 *f.* = 252 *f.* of one Brick-work, which reduced is =

168 $f.$ of one Brick and a half Work for the Content of the two Chimneys, according to the uſual Way of Meaſuring? We will try it our Way, the two Jaums added, make 10 $f. \times 12 f. = 120 f.$ of one Brick-work $=$ 80 $f.$ of one Brick and a half Work: The Back is 4 $f. \times 12 f. = 48 f.$ of two Brick and a half Work $=$ 80 $f.$ of one Brick and a half Work: The two outſide Breaſts, and inſide Breaſts, and Wings, and Weiths being added together, make 14 $f.$ 9 $i. \times 8 f.$ (for we ſuppoſe the Mantle-tree to lie 4 $f.$ high from the Floor) produces 118 $f.$ of half Brick-work, which reduced is 39 $f.$ of one and a half Brick-work, which three Contents being added together, make 199 $f.$ of one and a half Brick-work, out of which we muſt deduct the Funnel in each Story; the Funnel of the Ground Story, is 1 $f. \times 1 f.$ 2 $i. \times 12 f. = 12 f.$ of one Brick and a half Work, and abating one half of the Deduction for the Pargetting, there remains 6 $f.$ of one Brick and a half Work, to be deducted for the Funnel of the Ground Floor in this Story; alſo we muſt deduct the Funnel of this Story from the Mantle-tree, which is 1 $f.$ 2 $i. \times 1 f. \times 8 f. = 8 f.$ of one Brick and a half Work, which allowing one half for the Pargetting of the ſame, there remains 4 $f.$ of one and a half Brick-work; theſe two Deductions being added, make 10 $f.$ to be taken out of 199 $f.$ and then there remains 189 $f.$ of one Brick and a half Work, the true Content of theſe two Chimneys in this Story, which is 21 $f.$ of one Brick and a half Work more than the Contents, by the uſual Way of Meaſuring.

EXAMPLE X. Fig. 5.

The Invention of placing the Funnels without or beyond the Jaums of the Chimneys, as in this fifth Figure,



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Figure, hath been but of late Years; and indeed where the Breadth of the Room is but narrow, it is a good Contrivance, by Reaſon the Chimneys do not come ſo far into the Room as they would do, if the Funnels were placed behind them, there being the Breadth of the Funnels, or 16 *i.* ſaved in the Breadth, which is compenſated in the Length: Now to meaſure this Chimney, ſuppoſing the Wall it ſtands againſt, to be either an old Wall, or elſe to have been meaſured before by itſelf; the uſual Way is to girt the Chimney and Funnel about for the Length, and to multiply that Length by the Height of the Story, which we will ſuppoſe to be 10 *f.* the Girt then is 12 *f.* 2 *i.* × 10 *f.* = 121 *f.* 8 *i.* of one Brick-work = 81 *f.* 1 *i.* of one Brick and a half Work. Let's try it our Way, each Jaum is 2 *f.* from the Wall, that is 19 *i.* the Depth of the Chimney, and 5 *i.* for the Breadth of the Back, which two Jaums being added, make 4 *f.* × 10 *f.* = 40 *f.* of one Brickwork=26 *f.* 8 *i.* of one Brick and a half Work: The Back is 7 *f.* × 10 *f.* = 70 *f.* of half Brick-work =23 *f.* 4 *i.* of one Brick and a half Work; the Front of the Funnels, is 3 *f.* × 10 *f.* = 30 *f.* of Brick-work 16 *i.* thick, which muſt be reduced to Brick-work of 14 *i.* the uſual Thickneſs, by making it as 14 to 16, ſo 30 *f.* to 34 *f.* 3 *i.* of one Brick and a half Work; the outſide and inſide Breaſts, and Wing, and Wieth, are 7 *f.* 10 *i.* × 6 *f.* (that is from the Tarſels to the Top of the Story) = 47 *f.* of half Brick-work = 15 *f.* 8 *i.* of one Brick and a half Work, all theſe being added together, make 99 *f.* 11 *i.* of one Brick and a half Work for the true Content, it being 18 *f.* of one and a half Brick-work, more than the Content found according to the uſual Way of Meaſuring.

A TABLE of Brickwork from 20 s. to 50 s. per Rod.

	d.	q.	C.
20 *per* Rod, 1 Foot is ——	0	3	24
21 ——	0	3	68
22 ——	0	3	86
23 ——	1	0	04
24 ——	1	0	23
25 ——	1	0	40
26 ——	1	0	58
27 ——	1	0	76
28 ——	1	0	93
29 ——	1	1	11
30 ——	1	1	28
31 ——	1	1	46
32 ——	1	1	64
33 ——	1	1	81
34 ——	1	1	99
35 ——	1	2	17
36 ——	1	2	34
37 ——	1	2	52
38 ——	1	2	69
39 ——	1	2	87
40 ——	1	3	05
41 ——	1	3	22
42 ——	1	3	40
43 ——	1	3	58
44 ——	1	3	75
45 ——	1	3	93
46 ——	2	0	11
47 ——	2	0	28
48 ——	2	0	46
49 ——	2	0	63
50 ——	2	0	81

The

From 4 *l.* 10 *s.* to 6 *l.* 10 *s.* *per* Rod.

l.	s.		d.	q.	C.
4	10	*per* Rod, 1 Foot is——	3	3	87
4	11	——	4	0	05
4	12	——	4	0	23
4	13	——	4	0	40
4	14	——	4	0	58
4	15	——	4	0	76
4	16	——	4	0	93
4	17	——	4	1	11
4	18	——	4	1	28
4	19	——	4	1	46
5	00	——	4	1	64
5	1	——	4	1	81
5	2	——	4	1	99
5	3	——	4	2	17
5	4	——	4	2	34
5	5	——	4	2	52
5	6	——	4	2	69
5	7	——	4	2	87
5	8	——	4	3	05
5	9	——	4	3	22
5	10	——	4	3	39
5	11	——	4	3	57
5	12	——	4	3	74
5	13	——	4	3	92
5	14	——	5	0	10
5	15	——	5	0	28
5	16	——	5	0	46
5	17	——	5	0	63
5	18	——	5	0	81
5	19	——	5	0	99
6	00	——	5	1	17
6	1	——	5	1	34
6	2	——	5	1	52
6	3	——	5	1	69
6	4	——	5	1	87
6	5	——	5	2	05
6	6	——	5	2	22
6	7	——	5	2	40
6	8	——	5	2	58
6	9	——	5	2	75
6	10	——	5	2	93

A TABLE of Tyling, *from* 2 s. 6 d. *to* 40 s. per *Square*.

s.	d.		d.	q.	C.
2	6	*per* Squ. 1 Foot is——	0	1	20
2	7	——————	0	1	24
2	8	——————	0	1	28
2	9	——————	0	1	32
2	10	——————	0	1	36
2	11	——————	0	1	40
3	0	——————	0	1	44
3	1	——————	0	1	48
3	2	——————	0	1	52
3	3	——————	0	1	56
3	4	——————	0	1	60
3	5	——————	0	1	64
3	6	——————	0	1	68
3	8	——————	0	1	76
3	10	——————	0	1	84
4	0	——————	0	1	92
4	6	——————	0	2	16
5	0	——————	0	2	40
5	6	——————	0	2	64
6	0	——————	0	2	88
7	0	——————	0	3	36
8	0	——————	0	3	84
9	0	——————	1	0	32
10	0	——————	1	0	80
11	0	——————	1	1	28
12	0	——————	1	1	76
13	0	——————	1	2	24
14	0	——————	1	2	72
15	0	——————	1	3	20
16	0	——————	1	3	68
17	0	——————	2	0	16
18	0	——————	2	0	64
19	0	——————	2	1	12

The

The Table of Tyling continued.

s.	d.		d.	q.	C.
20	0	*per* Sq. 1 Foot is	2	1	60
21	0	——— ———	2	2	08
22	0	——— ———	2	2	56
23	0	——— ———	2	3	04
24	0	——— ———	2	3	52
25	0	——— ———	3	0	0
26	0	——— ———	3	0	48
27	0	——— ———	3	0	96
28	0	——— ———	3	1	44
29	0	——— ———	3	1	92
30	0	——— ———	3	2	40
31	0	——— ———	3	2	88
32	0	——— ———	3	3	36
33	0	——— ———	3	3	84
34	0	——— ———	4	0	32
35	0	——— ———	4	0	80
36	0	——— ———	4	1	28
37	0	——— ———	4	1	76
38	0	——— ———	4	2	24
39	0	——— ———	4	2	72
40	0	——— ———	4	3	20

A TABLE *of Perpendiculars for Gable-ends.*

Base.		Perpendicular.		
F.	I.	F.	P.	P.
10	0	5	7	6
11	0	6	0	7
12	0	6	8	7
13	0	7	4	9
14	0	7	10	2
15	0	8	4	9
16	0	8	11	6
17	0	9	6	4
18	0	10	0	1
19	0	10	7	8
20	0	11	2	5
21	0	11	9	3
22	0	12	3	9
23	0	12	10	7
24	0	13	5	4
25	0	14	0	2
26	0	14	6	9
27	0	15	1	6
28	0	15	8	3
29	0	16	3	0
30	0	16	9	8
31	0	17	4	5
32	0	17	11	3
33	0	18	6	0
34	0	19	0	7
35	0	19	7	4
36	0	20	2	0
37	0	20	8	9
38	0	21	3	6
39	0	21	10	2
40	0	22	4	3
10	0	05	7	6
9	0	05	0	5
8	0	04	5	8
7	0	03	11	1
6	0	03	4	1
5	0	02	9	6

OF MEASURING Superficies and Solids.

Concerning a Line of Sines *and* Cycloids.

DEFINITION I.

A Line of Sines, is of right Sines, bowing the Termination of the ordinate Applacates to the Arch of the Quadrant. Or it is, A crooked Line deſcribed from the extream Point of the Semidiameter, aſcending by an equal Motion through the bowed Arch of the Quadrant, approaching to the oppoſite Point, according to the Ratio of the verſed Sines of the Arch paſſed over, each Definition ſhall be explained in the firſt Propoſition.

DEFINITION II.

A Cycloid is the bounding or terminating of the compounded, from the Sines, and from the ſuperior Arches, by the ordinate Applicates to the Diameter

ter of the ſame Circle: Or it is a crooked Line deſcribed from a moveable Point, by a mixt Motion from equal Motions of the Orb, and of the Center: Both ſhall be explained in Prop. 23.

DEFINITION III. Fig. 1.

A Figure is ſaid to proceed by its Elements, in which Lines are taken or accepted parallel to the Baſe, if the Baſe is a Line, or parallel Superficies, if the Baſe is a Superficie.

We will explain it: Let there be a Rectangle MD, take how many ſoever Parallels to the Baſe MN, as PO, *&c.* the Rectangle MD is ſaid to proceed or increaſe by MN, PO, and the other Elements of the ſame Kind: In like manner the Cylinder MD proceeds or increaſes by the Circular Planes MN, PO, *&c.* Likewiſe the Triang. MAN by the Lines MN, PV: The Cone MNA by the Circle MN, PV, *&c.*

DEFINITION IV.

A Figure is Iſoparallel, which proceeds by Elements equal to the Baſe, as a Rectangle, a Cylinder, a Priſm, *&c.*

DEFINITION V.

Homogene Figures are thoſe which proceed by proportional Elements (for Example) the Rectangle MD, and the Cylinder MD, are Homogene Figures, becauſe as the Line MN to PO, ſo the Plane MN to the Plane PO.

DEFI-

DEFINITION VI.

The Center of Gravity is that from which Quantity being hung down by a Perpendicular, 'tis Equiponderate, or in Equilibrium.

DEFINITION VII.

A Ballance is a Line, to the Extreams of which Weights being annexed, and being hung perpendicular from ſome Point of the ſame Line, they make Equilibrium, or weigh equal.

DEFINITION VIII.

Moments are the Endeavour of Weights hanging, compared to one another.

POSITION I. Fig. I.

Homogene Figures, of the ſame Kind of Altitude and Baſe, are equal; being of the ſame Altitude, they are as their Baſes; being of the ſame Baſe, they are as their Altitudes; being of divers Baſes and Altitudes, they are in Compoſition, or they are compounded of their Baſes and Altitudes: Let there be two Homogene Figures of the ſame Kind, B A C, B C D, of the ſame Baſe and Altitude; they are equal: For as A C is equal to C D, ſo E I is equal to I K, and ſo of the reſt, if there were never ſo many Lines drawn parallel to the Baſes A C, C D; therefore they are equal. Let them be of the ſame Altitude, and not of the ſame Baſe, as A B C, A B D, then

then they are as their Bafes AD, AC; for as AD to AC, fo EK to EI, and fo of the reft. Therefore ABD is to ABC as AD to AC, that is, as 2 to 1. Let them be of the fame Bafe, but not of the fame Altitude, as ABC, AEC, then they are as their Altitudes AB, AE; for as AB to AE, fo GI to GI, and fo of the reft; therefore ABC is to AEC as AB to AE, that is, as 3 to 2. Laftly, let them be of divers Bafes and Altitudes, as ABD, AEC, then they are in Compofition, or they are compounded of the Bafes, and of the Altitudes, that is, as the Rectangles under BA, AD, and under EA, AC; *To wit*, AEC is to ABC, as the Rectangle under EAC to the Rectang. under BAC, *viz.* 2, 5: 6, 25:: 5 : 12, 5. But BAC is to BAD as the Rectangle under BAC to the Rectangle under BAD. Therefore EAC is to BAD, as the Rectangle under EAC to the Rectangle under BAD, that is in a compounded Ratio, from the Ratio's of AE to AB, and of AC to AD: If the Bafe is a Plane, the Figures will be as Ifoparallels under the Bafes and Altitudes: Alfo the fame manner of Demonftration may be applied to other Homogene Figures.

POSITION II. Fig. 2.

A Quadrant is to the Triangle, in Compofition, or compounded from the Ratio of the Semidiameter to the Altitude, and from the Ratio of the Arch of the Quadrant to the Bafe of the Triangle; Let the Quadrant be ADC, and any Triangle ACH, I fay, 'tis Homogene to the Quadrant; for as AC to CH, fo AB to BF: Therefore as AC to AB, fo

ſo CH to BF; but as AC to AB, ſo the Arch CD to the Arch BE; therefore the Figures proceed by proportional Elements, by Defin. 3. therefore they are Homogene by Definition 5. therefore they are compounded of the Baſes and Altitudes, by Poſition 1. and the Baſe of the Quadrant is the Arch of the ſame, but the Altitude is the Radius or Semidiameter of the ſame.

COROLLARY I. Fig. 2.

Hence aſſuming the Baſe of the Triangle CG, equal to the Arch CD, and the Altitude CA; the Triangle ACG, will be equal to the Quadrant ACD: Aſſuming the Baſe leſſer as CK, or greater as CH: ACK will be to ACD, as CK to CG; and ACH to ACD, as CH to CG; laſtly, BCK to ACH, will be as the Rectangle under BCK, to the Rectangle under ACG.

COROLLARY II.

Hence the Rectangle under the Radius AC, and the Half of CG, is equal to the Quadrant ACD; therefore the Rectangle under AC, and the Double of CG, is equal to the Circle; but under four times CG is double of the Circle: Hence laſtly, any Sector, ſuppoſe ACM, is to a Triangle, ſuppoſe ACK, as the Arch CM to CK.

POSITION III. Fig. 3.

A Cylinder is ſeſquialter, or once and a half of an Hemiſphere of the ſame Altitude, and Baſe: Let a Quadrant

Quadrant be ALB, a Rectangle AL, a Triangle BML, all turned about BL; there will be generated an Hemisphere from the Quadrant ABL; a Cylinder from the Rectangle AL; a Cone from the Triangle BML; these being granted, that begat from the Triline AML, is Homogene to the Cone, begat from BLM; For since Circles are as the Squares of their Radii, let any one be EG, the Square of GE or BD is equal to the Squares of GD and GB or GF; therefore that begat from GE equals those begat from GD and GF or DE, therefore that begat from GF, is equal to that begat from DE, therefore that begat from LM is to the begat from FG, as to that begat from DE: Therefore the begat from the Triline AML, and from the Triangle BML are Homogenes by Definition 5. and they are of the same Base, *viz.* of the begat from LM, and of the Altitude BL, therefore they are equal by Position 1. Then since that begat from the Triangle BML being a Cone, is one third of the Cylinder begat from the Rectangle AL, that begat from the Triline AML will be likewise one third of the same Cylinder; therefore that begat from ALB the Quadrant, *viz.* the Hemisphere, will be equal to two Thirds of the same Cylinder, therefore the Cylinder is Sesquialter, or one and a half of the Hemisphere.

Suppose the Radius AB = to 5 *f.* the Altitude AM equal, or = to 5 *f.* the Diameter = 10 *f.* Then for the Solidity of the Hemisphere, $10 \times 10 = 100 \times 10 = 1000$, the Cube of the Diameter $1000 \times 11 = 11000$, which divided by 21, gives in the Quotient 524 *fere* for the Solidity of the whole Sphere, the Half whereof 262 *f.* is the Solidity of the Hemisphere.

For

For the Cylinder whoſe Diameter is 10 *f.* and its Altitude 5 *f.*

The Area of the Baſe = 78 *f.* 54 × 5 *f.* the Altitude = 392 *f.* 70 for the Solidity of it.

The Cone, the Area of its Baſe 78 *f.* 54 × by $\frac{1}{3}$ of the Altitude 5 *f.* which is 1, 7 *fere* makes for the Solidity of the Cone 130 *f.* 9.

POSITION IV. Fig. 3.

The Superficie of the Hemiſphere is equal to the Superficie of the Cylinder, of the ſame Baſe and Height, the Baſes being taken away, *viz.* the Hemiſphere is Homogene to the Cone: For as that begat from LM to that begat from GF, ſo that begat from the Arch LA, to that begat from the Arch GP, to wit, as the Square of LM to the Square of GF. But the common Altitude is BL, therefore the Figures are as their Baſes; but the Hemiſphere is double of the Cone by Poſition 3. Therefore the Baſe of the Cone begat from LM, is half of the Superficie of the Hemiſphere, begat from the Arch LA: And the Superficie of the Cylinder begat from AM, is double of the Circle begat from LM, *viz.* equal to the Rectangle under AM, and the Quadruple of the Arch of the Quadrant by Coroll 2. of Poſition 2. therefore the Superficie of the Cylinder is equal to the Superficie of the Hemiſphere: Hence the Superficie of the Sphere is quadruple of the Area of the greater Circle.

Example, Diameter 10 *f.* the Periphery is 31 *f.* 417; the Area of the Circle, 78 *f.* 543.

The Area 78, 543 × 2 = 157 *f.* 086 the Superficie of the Hemiſphere.

The

The Periphery $31,417 \times 5 = 157 f. 085$, the Superficie of the Cylinder.

Another Way. The Hemiſphere is Homogene to the Cone, therefore the Superficie of it multiplied by one Third of the Height of BL, produces the Solid: But the Cylinder, foraſmuch as it proceeds by Cylindric Superficies, 'tis Homogene to the Triangle; therefore the Superficie begat from MA, drawn into half of ML or BL, produces the Solid; and the Ratio of the Products, or of the Solids is $\frac{3}{2}$, the other Ratio is $\frac{3}{2}$: Therefore the other which is of Superficies is $\frac{1}{1}$, therefore the Superficies are equal.

POSITION V. Fig. 3.

A Sphere being cut by parallel Planes, the Segments of the ſpherical Superficie are as the Segments of the Diameter, cut by the Planes at right Angles. Example, Let the Plane be EK; I ſay the Superficie begat from the Arch AD is to that begat from the Arch DL, as BG to GL, to wit that begat from ABM is equal to the Hemiſphere, by Poſition 3. And that begat from ABE proceeding by Cylindric Superficies, of which the firſt is begat from AE, is Homogene to that begat from ABD proceeding by Sphericals, of which the firſt is that begat from AD: And ſince that begat from GF is equal to that begat from DE, and ſince that begat from BFG is to that begat from BED, as the begat of the Baſe from FG to that begat from DE: the begat from BFG, will be equal to that begat from BDE: But that begat from BFG is equal to that begat from the Triline AED; and by

by taking away the common begat by the Triline VED, the Remainers will be equal, *viz.* the begat from BVD, and AEV, therefore the begat from BAD is equal to that begat from BAE, but theſe are Homogenes of the ſame Altitude AB, and therefore of the ſame Baſe, by Poſition 1. And the Baſes of the begotten from AE, and from the Arch AD: But that begat from AE, is to that begat from EM, as AE to EM, or as BG to GL; therefore that begat from the Arch AD, to that begat from the Arch DL, as BG to GL; the ſame being aſſumed, it may be demonſtrated in any other Point: Therefore the Segments of the Superficie of the Hemiſphere, are as the Segments of the Semidiameter for the ſame Planes; therefore the Segment of the Superficie of the Sphere, are as the Segments of the Diameter.

POSITION VI. Fig. 4.

If as AD the Radius of the Quadrant ADK, or the whole Sine, to DI the right Sine of the Angle BAL, ſo DI to DC; AD will be to AI, as AI to AC, of which alſo AD, the Difference is the ſame DC, which is the verſed Sine of the Angle DBI, the double of DAI in the Quadrant DBM, under the Radius BD ſupduple, or half of the former AD; to wit, the ſubtenſe DH, and all the other will be cut in the Middle by the Periphery DMA: Theſe Things appear from the Elements.

POSI-

POSITION VII. Fig. 5.

If there be any Fig. ſuppoſe the Semicircle AHM, which let be turned about AC; that begat from AMH is to that begat from the Rectangle AL, as the Iſoparallel under the Baſe AMH, and from the Altitude AM, to the Parallelipiped under the Baſe AL and the Altitude AM, to wit, that begat from AM is to that begat from DI, as the Square of AM to the Square of BI, leſs by the Square of BD, that is to the Rectang. under the Baſe DI, and the Altitude AM. The ſame may be demonſtrated by aſſuming any other. Therefore that begat from AMH is to that begat from AL, as the Iſoparallel under the Baſe AMH, and the Altitude AM, to the Parallelipiped of the ſame Altitude under the Baſe AL. If we aſſume any other Fig. (for Example) the Triang. AMH, or any other Triline, the ſame may be demonſtrated generally.

POSITION VIII. Fig. 6.

Homogene Figures of the ſame Altitude have the Center of Gravity equidiſtant from the Baſe. Let CAB, EAB be homogene Figures, and let DA be the Diſtance of the Centre of Gravity of the Fig. CAB from the Baſe AC; it will be the ſame of the Center of the Fig. EAB from the Baſe AE. For if to the Perpendicular DF you hang down CAB, it will equiponderate. Therefore the Moments of the Trapezium ACFD, and of the Triang. DFB, are equal; but they are compounded of the Quantities and of the Diſtances by exchanging. And ſince

the Trapezium ADGE is to DGB the Triang. as ADFC is to DFB, the Center of Gravity of DFB will be in OL; and it will be, as the Trapezium ADFC to DFB, ſo OD to DI; the Center of the Trapezium will be in IH. Therefore alſo the Center of GDB in OM, and of the Trapezium ADGE in IK. Therefore of the whole Fig. ABE the Center of Gravity will be in DG. The ſame may be demonſtrated in any other Homogenes, either Solids or Planes. I ſhew theſe in haſte, and briefly, which, in *Cavalerius* his 5th *Exerciſe*, at Propoſition the 9th, and others that have writ concerning *Staticks in Geometrical Rigour*, may be found demonſtrated more at large. Therefore ſince theſe things are ſuppoſed, it will ſuffice to ſhew them briefly.

POSITION IX. Fig. 7.

If two Figures (for Example) EA, BAC, do weigh on the common Axis BA, the Moments will be as the Solids begat from the ſame, being turned about BA; becauſe the Moments are compounded from the Ratio of the Quantities, and from the Ratio of the Diſtances of the Center of each Figure from the common Axis BA. Therefore the Moment of KG is to the Moment of GI, in the compounded Ratio of the whole to the whole, and of the $\frac{1}{2}$ to the $\frac{1}{2}$; that is, in duplicate of GK to GI; that is, as the Square of GK to the Square of GI; that is, as that begat from GK, to that begat from GI. The ſame may be ſhewn by aſſuming any other. Therefore the Moment of the whole EA, is to the Moment of BAC, as that begat from IA to that begat from BAC. Theſe things I ſhew briefly;

briefly; they may be ſeen demonſtrated more at large in the aforeſaid *Cavalerius*, and in *Torricellius*, concerning the meaſuring of the *Parabola*, *Lem.* 31.

COROLLARY.

Hence from a given Ratio of the Figures, and of the Diſtances from each Center (for Example) of GF, GH, the Ratio of the Solids may be known, or of the Generators or Begetters from the ſame Figures; and back again from the Ratio given of the Solids and of the Diſtances, the Ratio may be had of the Figures; and from the Ratio given of the Solids and of the Figures, the Ratio of the Diſtances is had.

PROP. I. Fig. 8.

An homogene Figure of a Figure of Sines, the Baſe of which is quadruple of the Axis, equals the Superficie of the Hemiſphere under the Radius, equal to the Baſe of the Fig. of Sines. That this Propoſition may be underſtood, ſomewhat of Conſtruction muſt be uſed. Let BAE be at right Angles, and let AE be quadruple of AB; let any Quadrant be APL; divide the Arch of it AL in the middle in S, the Sine SZ. Divide AB in the middle in D, and let it be as the whole Sine AP to SZ, ſo AE to the Ordinate applied DH. In the ſame manner may be found the other Ordinates applied from the Arch AL and the right Line AB proportionally divided. Laſtly, through the Extreams of the Applieds ſuppoſe the Curve Line BHE to be drawn: Let AQ be equal to AB, and let it be as AE to

 DB,

DB, ſo AE to DK; and do the ſame in the other Applieds, the Fig. ABKQ will be homogene of the Fig. ABHE. Laſtly, let it be as AC to AB, as Radius, or the Semidiameter to the Arch of the Quadrant; let AP be equal to AC, and let DI be equal to SZ; and in like manner apply the other Sines; BTP will be the Line of Sines; and call ABP the Fig. of Sines, AB the Axis, AP the Baſis, APL the generating Quadrant, CBT the right Segment, YTP the verſed Segment, CT, DI Ordinates applied; T Υ parallel to the Axis, ACTP the right Trapezium of the Fig. ABTΥ the verſed Trapezium; that begat from the Fig. about BA the right Solid; that about AP the verſed Solid, and it will appear that AE, the Quadruple of AB, is equal to the Periphery of the Circle under the Radius AC or AP: Alſo it appears, that the Fig. ABE is homogene of the Fig. of Sines: Laſtly, turn about PA in the Plane of the Quadrant, the Point A will deſcribe the Arch ASL, equal to the right Line AB, which if it be turn'd about LP, that A will deſcribe the Periphery under the Radius PA; ſo S will deſcribe his under the Radius SZ, and each other Point his under the Radius which is the Sine terminated at the generating or begetting Point: and at length the whole Arch ASL, the Superficie of the Hemiſphere. Theſe things being done, the Propoſition may be eaſily demonſtrated.

For the Fig. ABHE is homogene of the Fig. of Sines ABTP, by *Defin.* 5. the Baſe AE is quadruple of the Axis AB; and it is as AP to SZ, ſo AE to DH; AE is equal to the Periphery under the Radius AP of the Begat, *viz.* from A. Therefore DH is equal to the Periphery under the Radius

Radius SZ of that Begat, *viz.* from S; therefore the Fig. ABE, and the Superficie of the Hemisphere begat from the Arch ASL, are homogene Figures by *Defin.* 5. They are of the ſame Baſe, for AE is equal of the Periphery of the Begat from A; alſo of the ſame Altitude, for the Arch AL is equal to the right Line AB: Therefore the Figures are equal by *Poſition* 1.

PROP. II.

The Fig. ABHE is to ABKQ, as AE to AC; likewiſe to the Fig. of Sines as AE to AP, that is, as the Periphery to the Radius; becauſe, ſince they are homogene by Conſtruction, and of the ſame Altitude AB, they are as the Baſes by *Poſition* the 1ſt. Therefore as AE to AQ and AB. The ſame may be demonſtrated concerning all other Homogenes.

PROP. III.

Any Fig. homogene of the Fig. of Sines ABTP, and of the ſame Altitude AB, is equal to the Rectang. under its Baſe and AC or AP: For ſince ABHE is equal to two Circles under the Radius AC or AP, by *Poſition* 4. it will equal the Rectang. AF by *Corol.* 2. of *Poſition* 2. Therefore ſince ABHE is to ABTP as AE to AP, by *Prop.* 2. it will be as the Rectang. AF to the Rectang. AL. Therefore ABTP is equal to the Rectang. under the Baſe AP, and AC. In like manner it may be ſhewn, that ABKQ, is equal to the Rectangle AM; and ſo of the reſt.

COROL. I.

Hence the Fig. of Sines is equal to the Quadrate of its Baſe, or to the Radius of the generating Quadrant, ſo ABTP is equal to the Quadrate AL, for AL is the Quadrate, becauſe AP is equal to AC.

COROL. II.

The Triline BMP is equal to the Difference of the Square of the Radius, to wit, of AL, and of the Semicircle under the ſame Radius; for AM is equal to the Semicircle by *Corol.* 2. of *Poſition* 2. and ABTP is equal to the Square of the Radius.

COROL. III.

Hence the aforeſaid Triline is equal to the Rectangle EM under the Radius, and the Difference of the ſame Radius, and the Arch of the Quadrant.

COROL. IV.

The ſame Triline is equal to twice the Segment of the Circle under the Radius AP, contained by the Arch of the Quadrant and the Subtenſe.

COROL. V.

The Segment contained by the right Line BP, and the curve Line BTP, that is the Line of Sines, is equal to the Triline ACL.

COROL.

COROL. VI.

As AO the Square of the Arch of the Quadrant to AM, or the Semicircle; ſo AM to AL the Square of the Radius: Hence the Semicircle is a mean Proportional between the Square of the Radius, and the Arch of the Quadrant.

COROL. VII.

The Triline CBP equals the Triline ACL: Likewiſe BCT equals the Triline PLT: Alſo BCY equals the Section under the Arch YP, and the right Line YP.

SCHOL.

We muſt imagine in our Mind the Arch ASL unbent from Curvity, and deflected into the right Line AB; likewiſe each Periphery ordinately applyed at right Angles to the Arch ASL (for ſo the Radius falls in with the Arch) being deflected alſo into right Lines equal to AB, ordinately applyed at right Angles: So the whole Superficie of the Hemiſphere from the Arch ASL begat in the Plane Fig. goes ABHE away, and the Quantity remains the ſame, but the Round only and the Curve are changed into right Lines and Planes.

PROP. IV. Fig. 9.

If the Fig. of Sines be turned about the Axis, that begat or generated is ſubduple of the Cylinder

of the ſame Baſe and Altitude: Let the Fig. of Sines be ABC, the Quadrant ADC, the Arches DG, IC are equal: And let the Sine EG be tranſlated into OR, and KI into LV; OB will be equal to AL, alſo to the Arch DG or IE: And ſince the Square of AC or AI is equal to the Squares of KI, and IC, or EG, that is the Squares LV and OR; the Difference of the Squares LV, LM, will be equal to the Square of OR, and the Circles are as the Squares of the Radius's; therefore that begat from LV, with that begat from OR, is equal to that begat from VM: In like manner that begat from RS, is equal to that begat from LV or Z; therefore that begat from ABC, and from the Triline BSC are homogene; therefore they are equal ſince the Baſes are equal, and are in the ſame Altitude; therefore either of them is ſubduple to that begat from AS, to wit, of the Cylinder of the ſame Baſe and Altitude.

COROL. I.

Hence the Semiparabola under the Axis AB, and the Baſe AC, is indeed greater than the Fig. ABC, yet being revolved about the Axis, begets a Solid equal to that begat from the Fig. ABC: But if AB be divided in the middle in N, and NP be drawn, the aforeſaid Semiparabola will cut the Line of Sines in PI; and the Applicates or Applyeds above NP in the Parabola are greater, but beneath leſſer than in the Line of Sines.

COROL.

COROL. II.

That begat from any Fig. homogene of the Fig. of Sines, is ſubduple of the Cylinder of the ſame Altitude and Baſe, for they are Homogenes begat of homogene Figures.

COROL. III. Fig. 8.

That begat from the Quadrant A B Q, to that begat from the Fig. ABKQ, is as 4 to 3, becauſe that begat from the Quadrant, to wit, the Hemiſphere, is to the Cylinder of the ſame Baſe and Altitude, as 4 to 6.

COROL. IV. Fig. 8.

That begat from the generating Quadrant A L P or ACP, is to that begat from the Fig. ABP, is as half of the Radius CA to two Thirds of AB, equal to the Arch of the Quadrant ASL; that is according to *Archimedes*'s Cyclometry, as 28 to 3: But to that begat from the Triline CBP, as one Third of AC, to the Difference of two Thirds of A C and two Thirds of A B, that is as 28 to 5.

COROL. V.

If there be a mean Proportional between A B, AP, the Parallelipiped under the Square of it, and of the Altitude A B, will be equal to that begat from the Fig. ABTP; for it is ſubduple of the ſame Cylinder, ſince the Baſe of it is equal to the Semicircle.

CO-

COROL. VI.

That Begat from homogene Figures ABTP of the ſame Height or Altitude, are as the Square of the Baſes of the Figures, (for Example) that begat from ABTP is to that begat from ABKQ, as the Square of AP to the Square AQ.

COROL. VII.

That begat from the Lunula contained by the Arch of the Quadrant BVQ, and the Curve Line BKQ, is ſubduple of the begat from the Triangle ABQ: For let that begat from the Rectangle AO be 6, that begat from the Triangle ABQ will be 2; from the Fig. ABKQ, 3; from the Quadrant ABVQ, 4; therefore that begat by the Lunula will be 1: Therefore the Subduple of that begat from the Triangle ABQ, and equal to the begat from the Segment contained by the Curve Line BKQ, and the right Line BQ: Hence as the Superficie begat from the right Line BQ, will divide in the middle that begat from the Quadrant, *viz.* the Hemiſphere; ſo the Superficie begat from the Curve Line BKQ, will divide in the middle the Begat from the Segment of the Quadrant contained by the Arch BVQ, and the right Line BQ.

COROL. VIII. Fig. 10.

If you let the Quadrant be ATMB, and the Fig. homogene of the Fig. of Sines ATNB, the Rectangle AG, AT divided in the middle in

C;

C; CF Parallel to AB, the right Line begat from the four Segments CDEOF are equal: For let that begat from CF be 16, that begat from CD will be 4; from CE, 8; from CO will be 12; therefore from DE, 4; from EO, 4; from OF, 4: And therefore equal.

COROL. IX.

If you draw two Lines RP, LH Parallels to EF, and equidiſtant from the ſame, ſince that begat from RS and IH are equal, for AL, RT, RS are equal, likewiſe thoſe begat from LK, MP will be equal to the begat from RM and KH: And by ſubtracting the Equals begat from RS and IH, the Remainers will be equal, to wit, the begat from SM, KI: The ſame may be demonſtrated from any other being aſſumed: Hence from the Cone begat from ATB being ſubtracted from the Hemiſphere, the Remainer will be divided in the middle by the Plane CF.

COROL. X. Fig. 9.

If AL, OB are aſſumed equal, the Cylinder under the Baſe from the Circle from AC, and from the Altitude AL, is equal to the begat from the right Segment OBR, and from the Trapezium ALVC, to wit, the begat from the Triline CVM is equal to that begat from the Segment OBR.

COROL. XI. Fig. 9.

Hence that begat from LVRO is to the Remnant as NL to LA: Hence may be had a mean Fruſ-

Fruſtum in any given Ratio to the whole; if the ſeſquitertia Ratio be required, let N L be to N A, as 3 to 4, the begat from L V R O will be to the begat from the Fig. A B C, as 3 to 4, that is N L to N A.

C O R O L. XII.

Laſtly, hence that begat from OBR, and CMV are equal, likewiſe that begat from A L V C, and from BR σ S; alſo the begat from RTY σ, and from L V θ Υ are equal, &c.

P R O P. V.

Any right Segment of a Fig. of Sines is to the whole Fig. as the verſed Sine of the Arch of it, to which the Altitude of the Segment is equal to the whole Sine: For let any Segment be N B P whoſe Altitude NB is equal to the Arch DH, and therefore HF equal to N P: I ſay the Segment NBP is to the whole Fig. ABC, as the verſed Sine of D F I to the whole Sine D A: For ſince the Fig. A B C is homogene to the Superficie of the Hemiſphere, this will be divided in the Ratio of the Segments of the Radii, by Poſition 5. (for Example) If you revolve the Arch D C about D A, the Superficie begat by the Arch DH, is to that begat from the Arch D C, as the Segment N B P to the Fig. ABC: But that begat from the Arch D H is to that begat from the Arch DC, as DF to DA by Poſition 5. Therefore the Segment N B P is to A B C, as D F the verſed Sine of the Arch D H, equal of the Altitude of the Seg-

Segment N B to D A the whole Sine. The ſame may be demonſtrated in any other Segment.

COROL. I.

Hence may be given the Segment which is to the whole Fig. in any given Ratio, (for Example) let the Ratio given be of D E to D A; let O R be applied equal to E G, the Segment O B R will be to the whole A B C, as D E to D A: Hence if a Segment be required which ſhall be one quarter of the whole, let D F (for Example) be one quarter of D A, let N B be applyed equal to F G; N B P will be to A B C, as 1 to 4: Hence if A Υ be half of A B, Υ B θ will be one Third of A B C; and therefore Υ θ will divide the Fig. of Sines equally.

COROL. II.

Hence the right Segment is to the whole Fig. as the Applyed to the Triline of the Fig. conjoined equally diſtant from the Baſe of the Triline, alſo the Baſe of the Segment from the Baſe of the Fig. O A will be equal from the Baſe of the Triline S M; I ſay the Segment O B R is to the whole A B C, as the Applyed to the Triline V M to the Baſe B S: For let the Sine E G or Z I be equal to O R, and therefore O B equal to the Arch D G or C I, the Segment O B R is to the whole A B C, as D E or Z C, to D A or A C: But L V is equal to A Z or A E; therefore V M is equal to Z C; therefore O B R is to A B C, as V M to B S: The ſame may be demonſtrated concerning any other Segment.

PROP.

†

PROP. VI. Fig. 11.

The Ratio may be defined of the verſed Sine to the whole Fig. of Sines: Let the Fig. of Sines be APT, the Quadrant AMT, the Triline annex'd PVT, let AK be equal to AT, and therefore the Quadrate AN; let any verſed Segment be BRT, draw LR indefinitely, let LR be equal to IF; laſtly, through the Point X, in which BR the Axis of the Segment will cut KN the ſide of the Quadrate AN, draw AXS from S, let fall SC parallel to the Axis RB; ſince the whole Fig. APT is to the right Segment LPR, as AT to AC, it will be to the Remnant, to wit, to the verſed Segment BRT, as AI to CI; therefore the Ratio ſought is found.

COROL. I.

Hence the whole Fig. is to each Segment ART, LPR, as AT to CT.

COROL. II.

Hence the whole Fig. is to the verſed Segment, as the whole Sine to the Baſe of the Segment leſs by the Compound, whereby the whole Sine exceeds the right Sine of the Arch equal to the Axis of the Segment, and from the Line which is to the Exceſs, whereby the aforeſaid Arch exceeds the Radius, as the Sine of the Complement of the aforeſaid Arch to the Radius.

PROP.

PROP. VII. Fig. 12.

If a Fig. of Sines be turned about a Parallel to the Axis erected in the Extream of the Baſe, that begat from the Fig. will be to the Cylinder of the ſame height under the Circle of the Baſe, to thai begat from the Fig. of the Baſe, as the Compound from the Radius, and the Exceſs whereby the Radius exceeds half the Axis to the whole Axis: For let the Fig. of Sines be ABC inſiſting perpendicularly to the Plane of the Quadrate AE, and let the Solid proceed by the Squares AE, NI, &c. under the Baſe AE, and the Altitude AB, and therefore homogene to that begat from the Fig. ABC turned about the Axis AB, it will make the Iſoparallel under the Baſe ABC, and the Altitude AD, to wit, proceeding by the equal Planes ABC, DKE: Let AX be equal to AC, likewiſe AN equal to BN, and XV equal to XN; the Iſoparallel aforeſaid will be to the Parallelipiped AF, to wit, under the Quadrate AE, and the Altitude AB, as ABC to the Rectangle AL; for they are homogene Iſoparallels, therefore if they ſhall be of the ſame Altitude, they are as their Baſes by Poſition 1; therefore as AX to AB, the aforeſaid homogene Solid proceeding by the Squares AE, NI, is to the Parallelipiped AF, as AN to AB, by Prop. 4. therefore it is to the aforeſaid Iſoparallel, as AN to AX; therefore to the Difference of each, as AN to NX: And by taking away from the Parallelipiped AF the Solid proceeding by the Squares BF, IH, to wit, under the Applicates to the Triline KFE; and by taking from the Reſidue, the

the aforeſaid Homogene proceeding by the Squares AE, NI, under the Applicates of the Fig. the Reſidue will proceed by Supplements parallel to the Gnomons IG, IS: For as from the Square NH IH being taken away, and from the Reſidue NI being taken away, there will remain the Supplement of the Gnomon IG, IS: So it may be done in any other Square aſſumed; and ſince IG is equal to IS, and that from any other Quadrate aſſumed, the Exceſs whereby the Reſidue of the Parallelipiped by taking away the Solid, proceeding by the Squares BF, IH, will exceed the Iſoparallel under the Baſe ABC, and the Altitude AD, equal to the Exceſs whereby the aforeſaid Iſoparallel exceeds the aforeſaid Homogene proceeding by the Squares AE, NI: Therefore this Homogene it ſelf is to the aforeſaid Iſoparallel as AN to AX, and to this the Difference of both being added, as AN to AV; laſtly to the Solid, proceeding by the Squares BF, IH, as AN to VB.

But ſince the Solid proceeding by the Squares BF, IH, is homogene to that begat from the Triline KFE, or BLC turned about LC; this begat will be to the Cylinder of the ſame Baſe and Altitude, as BV to BA; therefore that begat from BAC, turned about CL, will be to the ſame Cylinder, as AV to AB; therefore as the Compound from the Radius AX and XV, to wit, the Exceſs whereby the Radius AX exceeds AN the half of the Axis AB, for XV is equal to XN.

COROL.

COROL. I.

These may easily be reduced to Calculation according to the Cyclometry of *Archimedes*, for A N will be to A X as 11 to 14; and to A V as 11 to 17; and to V B as 11 to 5: Hence if the Cylinder begat from AL turned about CL be 22, that begat from the Triline BCL will be 5, and from the Fig. AB 17; and the Parallelipiped AF, to the Isoparallel under A B C and A D as 22 to 14.

COROL. II.

If between N G, N P, there be a mean Proportional, and also between the Sides of other Rectangles Parallels of NM: The solid Proceeding by the Squares under the aforesaid mean Proportionals, is equal to the Isoparallel under A B C and A D: But if from the Applicates ordinately applied to BA, by the aforesaid mean Proportionals, the Fig. turned about AB, that begat will be to the Cylinder of the same Base and Altitude as AX to AB, that is, as 14 to 22.

PROP. VIII. Fig. 13.

If the Fig. of Sines is CAE, and the other common Axis EAB, and let the solid Isoparallel under EAB, and the Altitude AI be equal to A E: And lastly, the Solid be cut by the Plane B I E, the lower Frustum R I Q B is to the whole Solid, as the Radius C E to the Axis C A; but the upper Frustum, as the Difference of the Axis, and of the Radius

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dius to the Axis; and to the other Frustum as the aforesaid Difference to the Radius: We will demonstrate it; let I T be equal to the Radius E C, and let the Fig. of Sines be T O I, and the accompanying Triline OAI, the superior Frustum E A B I will proceed by right Segments the Parallels B A E, NDM, &c. And BAE is to NDM, as AO to DF, by Coroll. 2. of Prop. 5. The same may be demonstrated concerning any other Segment; therefore the Frustum EABI is Homogene to the Triline AOI; therefore as the Triline AOI to the Rectangle AT, so the Frustum EABI to the Isoparallel under the same Base EAB, and the Altitude AI: But the Rectangle AT is to the Triline A O I as the Axis A I to the Difference of the Axis AI or AC, and of the Radius CE: Therefore the aforesaid Isoparallel is to the aforesaid Frustum in the same Ratio, therefore to the Frustum RIQB as the Axis to the Radius; therefore the Frustum R I Q B, to the Frustum E A B I, as the Radius to the aforesaid Difference.

C O R O L. I.

Hence the Frustum R I Q B, so far as it proceeds by right Trapeziums, parallels to the Plane R I Q, 'tis Homogene of the Fig. of Sines T O I; to wit, as T I to π F, so R I Q to Z M N V.

C O R O L. II. Fig. 9.

Hence any right Trapezium is to the whole Fig. as the Applicates of the Fig. equally distant from the Base of the Fig. and of the Base of the right Segment joined to the Trapezium, distant from the Vertex

Vertex of the Fig. to the Radius; (for Example) let the right Segment be O B R, let L V be applied equally distant from A C, as O R from the Vertex B, the Trapezium A O R M will be to the whole A B C, as L V to A C; because since the whole A B C is to O B R, as L M to V M, it will to the Remnant, *viz.* A O R C, as L M to L V.

C O R O L. III. Fig. 13.

Hence as the Rectangle T A will be divided by the Curve O I F I, so the aforesaid Isoparallel by the Plane BIE; therefore according to *Archimedes*'s Ratio, the upper Frustum is to the lower, as 4 to 7; but to the whole as 4 to 11; but the lower to the whole, as 7 to 11.

P R O P. IX.

The Center of Gravity of the whole Fig. of Sines, or of the double, with the common Axis, will divide the Axis so, that the Segment towards the Vertex of the Fig. is to the whole Axis, as the Radius to the Arch of the Quadrant: Let the Fig. of Sines be C A E, with its sociate C A B, the common Axis A C, so divided in K, that the Segment K A be made to the whole C A, as the Radius to the Arch of the Quadrant; K will be the Center of Gravity of the Fig. E A B; that it will be in the Axis A C, appears, since A C divides in the Middle all the Applicates EB, HG, &c. which is in K, shall be demonstrated: Let fall K X parallel to A I divided in the Middle in S, and draw S ♂ parallel to A C; draw I ♂, S C; these are parallel, in I ♂ is the Center of Gravity of the Frus-

 tum

tum RIQB, becauſe it will paſs by the Center of Gravity of all the Rectangles of the Parallels ER: Alſo in SC will be the Center of Gravity of EABI becauſe it paſſes through the Center of Gravity of all the rectangled Parallels HA. For Example, let the Center of the Fruſtum EABI be in P, and let the Point Y be in which S ♂ cuts KX, and draw PY θ; ſince the Triangles SPY, ♂ θ Y are Proportional, it will be as PY to θ Y, ſo SY to Y ♂, that is, AK to KC; but PY is to Y θ, as the Fruſtum RIQB to the Fruſtum EABI; for the Diſtances are as the Weights by exchanging: Therefore AK is to KC as the lower Fruſtum to the upper, therefore to AC as to the whole; therefore as the Radius to the Arch of the Quadrant. Now this hath been demonſtrated in general; concerning each Iſoparallel you may conſult *Torricellius* with *Cavellerius* at Exercife the 5th, Propoſition the 17th.

COROL. I.

Hence the Center of Gravity of the Iſoparallel will be Y; becauſe KX will be drawn through the Centers of all the parallel Planes BAE, RIQ, therefore it will be in KX: Moreover, S ♂ will paſs through the Center of Gravity of all the parallel Planes of BQ; therefore it will be in S ♂, therefore in Y.

COROL. II.

Hence the Center of Gravity of the Fig. of Sines CAE, is in the right Line KH, as appears: Likewiſe of the Fig. CAB in the right Line KG, to wit,

wit, the Center of both equally diſtant from EB, or from the Vertex A, ſince the whole EAB is Homogene of each aſunder.

PROP. X. Fig. 14.

The Center of Gravity of the Fig. of Sines is diſtant from the Axis one quarter of the Axis, and from the Baſe, by the Exceſs it ſelf, whereby the Axis exceeds the Baſe: Let the Line of Sines be AMC, with the Rectang. CY; the Moments of both Figures librate in CM, they are as the Solids begat from the Figures turned about CM, by Poſition the 9th. And that begat from CY is double of that begat from ACM by Prop. 4. but the Ratio of Solids begat from Figures is compounded from the Ratio of the Figures, and from the Ratio of the Diſtances of the Center of both from the common Axis by Poſition 9; therefore the Moment of the Rectangle CY is to the Moment of the Fig. CMA, in the Compoſition from the Ratio CM to CO, or its equal MD, which is the Ratio of the Figures, and from the Ratio of EQ, or ST, the half EX; for this is the Diſtance of the Center Q from the Axis CM, to the Diſtance of the Center of the Fig. CMA from the ſame Axis CM; that is, the Moment of the Rectangle is to the Moment of the Fig. as the Rectang. under CM, CT to the Subduple Rectangle, under one of the Sides equal to CO, and under the other, which may eaſily be found by drawing OR, alſo CR, PVS; for CP is equal to CQ, and EV is one quarter of MC; for ſince EP, CQ are equal, as CO to CE, the half of CM, ſo CT, the half of CO to CS, the

half of CE; therefore CS or EV is half of CE; therefore one quarter of CM; therefore CS is the other Side of the Rectang. sought, therefore 'tis the Distance of the Center of Gravity of the Fig. CMA from the Axis MC. Lastly, since CD is the Excess whereby the Axis CM exceeds the Base CA, by drawing DF parallel to the Base, and assuming DF equal to CS, the Center of Gravity of the Fig. CMA is in F.

COROL. I.

Hence since ME is to DE, as 4 to 1, that is, as the Rectang. CY to the Rectang. under one quarter of CM, and under CA, that is, as the Semicircle to the Semiquadrant, CA will be to DF as the Square of the Radius to the Semiquadrant: Therefore equal weighing being made, and assuming the Weight to the Arm CX, with the Perpendicular CM, the Semiquadrant will equiponderate the Fig. CMA, for the Moments are equal, *viz.* compounded from the Ratio of CX to CY, and from the Ratio of CY to CX.

COROL. II.

Hence is had the Center of the Triline MKA, *viz.* by drawing EG equal to EQ, and through G draw FGH: It will be as CD to DM, so FG to GH; H will be the Center sought, and the thing may easily be reduced to Calculation; let CM be 11, CA will be 7, likewise CO, MD will be 7, therefore CD will be 4, DE will be $1\frac{1}{2}$; EV $2\frac{5}{8}$; NM, or IK, $2\frac{7}{8}$; LK will be $2\frac{3}{10}$.

COROL.

COROL. III. Fig. 13.

Hence the Center of Gravity of each Fruſtum of the Iſoparallel in Prop. 8. may be found; for ſince the upper Fruſtum EABI is Homogene to the Triline OAI, by Prop. 8. the Center of the Fruſtum will be equally diſtant from the Baſe EAB, as the Center of the Triline from the Baſe AO: And theſe Diſtances are had by Coroll. 2. let it be AD, draw DP parallel to AC; I ſay, the Center of the Fruſtum is in P; for ſince 'tis in CS, alſo in DP, it will be in P, in the Point of the common Section. In like manner, the Center θ of the other Fruſtum is had, as well by drawing the right Line P θ through Y, as alſo by the ſame Method, ſince this Fruſtum is Homogene of the Fig. of Sines TIΘ, by Coroll. 1. Prop. 8.

COROL. IV.

Hence may be known the Quantity of the Line DP; for ſince AC is to AS, as DP to DS; and ſince AC is double of AS, DP will be double of DS: Therefore by putting the Axis AI to be 11, DS will be $2\frac{5}{8}$, and conſequently DP will be $5\frac{1}{4}$ becauſe 'tis the Double of $2\frac{5}{8}$, or of DS.

PROP. XI. Fig. 14.

If the Fig. of Sines be turned about the Baſe, that begat from it is to the Cylinder of the ſame Baſe and Altitude, as the Rectang. under the Baſe,

and the Exceſs whereby the Axis exceeds the Baſe; to the Rectang. under the Axis and half of the Axis; for let the Fig. be CMA, the Rectang. CY; turn both Figures about AX, the Solids begat are in Compoſition from the Ratio of the Figures, which is CO to CM, and from the Ratio of the Diſtances of the Center of both from AX, which is CD to CE by Poſition 9. and Prop. 9. Therefore that begat from CMA is to that begat from CY, as the Rectang. under CO, CD, to the Rectang. under CM, CE. Therefore as the Rectang. under the Baſe, and the Exceſs whereby the Axis exceeds the Baſe, to the Rectang. under the Axis, and half the Axis.

COROL. I.

Hence is had that begat from the Triline MAK; for by taking away that begat from CMA being known, from that begat from CY being known, the Reſidue will be the begat from the Triline MKA alſo known; and that the thing may be reduced to Calculation, let that begat from CY be 121, that begat from CMA will be 56; therefore that begat from the Triline MKA will be 65.

COROL. II.

And that we may find that begat from the Fig. CMA turn'd about MK, let that begat from the Fig. CMA turn'd about CA, be an Homogene Solid under the Square of the Baſe CM, and the Altitude CA be called B: Let the Parallelipiped of the ſame Baſe and Altitude be called C: Let the Iſoparallel

Isoparallel under the Fig. of the Base CMA, and the Altitude CM be called D; then C will be to D, as MC to MD: Add to D the Excess whereby D exceeds B; C will be to the Aggregate from D, and the aforesaid Excess, as that begat from CY to that begat from the Fig. CMA turn'd about MK: And by taking away this begat from that, the Residue will be the begat from the Triline MKA turn'd about MK. These may be demonstrated in the same Manner as is demonstrated at Prop. 7. but shorter. That begat from MA is to that begat from MCA turn'd about MK, as the Rectang. under MC, ME to the Rectang. under MD, MD; that is, as 121 to 98.

PROP. XII. Fig. 15.

If you let any right Segment of a Fig. of Sines be of the same Altitude with the versed Segment of the same, it will be to the whole Fig. as the Base of the versed Segment to the Base of the whole Fig. For let the Fig. of Sines be ARB, the Quadrant ABD; the right Segment ILR, the versed Segment of the same Height CNB: I say, ILR is to ARB, as CB to AB; for since MN and AC are equal, also AC and VC; TS will be equal to IL; because IA is equal to MR, therefore IR will be equal to the Arch DS; therefore the Segment IRL is to the whole Fig. ARB, as TD equal to VF, or CB to the whole AC, by Prop. 5.

COROL.

COROL. I.

Hence the right Trapezium AILB is to the whole ARB, as the right Sine of the Arch equal of Altitude to the Trapezium, *viz.* AI to the whole Sine AB.

COROL. II.

Hence the whole Fig. ARB is to the Segment MRN, as AB to SX; but to the Segment ILR, as AB to TD: Likewiſe to the Trapezium AMNB, As AB to TS; and to the Trapezium AILB, as AB to AT.

PROP. XIII. Fig. 16.

The Solid under the Altitude of the Baſe of the Fig. of Sines proceeding by right Segments under the Altitudes ordinately applied, parallel to the Axis, is Homogene to the Triang. Let ABC be the Fig. of Sines, for Example, the Baſe; let alſo the Fig. of Sines be ABD under the right Angle BAD, and under the ordinate Applicates Parallels BA, OH; let the right Segments, as OHG be equal to EAF: I ſay, this Solid under the Baſe ABC, and the Altitude AD proceeding by the aforeſaid right Segments, of which the Vertexes are terminated at the right Line AD, is Homogene to the Triang. (for Example) ASD. For ABC is to AEF, or HOG, as AD to HD, or AS to HK, by Prop. 12. The ſame may be ſhewn by aſſuming any other right Segment. Therefore the whole Solid ABCD is Homogene to the Triangle ASD.

COROL.

COROL. I.

Hence the aforeſaid Solid ABCD is half the Iſoparallel under the Baſe ABC, and the Altitude AD: For as the Triangle ASD is to the Square AI, ſo is the Solid ABCD to the Iſoparallel ABCD.

COROL. II.

Hence the Solid, which proceeds by Rectangle Parallels under the right Sine, and of the Complement, being applied to AB in the two Figures ABC, ABD is equal to the former Solid. For let there be any Applicate EO, to which another EF approaches at a right Angle OEF; EO is the right Sine of the Arch equal to EB; and EF the right Sine of the Arch equal to EA; therefore 'tis the Sine of the Complement of the firſt Arch; therefore the Rectang. E. F. is under the right Sine, and of the Complement: The ſame may be ſhewn by aſſuming any other right Angle. Therefore that Solid, which proceeds by the Rectangles of this ſort is equal to the Solid ABCD; therefore Subduple of the Iſoparallel under the Baſe ABC, and the Altitude AD.

PROP. XIV. Fig. 17.

The Solid under the Baſe from the generating Quadrant, and from the Altitude of the Axis of the Fig. of Sines, proceeding by Sectors parallel to the Baſe, which ſhall be in the Ratio of Altitudes, is

is Homogene to the Triang. Let the Quadrant be ALC, the Altitude AF, equal to the Arch of the Quadrant CL, and let any Sector be GID, under the Arch ID, equal to GF, since the right Sine of it is GH; I say, the aforesaid Solid is Homogene to the Triang. AFC: Because the Sector ACL is to the Sector GID, as the Arch CL to the Arch DI; that is, as AF equal to the Arch CL to GF, equal to the Arch DI; that is, as AF equal to the Arch CL to GF, equal to the Arch DI, that is, as AC to GO; therefore the Solid, and the Triangle proceed by proportional Elements, therefore they are Homogene.

COROL. I.

Hence the aforesaid Solid, proceeding by Parallel Sectors, is one Eighth of the Cylinder under the Base of the Circle, whose Radius is AC, and under the Altitude AF.

COROL. II.

Hence, if a Solid happens of the same Altitude AF, proceeding by Rectangle Triangles, under the Base and Altitude, from the right Sine and the Complement, it will be $\frac{1}{4}$ of the Isoparallel Solid, whose Base is the Fig. of Sines, and the Altitude the same of the Fig. of the Base; (for Example) let the Triang. be GHD under GH the right Sine of the Arch DI, or KL equal to GF, and under HD, equal to BK, the right Sine of the Arch KC equal to GA. Therefore GHD is under the right Sine, and the Sine Complement: The same

may

may be ſhewn in any other aſſumed. And ſince this Solid proceeding by the aforeſaid Triangles, is Subduple of the Solid proceeding by Rectangles under the ſame right Sine, and the Sine of the Complement; for they are Homogenes, and of the ſame Altitude, and every Rectangle is double of its Triangle. And ſince the Solid proceeding by the aforeſaid Rectangle is ſubduple of the Iſoparallel Solid under the Baſe of the Fig. of Sines, and the Altitude the ſame as the Fig. from the Baſe, by Coroll. 1. of Prop. 13. the aforeſaid Solid proceeding by Triangles will be $\frac{1}{4}$ of the aforeſaid Iſoparallel.

COROL. III.

Hence is had the whole Solid ACL, *viz.* the Aggregate from the Procedents by the Sectors, and from the Procedents by the Triangles, for the Procedents by the Triangles is $\frac{1}{4}$ of the aforeſaid Iſoparallel, which is equal to the Cube under the Side AC. Hence the Ratio's may be reduced to Calculation, let the Parallelipiped under the Square of the Baſe AC, and the Altitude AF be 14, it will be to the Cube under the Side AC as 14 to $8\frac{5}{22}$; therefore to the Procedents by Triangles as 14 to $2\frac{10}{11}$; therefore to the Quadrant, or Fourth of the Cylinder, under the Baſe from the Quadrant ALC, and the Altitude AF, as 14 to 11; therefore to the Procedents by the Sectors, as 14 to $5\frac{8}{11}$; therefore to the Aggregate aforeſaid, as 14 to $7\frac{1}{2}$.

COROL.

I

COROL. IV.

Hence, if from the aforeſaid Aggregate be taken away the Solid proceeding by parallel Squares AI under the Altitude AC, which is equal to $\frac{2}{3}$ of the Cube, and therefore 5 $\frac{31}{33}$ of the Reſidue; that which proceeds by Rectangles parallel to GE will be 1 $\frac{26}{33}$, which if it be added to the Solid AEC, the Aggregate will be 9 $\frac{17}{33}$.

COROL. V.

Hence, if there be a Parallelipiped under the Altitude AC, and the Square of the Baſe AF, it will be to the aforeſaid Parallelipiped in the 3d Coroll. as 22 to 14; ſince the Parallelipiped under the Square of AF, and the Altitude AC, is to the Solid, which proceeds by the Squares under AF, BH; that is, under Arches of the ſame Altitude, as 121 to 56, by Prop. 11. Coroll. 1. that is, as 22 to 10 $\frac{2}{11}$; if from 10 $\frac{2}{11}$ we take away the Aggregate 9 $\frac{17}{33}$, as in Coroll. 4. the Reſidue will be $\frac{22}{33}$, that is, the Solid under the Altitude AC, proceeding by Squares, under the Differences of the Sines, and of the Arches, which is to the Solid, proceeding by the Squares under the Sines, as 11, to 98.

COROL. VI.

The Solid AEC proceeds by Rectangles parallels of AE, under the Arches, and right Sines: So AE is under AL the whole Sine, and AF equal to the Arch of the Quadrant. Alſo BD under BH, equal

equal to the Arch KC, and BK the right Sine of the Arch CK: The ſame may be ſhewed in any other aſſumed.

COROL. VII.

Hence if a Solid be made of the Altitude AC, proceeding by Rectangles under the Arches, and under the Compounds from the Arches and the Sines, from which take away a Solid of the ſame Altitude, proceeding by Squares, under the Arches, and which we appoint at Coroll. 5. to be to the Parallelipiped of the ſame Baſe and Altitude AC, as $10\frac{2}{11}$ to 22: Without doubt the Solid remaining of the ſame Altitude AC will proceed by Rectangles under the Arches, and the Sines; therefore 'tis equal to the Solid AEC: Therefore ſince AEC is $7\frac{8}{11}$, the aforeſaid Solid, proceeding by Rectangles under the Arches, and the Compounds from the Sines, will be $17\frac{10}{11}$; to which if we add the Solid proceeding by the Squares of the Sines, likewiſe proceeding by the Rectangles under the Arches and the Sines, the Aggregate will be $31\frac{19}{33}$, and this is the Solid of the Altitude AC proceeding by the Squares under the Compounds from the Arches and the Sines. Alſo the Parallelipiped is had under the Altitude AC, and the Square of the Baſe under the compounded from the Arch of the Quadrant, and the Radius; for it is to the Parallelipiped of the ſame Altitude under the Square of the Baſe AF, as 324 to 121, or as $58\frac{110}{121}$ to 22.

PROP.

I

PROP. XV. Fig. 18.

If the Fig. of Sines be doubled or whole, they will resemble two Trilines joined by the Bases, the whole Fig. is Homogene to the Solid begat from the Fig. of Sines turned about the Axis; for ERM will be whole or double of the Fig. of Sines, to which the two Trilines EDR, ADR are alike, being joined by the Bases in AR: I say, the Fig. EARM is Homogene to that begat from the Fig. of Sines (for Example) of EAM turn'd about EA; to wit, as EM to DT, so DT to DR; and accepting DF, DC equal; as EM to CQ, the right Sine of the Arch equal to CA, so CQ to CX, the versed Sine of double the Angle in the Quadrant, whose Arch is equal to AD, the Half of AE; and as EM to FO, the right Sine of the Arch equal to FA, so FO to FZ, of which also FY the Difference is ZY equal to CX, by Position the 6th: But as EM to DR, so that begat from EM to that begat from DT; and as EM to FZ, so that begat from EM to that begat from FO; and as EM to CX, so that begat from EM to that begat from CQ, for they are begat as the Squares; therefore the Fig. EARM is Homogene to that begat from EAM turn'd about EA.

COROL. I.

Hence EARM is subduple of the Rectan. ES, because the aforesaid begat is Homogene; yea, since the Triline ADR is equal to the Triline EDR, the Rectang. EK will be equal to the Fig. EARM, and

and ſince this is Homogene to the aforeſaid begat, from thence alſo it follows, that the aforeſaid begat is ſubduple of the Cylinder of the ſame Baſe and Altitude.

COROL. II.

Hence the Center of Gravity of the Fig. EARM is had; for ſince the Center of ERM is had, ſuppoſe γ, and of the Triline AER, ſuppoſe δ; draw $\gamma\delta$, and ſo divide it in H, that H γ be to H δ, as the Triline ERA to ERM, that is, according to *Archimedes*'s Ratio, as 7 to 4; H will be the Center of Gravity of that you ſeek, as appears from the aforeſaid.

COROL. III.

Hence by uſing Calculation, and by drawing $\gamma\pi$, H θ; E π is to ED, as 4 to 11, and to EA, as 4 to 22; and $\pi\theta$ to θD, as 4 to 7: Therefore let D π be 11, and $\theta\pi$ will be 4; E π $6\frac{2}{7}$; ED $17\frac{2}{7}$; EA $34\frac{4}{7}$: Therefore E θ to θ A, as $10\frac{2}{7}$ to $24\frac{2}{7}$.

COROL. IV.

The Center of Gravity of the begat from the Fig. of Sines EATM turn'd about AE is θ, for the aforeſaid begat is Homogene of the Fig. EARM; therefore the Center of both will be equally diſtant from the Baſe.

PROP. XVI. Fig. 19.

If we turn about the Axis Plane Figures Homogene to the begat from the Fig. of Sines turned about the Axis, the Ratio is had of the begat to the Cylinder of the ſame Baſe and Altitude; for let EARM be Homogene to the begat from the Fig. of Sines, as aforeſaid, turned about EA; that begat from it, is to that begat from ES in the Ratio compounded from the Ratio of the Fig. EARM to ES, that is, half; and from the Ratio of IH to DR: For I ſuppoſe in H to be the Center of Gravity of the Fig. EARM, for the aforeſaid Solids are as the Moments of each Fig. vibrating in EA, by Poſition 9; and the Moments are in the aforeſaid compounded Ratio, as hath been already ofttimes repeated.

COROL. I.

Hence is found the begat from ERM; now becauſe we have the begat from the Triline EDR, by Prop. 7; therefore equal to this is the begat from the Triline ADR, both being taken away from the begat from EARM being known, the Reſidue will be the begat from ERM. This may alſo be demonſtrated otherwiſe; for the begat from ERM is to the Cylinder of the ſame Baſe and Altitude, as the Iſoparallel under the Baſe ERM, and the Altitude EM, to the Parallelipiped under the Baſe EP, and the Altitude EM, by Poſition 7.

COROL.

COROL. II.

Hence alſo is had that begat from MPR; to wit, by taking away the begats from ERM, and from EDR known, from the begat from EP known.

PROP. XVII. Fig. 12.

The Center of Gravity is had of that begat from the Triline annex'd to the Fig. of Sines turn'd about the proper Axis. Let the Triline be CBL turned about CL, from it the Center of Gravity of the begat is had from the Centers of the other Solids being known, of which the Axis is common; to wit, we have the Center of the Parallelipiped under the Baſe AE, and the Altitude AB, equal to QY in the Scale. Alſo of the Solid proceeding by the Squares AE, NI, under the Sines, Homogene to that begat from ABC turn'd about AB; let it be T. Alſo of the Iſoparallel under the Baſe ABC, and the Altitude AD of the proceeding by the Rectangles AE, NM, for it is Homogene of the Fig. ABC; let it be X. And as the Difference of each to the Solid proceeding by the Squares AH, NI, ſo TX to XZ in the Scale annex'd to the 12 Fig. Z will be the Center of the aforeſaid Difference; to wit, of the Solid proceeding by Rectangles parallel to GI; alſo in Z is the Center of the other equal Solid proceeding by Rectangles parallel to IS; alſo the Center of the Iſoparallel Solid is had, under the Triline Baſe CLB, and the Altitude CE; for let QY the Scale, be divided in the Middle in θ, and

as the foresaid Isoparallel under the Base CLB, to the Isoparallel as above under the Base ABC, so X θ to $\theta\delta$; δ will be the Center of the Isoparallel under the Base CLB; lastly, let it be as the Solid proceeding by Rectangles parallel IS as above, also known, of which the Center is Z, so Z δ to $\delta\,\gamma$, the Center of the Solid proceeding by the Squares parallel BF, IH will be in γ.

But this is Homogene to the begat from the Triline EFK turned about EF; therefore if QY is the Axis of the foresaid begat, the Center of Gravity of it will be in γ; lastly, if as $\gamma\,\theta$ to $\theta\,\pi$, so the begat from CAB turned about CL, to the begat from the Triline CLB; π will be the Center of gravity of the begat from CAB: In like manner the Center of Gravity may be had of the begat from the Triline CLB turned about AB, so T θ to $\theta\,\beta$; β will be the Center sought.

PROP. XVIII. Fig. 19.

The Center of Gravity is had of that begat from a Fig. Homogene, to the begat from the Fig. of Sines turned about the Axis; for let the foresaid Homogene Fig. be EARM turned about EA, we have the Center of the begat from both the Trilines EDR, ADR which is in D; likewise of the begat from ERM, to wit, of the Homogene to the Isoparallel Solid under the Base ERM, and the Altitude EM by Position 7. which is in Z, by granting that the Center known of the Fig. ERM is in X; so lastly, ED will be divided in I, that ZI is to ID, as the begat from both the Trilines EDR, ADR, is to the begat from ERM; I will be the Center of the begat from the Fig. EARM.

PROP.

PROP. XIX.

If the foreſaid Fig. Homogene to the begat from the Fig. of Sines as aforeſaid, be turned about the Baſe, the Ratio is had of the Solid begat to the Cylinder of the ſame Baſe and Altitude; let them be the ſame as above, and turn the Fig. E A R M about E M; let the Center of the Fig. be H, the Solid begat is to the Cylinder in compoſition from the Ratio of E D to E A, and from the Ratio of L H to E D, that is, as the Rectangle under E D, L H, to the Rectangle E A, E D, that is as L H to E A; for the Solids begat are compounded of the Figures, and of the Diſtances of the Centers from the common Axis about which the Revolution is made by Poſition 9.

COROL. I.

Hence alſo are had other begats, to wit, from E D R M; becauſe we have the begat from E R M, by Prop. 11. alſo the begat from the Triline EDR, by Corol. 1. of Prop. 11, therefore the Aggregate from both, therefore the begat from EDRM: Alſo we have the begat from the Triline ADR, for by taking away the begat from A D R M from the begat from E A R M, the reſidue will be the begat from A D R; likewiſe we have the begat from S M R A, for by taking away the begat from E A R M from the Cylinder begat from ES, the reſidue will give the begat from S M R A, from which if we take away the begat from the Triline M P R

known, the reſidue will give the begat from SPRA.

COROL. II.

Alſo we have the begat from EARM turned about MS, for by taking away the begat from SMRA known, from the Cylinder begat from MA, the reſidue will give the begat from EARM.

PROP. XX. Fig. 19.

The Center of Gravity is had of the begat from EARM turned about EM, for we have the Center of the begat from ERM, which is N; alſo of the begat from both Trilines EDR, ADR Homogenes to the Iſoparallel Solid under the Baſe EAR, and the Altitude EA; and ſince the Center of the Triline EAR, is known by Corol. II. of Prop. 10. let it be in Q, draw QV, the Center of the begat from EAR is in V: So laſtly, divide VM in O, that OV be to OM, as the begat from ERM is to the begat from EAR, E will be the Center of Gravity of the begat from EARM, the thing may eaſily be reduced to Calculation.

PROP. XXI. Fig. 20.

We may find the Ratio of the begat from any Segment of a Fig. of Sines; let the Fig. of Sines be DAS, any Segment AQT; let the Fig. DAFL be Homogene to that begat from the Fig. of Sines, as above; the begat from AQT, will be to the begat from DAS, as the Triline TAX to the whole DAFL;

DAFL; let DC be equal to AT, let alſo the Fig. of Sines be DFV, the Ratio of DFV to DAS being known, which is $\frac{1}{3}$, the Ratio alſo will be known of DAS to AL, which is $\frac{7}{11}$; therefore from theſe is had the Ratio of DAS to DAFL, which is $\frac{1}{2}$ AL, therefore the Ratio is known of DAFL to the Triline DCP; therefore alſo of the begat from DAS turned about DA to the begat from the Segment TAQ.

In another manner, the Ratio being known of the Rectangle DK to DG, equal to the Fig. DAFL, alſo to the Trapezium DPIL, which being taken from DK, the reſidue are the Trilines DEP, LKI; therefore the Ratio is had of the Rectangle DG to the Triline DCP.

COROL. I.

Hence by tranſlating the Segment TAQ into DCZ, and by making the Fig. of Sines EMK by producing the Axis FM, and then dividing VN in the Middle, draw RY parallel to LS; if the two Trilines LKI, DCE librate in YR, it will be equal weight; For as IK to DE, ſo any Parallel of IK to any Parallel of DE equidiſtant from RY; but as DCE to LKI, ſo the begat from TAQ or DCZ to the begat from S$\gamma\delta$, to wit, as the begat from DZ is equal to the begat from $\delta\gamma$, ſo the begat from any Parallel to DZ, is equal to the begat from the Parallel to $\gamma\delta$ equidiſtant from YR: Therefore the Moments of the Weights of thoſe begat from DCZ, and from S$\gamma\delta$ in YR are equal; likewiſe the begat from CδSZ, to wit, as PI is equal to EH, and any other Parallel to PI, equal to another

ther Parallel equidiſtant from OR, ſo the begat from C♂ equal to the begat from SZ; And any other begat from a Parallel to C♂ equal to the begat from another equidiſtant from YR.

COROL. II.

Hence is had the Diſtance of the Center of Gravity of the Segment TAQ from the Axis TA, for ſince the Ratio is had of the begat from TAQ, to the begat from the Rectangle under TA, TQ; alſo the Ratio of the Segment TAQ to the foreſaid Rectangle, ſince thoſe begat are in compoſition, or are compounded from the Ratios of the Figures, and of the Diſtances of the Center of each from the common Axis TA: Laſtly, ſince we have the Diſtance of the Center of the aforeſaid Rectangle, which is $\frac{1}{2}$ TQ, from thence we have the other Diſtance, *viz*, of the Center of TAQ from the ſame TA; which that it may appear more clear, ſee Fig. 21. let the Ratio of thoſe begatters be AB, AD, let the Ratio of the Figures be AB, AC, let BC be a Diſtance known, to wit, of the Center of the Rectangle common from the Axis; let the Rectangles be AE, AG, AB, draw KC, FIL; FG will be the Diſtance ſought; and the Rectangles are AE, LG, as AB, AD, and in compoſition from the Ratios of AB to AC, and of BE to FG.

COROL. III. Fig. 20.

Hence is had the Diſtance of the Center of Gravity of the Trapezium DAQZ from the Axis AD; becauſe we know the Diſtance of the Center of the Rectangle

angle DQ, alſo of the Segment TAQ; therefore we know the Aggregates from both; likewiſe we have the Diſtance of the Center of Gravity of the Trapezium DTQS from the Axis DA, ſince we have the Diſtances of the whole DAS, and of the Part TAQ, of the other Part alſo may be had.

COROL. IV.

Hence is had the Diſtance of the Center of Gravity of the verſed Segment ZQS from the ſame DT, to wit, ſince we have the Diſtances of the whole DTQS, and of the other Part, to wit, of the Segment ZQS, therefore alſo the Diſtance of the Center of the ſame ZQS from the Axis ZQ; therefore we have the begat from the Segment ZQS turned about ZQ, for it is to the Cylinder of the ſame Baſe and Altitude in the compounded Ratio from ZQS to ZT, and from the Ratio of the Diſtance of the Center ZQS from ZQ to the half of ZD.

COROL. V.

Hence from deſcribing the Quadrant DST ſince the Perpendicular is had falling on ZS from the Center of Gravity of the Segment ZQS, likewiſe the Perpendicular falling on ZS from the Center of Gravity, Z β S is had alſo the Perpendicular falling on the ſame DS, from the remaining Fig. SQT.

PROP. XXII. Fig. 22.

The Homogene Fig. begat from the Fig. of Sines turned about the Axis, is equal to the Circle whoſe

Diameter

Diameter is the Baſe of the Fig. Let the foreſaid Homogene Fig. be ASIH, let the Semicircle be AKH; ſince this is equal to the Rectangle AI, and therefore the whole Circle equal to the Rectangle AL, to which ASIH is equal, this will be equal to the Circle.

COROL. I.

Any Parallels being drawn to AS, as CT, FV, XZ: As AS is equal to the Arch AKH, ſo CR to the Arch ONH; and XM to the Arch NH; from XM appears from conſtruction of the Line of Sines IMH; for ſince FX is the Sine of the Arch KN, *viz.* equal to MZ; MX will be equal to NH, to wit, FI is ſuppoſed equal to the Arch KH; and ſince RT is equal to the Arch ω A, CR will be equal to the Arch ω KH; the ſame may be ſhewn of any other aſſumed.

COROL. II.

Hence by taking away the Semicircle AKH, the remaining Fig. is equal to the Semicircle; and the Fig. of Sines ASZH is equal to the Quadrate under AH. Laſtly the Leaf HISZH is equal to the Difference of the foreſaid Square, and of the Circle under the Diameter AH.

COROL. III.

If from the Center of Gravity of the Fig. ASIH you let fall a Perpendicular on AH, ſuppoſe on D, AD will be to DH, as 3 to 5, to wit, from the Center

Center A I H it will fall on F, likewiſe from the Center of the Triline A P I it will fall on B; let FA be 7, A B will be 2 $\frac{3}{16}$; ſo BF being divided in D, that B D be to D F as A P I to A I F, that is, as 4 to 7, or as 16 to 28; DF will be 1 $\frac{3}{4}$, therefore DA 5 $\frac{1}{4}$; D H 8 $\frac{3}{4}$, therefore AD to D H, as 5 $\frac{3}{4}$ to $\frac{3}{8}$, 8 $\frac{3}{8}$, or as 21 to 35, that is as 3 to 5, hence if it be divided into 8 equal Parts, A D will be 3, and D H will be 5 of the ſame Parts.

COROL. IV.

If from the Center of the Fig. of Sines F I H, we let fall a Perpendicular on G I, F G will be $\frac{1}{4}$ of the whole F I; and as F H to F G, ſo the Square under FH, to the Semiquadrant of the Circle under the Radius F H; therefore by hanging from F the Fig. FIH in the Perpendicular IF to the Arm of the Beam or Ballance A F, the Semiquadrant in A will make equal Weight, or equal Moment; for ſince the Moments are compounded of the Quantities and of the Diſtances, the Moment in G is to the Moment in A, as the Rectangle under F H, F G, to the Rectangle under F G, F H; to wit, in G is the Quantity F H, the Diſtance G F; but in A the Quantity F G, the Diſtance F A or F H, therefore both Moments are equal.

COROL. V.

If H ♂ be equal to AS, alſo FI equal to FY, and you make Y ß ♂ the Line of Sines, the Fig. IH ♂ Y will be Equal and Homogene to the Rectangle LY; for as H ♂ is equal to L θ, ſo BM is equal to π Z; the ſame

ſame may be ſhewn of any other aſſumed; therefore the foreſaid Fig. is Homogene to the Rectangle LY, therefore equal to the ſame, ſince 'tis of equal Altitude HF, and of equal Baſe H θ, therefore the foreſaid Fig. is equal to the Circle under the Radius FH, therefore alſo of the Fig. ASIH.

COROL. VI.

If the foreſaid Fig. be hung from F in the Perpendicular IF, on the End or Arm of the Ballance AF, the Semicircle AKH in A will make equal Weight, for divide HF in the Middle in X, and draw XZ β, it will go or paſs through the Center of the Fig. therefore the Moment in X is equal to the Moment in A, ſince the Ratio of the Weights is $\frac{1}{2}$, and of the Diſtances $\frac{2}{1}$ therefore compounded of $\frac{2}{2}$, therefore the Moments are equal; the ſame will be done if we hang the Perpendicular LH on the Arm of the Beam or Ballance H γ equal to HF.

COROL. VII.

Let there be the ſame Perpendicular LH, and the Arm of the Ballance H γ, for the Fig. of Sines FIH being hung, the Square under HF leſs $\frac{1}{8}$ of the Circle will make Equilibrium, as is manifeſt; but for the other Fig. FY δ H, they will equiponderate $\frac{5}{8}$ of the ſame Circle, leſs by the Square under HF.

COROL.

COROL. VIII.

But if the Perpendicular be AP, the Arm of the Ballance AE equal to AF, for the Fig. ASIF being hung; they will make equal Weight $\frac{5}{8}$ of the ſame Circle, leſs by the Square under AF; becauſe 'tis equal to the Fig. H ♂ YF already hung to the Perpendicular LH, and of the ſame Poſition; and for the Fig. of Sines FIH hung to the Perpendicular AP, it will make Equilibrium with $\frac{1}{8}$ of the ſame Circle, more by the Square under AF; therefore from the Ratio of the whole Fig. ASIH being hung, they will make Equilibrium $\frac{6}{8}$ of the foreſaid Circle, becauſe if we add $\frac{5}{8}$, leſs by the Square of $\frac{1}{8}$ more by the Square, the Sum will be $\frac{5}{8}$; therefore as 8 to 6, or as 4 to 3, ſo EA to AD, for from the Center of the Fig. ASIH the Perpendicular will fall in D; but AF is equal to AE, therefore AD is to DH, as 3 to 5, as is already ſhewn above.

COROL. IX.

If we ſuppoſe the Circle whole, the ſame Center A, ſince the Diſtances AF, EF are equal according to Equilibrium, the ſame thing will happen in the Moment E, therefore $\frac{4}{8}$ of the Circle; therefore thoſe of the Moment E compound $\frac{10}{8}$ of the Circle, but the oppoſite equal Moment compounds $\frac{12}{8}$ of the Circle; therefore the Diſtances are in the ſame Ratio by changing, therefore as 12 to 10, or 6 to 5, ſo EA to another (for Example) to AO, in O will fall the Perpendicular from the Center of the Fig.

And

And ſince A F is equal to AE, let AH be 12, AO will be 5, and OH will be 7.

COROL. X.

Hence if the Fig. ASIH be turned about A S, that begat is to the Cylinder of the ſame Baſe and Altitude, as 3 to 8, for it is compounded from the Ratio of the Fig. ASIH to the Rectangle under A S, AH which is $\frac{1}{2}$, and from the Ratio of A D to AF which is $\frac{3}{4}$, therefore 'tis compounded of $\frac{3}{8}$.

COROL. XI.

If we ſuppoſe an whole Circle from the Center A, and the Fig. turned about as above, or before the Ratio of the begat to that of the Cylinder is had; to wit, being compounded from the Ratio of the Fig. ASIH more by the Semicircle, to the Rectangle under AH and AS more by AF; and from the Ratio of AO to AF.

COROL. XII.

If from the Fig. ASIH be taken away the Semicircle, that it remain in Equilibrium, the Remainers as ſtanding above, the Semicircle ought to be taken away, that is, $\frac{4}{8}$ to the Moment E; therefore there remains $\frac{2}{8}$ for the ſame Moment E at Equilibrium; therefore ſince the Ratio of Weights is $\frac{1}{2}$, the Ratio of Diſtances will be $\frac{2}{1}$; therefore if A C be ſubduple of AE, or AF, the Perpendicular will fall in C, from the Center of Gravity of the Fig. ASIH, by taking away the Semicircle AKH.

COROL.

COROL. XIII.

If we turn about the Fig. ASIH, by taking away the Semicircle AKH, that begat is to the Cylinder, as 1 to 8, *viz.* in Compoſition from the Ratio of the Fig. to the Rectangle under AS, AH, which is $\frac{1}{4}$; and from the Ratio of AC to AE which is $\frac{1}{2}$; therefore the compounded is $\frac{1}{2}$; Hence that begat from the whole ASIH is triple of the foreſaid begat; and the begat from the Semicircle AKH double; hence by aſſuming the whole Circle, that begat from the Fig. is to the Cylinder under the ſame Baſe and Altitude AS, as 5 to 8, to wit, the begat from the Semicircle is to the aforeſaid Cylinder, as 2 to 8; therefore that begat from the Circle, as 4 to 8, that begat from the Reſidue, as 1 to 8, therefore that begat from the whole, as 5 to 8.

COROL. XIV.

The Center of the Fig. ASIH is had, for by drawing from the Center of the Fig. AIH known, the right Line to the Center of the Triline AIS known, and that will be the Center of the whole Fig. in which the foreſaid right Line will cut the Perpendicular by falling in D, ſuppoſe ω.

COROL. XV.

Hence is had that begat from the Fig. ASIH turned about AH, for it is to the Cylinder of the ſame

ſame Altitude and Baſe, as the Rectangle under AF, D ω, to the Rectangle under AH, AP, to wit, in the Compoſition from the Ratio of the Fig. to the Rectangle and of the Diſtance D ω, to the Diſtance AP.

Therefore that I may compriſe the whole in brief, let BEFC be like to the former, alſo BESA, they will reſemble BGHC, HGIA, the two Semicircles are under the Diameters AB, BC; likewiſe the Fig. of Sines BFC, BSA; let S δ be equal to ML, and the other in like manner to the other Sines, bein terminated with the Line A δ E; laſtly let the Fig. of Sines be AIB, the Semicircle BXA, the Parallels OR, ZV, &c. theſe being granted or ſuppoſed, if about EG we turn BEFC, that begat is to the Cylinder of the ſame Baſe and Altitude, as 3 to 8; if by taking away the Semicircle the Remainer of the Fig. be turned about, that begat is to the Cylinder, as 1 to 8; if we turn about the Fig. of the Semichord BGIAL, that begat is to the foreſaid Cylinder, as 5 to 8; in like manner is had that begat from BFC, which is to the Cylinder of the ſame Altitude, as the Square of the Radius to the Semicircle, alſo the begat from the Triline BFC.

COROL. XVI.

If we turn the Fig. BESA about the Tangent in A, that begat is to the Cylinder of the ſame Baſe and Altitude, as 5 to 8, *viz.* in Compoſition of the Figures $\frac{1}{2}$, and of the Diſtances from the Center $\frac{1}{4}$: If we turn about the whole AEC, that begat will be $\frac{1}{2}$ of the Cylinder under the ſame Baſe and Altitude, for the Ratio of Diſtances is of Equality; there-

therefore thoſe begat are as the Fig. If we ſubtract the Semicircle, and the remaining Fig. BESA be turned about, that begat will be to the Cylinder as 3 to 8, to wit, compounded from the Ratio of the Figures $\frac{1}{4}$, and of the Diſtances $\frac{3}{2}$; if we ſubtract each Semicircle, and turn about the Reſidue of the whole Fig. AEC, which is like to a Lilly, that begat will be to the Cylinder as 1 to 4, which is the Ratio or Proportion of the Figures, for they are equal of diſtance; if we turn about the Fig. of the half Chord, it will be to the Cylinder under the Altitude BG, and of the Baſe begat from AB, as 7 to 8: If laſtly the whole Heart, that begat will be to the Cylinder under the Altitude BG, and of the Baſe begat from AC, as 6 to 8, *viz.* the begat from the Fig. AEC, is to the aforeſaid Cylinder as 4 to 8, that begat from the Lilly, as 2 to 8, therefore that begat from both Semicircles as 2 to 8, therefore that begat from the Heart, that is from the Fig. together and the double Semicircle, is as 6 to 8; hence that begat from the Semi-heart turned about BG, is equal to that begat from BCSA about the Tangent A; alſo that begat from the Fig. BCSA about BE, is equal to that begat from the Reſidue of the ſame Fig. from which the Semicircle is taken away, about the Tangent A.

COROL. XVII.

If the Revolution be made about AC, ſince the Ratio of the Figures, and of the Diſtances is had from the Center, the Ratio of thoſe begat is had which is compounded from both, and we have the Center as well of the Fig. AEC, as of the Lilly, as appears: Alſo from the turning abont being made about the

Tangent is had that begat from the Fig. BESA; for by taking away from the Cylinder the begat from the ſame turn'd about BA, the Remainer will give that begat from the ſame Fig. about the Tangent E. Likewiſe is had that begat from the remaining Fig. BESA from the ſubtracted Semicircle; to wit, is had that begat from the Semicircle ALB being turned about the Tangent E, for it is half of the Cylinder under the Altitude BA, and of the Baſe begat from EB, leſs the Sphere under the Diameter AB; but by taking away the begat from the Semicircle is had that begat from the Remainer, therefore that begat from the Lilly; therefore alſo the begat from the Triline BSE, likewiſe that begat from the Fig. ESAIG, which is half of the Cylinder, for ſince the Diſtances are equal, thoſe begat are as the Figures.

COROL. XVIII.

If we turn about the Tangent G the Fig. of the half Chord, the begat from it is had; to wit, the begat from the Semicircle ALB is had; and as the Parallelipiped under the Square of GB, and the Altitude BA to the Cylinder under the Altitude GB, and of the Baſe from the Circle AXBL, more by the ſolid Homogene to the Hemiſphere, proceeding by the Squares of the Sines under the Altitude AB, ſo that begat from the Rectangle GA turned about G, to that begat from the Semicircle ALB; therefore by taking away the begat from BESA turned about BA, from the begat from the Rectangle GA turned about G, the Remainer ſince being begat from a Semicircle, will give that begat from the half

half Heart about G, hence are had those begat from the whole Circle AXBL, from the whole Figure of the Heart, from the Lilly, from the double Lilly by taking away the Circle on both Sides; all these may be reduc'd to Calculation, supposing *Archimedes* his Ratio.

COROL. XIX.

The Center of Gravity of that begat from BESA, turned about BE, may easily be found; to wit, the Center of that begat from the Triline BSE is in D, but of that begat from BSA 'tis equally distant from AC, as the Center of the Fig. BSA; therefore the Centers of the Begats ABSA and BSE are had in EB; therefore is had the Center of the begat from BXA, therefore from the common of the begat from BXAS; therefore of the begat BESAX, also of the begat from the Residue BESA, to which the taken away is BLA; therefore also the Center of that begat from the Figure of the Heart.

COROL. XX.

The Fig. contained by the Curve Line A δ E, and by the Curve Line ASE is Homogene to the Semicircle ALB, and equal to the same, as appears; also that begat from BSEAX and BE δ A are equal; likewise the Residue of the Fig. BE δ A, by taking away the Semicircle BLA, is equal to the Fig. BESA; also that begat from both are equal; likewise that begat from MSA about MA is had, which is Homogene to the Solid of that which proceeds by the Squares of the Arches known; also

that begat from M δ A about MA, which is Homogene to the Solid which proceeds by the Squares of the Compounds from the Arches and Sines alſo known by Corol. 7. Prop. 14. But the Solid proceeding by the Squares of the compounded from the Arches and Sines IL, TV, is equal to the proceedent by the Squares M δ, and of the Parallels terminated at the Curve Line A δ E; alſo equal to the proceedent by the Squares of IL and QR, &c. Laſtly, if from the Parallelipiped under the Square of GB, and the Altitude BM, be taken away twice, the Iſoparallel under the Baſe from the Triline LSB, leſs by the ſolid proceeding by the Squares under LS, and the other Differences of the Arches and Sines under the Altitude BG, being known by Corol. 5. Prop. 14. the Reſidue will give the Solid under the Altitude BM, proceeding by the Squares GB or IL equal to the proceedent by the Squares BE, M δ under the ſame Altitude; hence is had the whole ſolid under the Altitude AB, proceeding by the Squares BE, M δ, &c. and this is to the Parallelipiped under the Altitude AB, and of the Baſe from the Square of BE, as that begat from the Fig. BE δ A turned about BA to the Cylinder of the ſame Height and Baſe. Theſe Things have been ſo often demonſtrated that it were a Shame to repeat them.

PROP. XXIII. Fig. 24.

The Semicycloid terminates the compounded from the Arches and Sines ordinately applied to the Diameter of the Circle; that theſe Things may be underſtood with thoſe which are ſaid and demonſtrated,

ſtrated, ſomewhat of new Conſtruction is to be uſed; let CB be the Diameter of the Semicircle BDC inſiſting at Rectangles to CG, and draw how many ſoever Parallels OT, AI, KX, let LN be equal to the Arch LB; alſo DF equal to the Arch DLB, OS to the Arch PLB; Laſtly, CG equal to the Arch of the Semicircle CDB, and through the extream Points BNFG, ſuppoſe the Curve drawn; moreover, ſuppoſe the Semicircle CDB, ſo moved in CG by a right Motion of the Center through AI, and by the Motion of the Orb about A, that each Motion be equal and made together: Furthermore aſſume LM equal to KL; DE equal to AD; PQ equal to OP; and the ſame may be done in any aſſumed, and through the extream Points BMEQC draw the Curve; Laſtly, Let the Curve CRFH be like to the former BNFG, and the Fig. of Sines IYH; theſe being poſited, I will demonſtrate the Line BNFG to be the Semicycloid, that is the Curve Line deſcribed from the Point B, in the aforeſaid Motion of the Semicircle; by Definition 2. move B by the Motion of the Orb through the Arch BD, it will recede from BC by the Space AD, that is the Sine of the Arch DB; and together or at the ſame time let the Motion of the Center move with an equal Motion to the Orb, it will recede from BC by the Space DF, equal to the Arch DB, for equal Spaces ſuppoſe equal Motions in equal Time, therefore from both Motions together the Point B will recede from BC, by the Space AF; and ſince the Arch BD paſſing over is in the Line AI, it will be in F: In the ſame Manner it may be proved by the Arch BL paſſing over, that the Point B is in N; from the Arch PLB paſſing over 'tis in

S; from the Arch CDB passing over, 'tis in G; Lastly, From any other Arch passing over, to terminate the Applicate.

And that all Ambiguity may be taken away, I shall subjoin some Definitions of Words; the Curve Line BNG is called a Semicycloid; the Fig. of half the Cycloid CBFG; the Axis BC; the Base CG; the generating or begetting Circle BDC; the ordinate Applicate AF, the right Segment KBN; the versed Segment γ FG; the right Trapezium CKNG; the versed Trapezium CBF γ.

COROL. I.

Any Segment ordinately applyed, intercepted between the Arch of the Semicircle CDB, and the Curve BFG, is equal to the Arch cut of (for Example) DF is equal to the Arch cut of DB; LN equal to the Arch LB; PS equal to the Arch PLB.

COROL. II.

Hence the Curve Line A δ E, in Fig. 23. is an half Cycloid; and B A δ E the Fig. of the Cycloid.

COROL. III.

Hence the Semicycloid above, being rightly determined or limited, as well by the Motion of the Circle or Wheel, or of the Circle on a Plane, so that the Motion of the Orb, and of the Center are equal, as by the ordinate Applicates to the Diameter of the

the Circle, compounded from the Sines and the Arches.

COROL. IV. Fig. 24.

The Fig. BEC is Homogene to the Semicircle BDC; for as AD is to AE, ſo KL to KM, &c. 'tis called the Terminate of the double of the Sines, and it is an Ellipſis, as appears; likewiſe the Fig. contained by the Curves BCD and BEC is Homogene on both Sides.

COROL. V.

The Fig. contained by the Curves BEC and BFG is Homogene to the Triline BHG, to wit, EF is equal to FI, for EF is the Difference of the Sine AD, and the Arch DB; alſo FI more DA equals the Arch DC, therefore FI is equal to EF, likewiſe ST more OP equals the Arch PC; therefore by ſubtracting from OT, the Arch PC more PO, the Reſidue is QS; and aſſuming KB equal to OC, by ſubtracting from KX, the Arch LB equal to PC, or LN more KL equal to CP, that is, by ſubtracting the whole KN, the Reſidue NX will be equal to QS, the ſame may be ſhewn in any other aſſumed; therefore they are Homogene Figures; it may be ſhewn otherwiſe thus, ſince OP more ST is equal to the Arch PC, and conſequently of the right Line PR, and let PQ be equal to PO, QR will be equal to ST; in like manner MN is equal to VX, therefore MV equal to NX; therefore the Triline GHB Homogene of the Fig. contained by the Curves CEB, and CFB,

and the right Line HB, therefore alſo equal by Poſiton 1. hence alſo equal to the Semicircle CDB, is the Fig. contained by the Curve Lines CDB, CEB.

COROL. VI.

Hence the Triline GIF is equal to the Triline IFH, alſo the Trapezium CEFG is equal to the Trapezium CFIG.

COROL. VII.

The Triline DEB is Homogene to the Fig. of Sines; for as DF is equal to IY, ſo LN is equal to XZ, *viz.* to the Arch LB; I ſay the ſame by any other aſſumed, therefore the Triline is Homogene to the Fig. therefore alſo equal.

PROP. XXIV.

The Fig. of the Cycloid is equal to three generating Circles, or, which is the ſame, the Fig. of the half Cycloid is equal to three half Circles; this Propoſition ſome Men have already demonſtrated, as *Torricellius* in his Appendix concerning the Dimenſion of a Cycloid; I ſhall not retrude from his Invention, but after my Manner ſhall explain the Thing by a four-fold Demonſtration.

1. The Rectangle CI is equal to the Circle of the Generator or Begetter by Corol. 2. Poſition 2; the Triline GIF is equal to the Triline IFH by Corol. 6. of Prop. 23; ABE is equal to the Semicircle by Corol. 5. therefore ABE more CI, or, which

which is the ſame, the Fig. of the Semicycloid CBG is equal to three Semicircles.

2. CH is equal to two Circles; CEB equal to the Circle, therefore the Reſidue is equal to the Circle; but the half of the Reſidue is the Triline GHB by Corol. 5. therefore equal to the half Circle; therefore the other half contained by the Curve Lines BEC, BFG, and the right Line CG, is equal to the half Circle, therefore the whole Fig. CBG is equal to three Semicircles.

3. The Triline DFB is equal to the Fig. of Sines IYH by Corol. 7. of Prop. 23. therefore to the Square under AB, by Corol. 1. Prop. 3. the Trapezium CAFG is equal to the Circle leſs by the Triline GIF; likewiſe the Triline EFB is equal to the Difference of the Quadrant, and of the Quadrate; therefore ABF is equal to the Semicircle more by the foreſaid Difference; the Trapezium is equal to the Circle leſs by the ſame Difference; therefore both together, that is, the whole Fig. CBG is equal to three Semicircles.

4. The Fig. of the half Heart in Fig. 23 is equal to three Semicircles, as hath been ſhewn at large in Prop. 22. and this is equal to the Fig. CBG, becauſe Homogene; for both proceed by Applicates compounded from the Arches and Sines, by Corol. 2. therefore the Fig. of the half Cycloid is equal to three Semicircles, therefore the whole Fig. of the Cycloid is equal to three generating Circles.

COROL. I. Fig. 24.

The Section contained by the right Line BG, and the Curve Line BFG, is equal to the Semicircle, becauſe the Triangle CBG is equal to the Circle.

COROL.

COROL. II.

The Triline δ F B is equal to the Quadrate under AB; becauſe the Triangle A B δ is equal to the Quadrant, and the Fig. ABF equal to the Quadrant, more by the Quadrate.

COROL. III.

The Triline L B N more by the Trapezium C P S G is equal to the Rectangle KH, becauſe P Q C is equal to K B L; Q R C is equal to X H V; Laſtly, The Trapezium C R S G is equal to the Trapezium B N V H: The ſame may be ſhewn if any other be aſſumed: And K H is to the Circle as B K to B A; hence the Trapezium D L N F more D P S F is to the Circle, as A K to A B; hence the Cylinder under the Altitude K B, and of the Baſe from the Circle under the Radius A B, is equal to the Iſoparallel under the Baſe K B and the Altitude A B, for they are compounded from the Ratio of the Baſes which is B A to B K, and of the Altitudes, which is B K to B A.

COROL. IV.

δ E B is equal to the Quadrant, becauſe the Triline EFB is equal to the Difference of the Quadrant and the Quadrate; alſo δ F G is equal to twice the Segment of the Circle contained by the Arch of the Quadrant, and the Subtenſe ED, likewiſe the Curve Line BFG will divide in the Middle the Triangle BGH; alſo the Triline C F G will contain four

times

times tne aforesaid Segment, therefore 'tis double of the Triline HFG.

PROP. XXV.

The Ratio is had of any given right Segment of the Fig. of the Cycloid to the whole Fig. Let first of all the Segment be ABF, it is to CBG, as the Quadrant more by the Square under AB to three Semicircles, that is, according to *Archimedes*'s Ratio, as 25 to 66. Moreover, let another Segment be KBN, the Ratio is known of LBN, or XZβ to the Square under AB, by Prop. 6. therefore to the Quadrant; also the Ratio will be known of the $\frac{1}{2}$ Segment KBL to the Quadrant; for it is equal to the Rectangle under $\frac{1}{2}$ LV and AB, less by the Triang. BKL; therefore the Ratio is had of the Segment KBN to the Quadrant; therefore to the whole CBG, which contains 6 Quadrants: In the same manner is had the Ratio of the Segment OBS, because CT is had, also GTS equal to MNB, which being taken from CT, and the Residue being taken from the Whole CBG, the Residue will be OBS: Or shorter thus, Since EMNF is equal to FITS, from the given OI, ABE, and MNB, is had OBS equal to the aforesaid taken together.

PROP. XXVI.

The Center of Gravity of the Cycloid may be had; let the Fig. of half the Cycloid be CBG, by taking away the Semicircle CDB, the Remainer is an Homogene Fig. Homogene to that begat from the

the Fig. of Sines turned about the Axis, ſince each Fig. proceeds by Applicates equal to the Arches; but the Perpendicular falling from the Center of the aforeſaid Homogene Fig. will ſo divide the Baſe that the Segments ſhall be in Ratio $\frac{3}{5}$, by Corol. 3. of Prop. 22. Therefore from the Center of the Fig. contained by the Curves BDC, BFG, and the right Line CG, the Perpendicular falling on the Axis BG, it will cut it ſo, that the verſed Segment B is to the verſed Segment C, as 5 to 3. But if the aforeſaid Homogene Fig. reſemble a Semicircle in that Manner which is ſaid above, the Perpendicular falling from the Center on the Axis will ſo cut it that the Segments are in Ratio $\frac{5}{7}$, by Coroll. 9. of Prop. 22. but the Fig. added to the Semicircle is Homogene of the Fig. CBG, by Coroll. 2. of Prop. 23. Therefore a Perpendicular falling from the Center of the Fig. CBG on the Axis, will ſo cut it, that the verſed Segment B will be to the other verſed C, as 7 to 5. Hence if CB be of 12 equal Parts, by aſſuming or taking from the Vertex B 7 of them, there will be the Center of the Fig. of the Cycloid.

COROL. I.

Hence the Center of Gravity of any right Segment of a Fig. of a Cycloid is had; for Example, let it be ABF: The Center of Gravity of the Fig. IYH is had by Prop. 10. therefore by drawing from this Center the Perpendicular on BA, it will paſs through the Center of the Homogene Triline DFB, by Poſition 8. In like manner is had the Center of the Quadrant ADB, and by drawing

ing from it a Perpendicular on AB in the End it will ſo divide the Segment of the Axis intercepted by the Perpendiculars, that the verſed Part B will be to the other verſed A, as the Quadrate to the Quadrant, there will be the Center of the Segment BAF. And that we may ſhew ſomewhat of Calculation, the Center of ABD is eaſily had, for ſince the Ratio of the Figures, that is, of the Quadrate under AD, and of the Quadrant ADB is $\frac{14}{11}$: Likewiſe the Ratio of the Solids of thoſe begat being made by the Revolution about AD $\frac{3}{2}$, and let the other of the Diſtances be half of AB, (for Example) 7, the other Diſtance will be $5\frac{31}{33}$, this is the Diſtance of the Center of Gravity of the Quadrant from AD. Moreover, the Center of the Triline DFB will be diſtant from AI, $\frac{1}{4}$ of the whole DF, which ſince it is 22 by ſuppoſing that AB is 14, therefore the Diſtance will be of Parts $5\frac{1}{2}$,

Let another Segment be KBN, we have the Diſtance of the Center of Gravity of the Segment XZH from the Axis XZ, by Coroll. 4. of Prop. 21. therefore alſo of the Homogene Fig. LNB. Likewiſe the Diſtance of the Center of the Segment KLB from the ſame KL will be known; therefore if it be made as above, we ſhall have the Diſtance of the Center of the whole KBN from the ſame KN. Laſtly, Let the Segment be OBS, the Center of ABF will be had, likewiſe of KBN, therefore alſo of the other Part AKNF, alſo of ABE, and of KBM, therefore of EFB, likewiſe of EMNF, therefore alſo of FITS, likewiſe of OI, therefore of OAFS, therefore alſo of the whole Segment OBS: And that theſe may be reduced to Calculation, ſuppoſe the Quadratures of the Circle.

COROL.

t

COROL. II.

Hence alſo the Center of Gravity is had of the other Parts of the Fig. of the Cycloid in the Axis BC, for Example, of MNB; for ſince the Center of the whole Segment KBN is had, alſo of KBM, we ſhall have alſo the Center of the other Part MNB, likewiſe of STG, and of XHV, which are Homogene Figures to the Triline MNB: We have alſo the Center of the Triline CFG, ſince we have as well the Center of the whole CBG, as of the Part CEB, alſo of COSG. Alſo of the Fig. contained by the Curve Lines BEC, BFG, and the right Line CG being had; to wit, from the Center of the whole CBG, and of the Part CEB, therefore alſo the Triline GHB, alſo of CEFG, likewiſe of FIHB, alſo of the Section contained by the right Line BG, and the Curve Line BFG, and laſtly of any Trapezium.

PROP. XXVII.

If we turn about the Fig. of the Cycloid about the Baſe, that begat is to the Cylinder of the ſame Altitude, and the Baſe of the Circle under the Radius of the Axis of the equal Fig. as 5 to 8. Let the Fig. of half the Cycloid be CBG turned about the Baſe CG, that begat is to the Cylinder begat from the Rectang. CH, as 5 to 8; to wit, in a compounded Ratio from the Ratio of the Figures $\frac{3}{4}$, and of the Diſtances $\frac{5}{6}$, which is $\frac{15}{24}$ or $\frac{5}{8}$, *viz.* the Diſtance of the Center of the Fig. CBG from CG, is to CA, as 5 to 6, by Prop. 26: Add that the

Fig.

Fig. CBG is Homogene to the Fig. of $\frac{1}{2}$ the Heart, and that begat from it to that begat from theſe is Homogene; and that begat from the Fig. of the $\frac{1}{2}$ Heart to the Cylinder, as 5 to 8, by Coroll 15. of Prop. 22. therefore alſo that begat from CBG to the Cylinder begat from CH, as 5 to 8; the ſame may be ſaid concerning that begat from the whole Fig. of the Cycloid to the Cylinder under the Baſe from the Circle begat from CB, and from the Altitude of double CG.

COROL. IV.

Hence that begat from the Triline GHB is to that begat from the Fig. CBG, as 3 to 5; but to the Cylinder as 3 to 8.

PROP. XXVIII.

That begat from the Fig. of half the Cycloid turned about the Baſe, by taking away the generating Semicircle, is to the aforeſaid Cylinder as 3 to 8; for this is an Homogene of the Homogene Fig. begat from the Fig. of Sines turned about the Axis, therefore thoſe begat from both are Homogenes: But that begat from the other is to that Cylinder as 3 to 8, by Coroll. 15. of Prop. 22. therefore alſo that begat from the other to its Cylinder, as 3 to 8; for let the aforeſaid Fig. contained by the Curves BDC, BFG, and the right Line CG be turned about CG, that begat is to the Cylinder, compounded from the Ratio of the Diſtances, which is $\frac{3}{4}$, by Prop. 26. and Coroll. 3. of Prop. 22. and from the Ratio of Figures, which is $\frac{1}{2}$; but from theſe

these the Compound is $\frac{3}{8}$, therefore the aforesaid begat is to the Cylinder as 3 to 8.

COROL. I.

Hence that begat from the aforesaid Fig. is equal to that begat from the Triline GHB, which is to the Cylinder as 3 to 8, by Coroll. of Prop. 27.

COROL. II.

Hence that begat from the Semicircle CDB is to that begat from the aforesaid Fig. as 2 to 3, and to the Cylinder as 1 to 4.

COROL. III.

And since that begat from CEB is double of that begat from CDB, for those begat are as the Figures; that begat from the Fig. contained by the Curve Lines BEC, BFG, is subduple of that begat from the Semicircle CDB; for that begat from the Fig. contained by the Curve Lines BDC, BEC, is equal to that begat from CDB, therefore as 2; but that begat from the Fig. contained by the Curve Lines BDC, BFG, and the right Line CG, is as 3; therefore that begat from the Fig. contained by the Curve Lines BEC, BFG, and the right Line CG, is as 1; therefore $\frac{1}{8}$ of the Cylinder, and $\frac{1}{2}$ of that begat from CDA.

COROL. IV.

That begat from the aforesaid Fig. to wit, that contained by the Curve Lines BEC, BFG, is equal to that begat from the Triline CGH; for since all the Applicates are equal, the Superficies in both Figures of the begat are equal; for Example, that begat from QS equal to that begat from RT, that begat from EF equal to that begat from FI, therefore the Solids begat are equal: Hence that begat from the Triline GHB is triple of that begat from the Triline CGH.

COROL. V.

That begat from BCH turned about CG is to the Cylinder begat as 7 to 8, for since that begat from the Triline CGH is to the Cylinder as 1 to 8, that begat from BCH will be to the Cylinder as 7 to 8.

COROL. VI.

Hence at last are had others begat, suppose from the Triline CFG; for since we have that begat from the Triline CGH, also that begat from the Triline, GFH, which is to that begat from the Rectangle under GH, IF, as the Triline to the Rectangle we have that begat from CFG, also from the Fig. contained by the Curves CDB, CFH, and by the right Line BH: For since that begat from the Semicircle CDB is to the Cylinder as 2 to 8, and that begat from BCH as 7 to 8; that be-

gat from the aforesaid Fig. to the Cylinder as 5 to 8, therefore equal to that begat from CBG, and since that begat from CEB is as 4, that begat from the Fig. contained by the Curves CEB, CFB, and the right Line BH will be to the Cylinder as 3 to 8, therefore equal to that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG. Hence those begat from CDA, and from the Triline CFG, will be equal to that begat from the Triline BHF.

PROP. XXIX.

That begat from any right Segment of a Cycloid turned about the Base may be had; let ABF be turned about AF, that begat from DFB is equal to that begat from IYH, for they are homogene Figures, therefore the $\frac{1}{2}$ of the Cylinder of the same Base and Altitude by Prop. 4. But that begat from the Quadrant ADB is to the Cylinder of the same Base and Altitude as 2 to 3, therefore let AD be 7, DF 11; that begat from ABF will be to the Cylinder under the Altitude AF, as 10 $\frac{1}{6}$ to 18, or as the Cylinder under the same Base and Altitude compounded from $\frac{2}{3}$ of AD, and $\frac{1}{2}$ of DF, that is, as 61 to 108.

Let another Segment be KBN, since we have the Ratio of KBN to the Rectang. under KM, KB, and the Distance of the Center of Gravity of both from KM; we have the Ratio compounded from the Ratio's of the Distances, and of the Figures, but this is the Ratio of the Solids begat by Position 9; the same may be said of any other Segment; for Example, of OBS.

COROL.

COROL.

Hence are had thoſe begat from ADB, DEB, EFB apart; and that begat from EFB is equal to that begat from FIH; alſo that begat from BAF turned about BH, for we have the Diſtance of the Center of ABF from BH: Alſo of thoſe begat from DEB, EFB turned about BH; alſo from the Trapezium BAFH, and from any other from the general Rule delivered.

PROP. XXX.

We have the Center of Gravity of the begat as well from the Fig. of the Cycloid, as from any whole Segment turned about the Baſe, for it is in the Point in which the Baſe and the Axis are cut croſs-wiſe; to wit, the Center of the Solid begat is in the Axis, is drawn through the Centers of all the Planes; alſo in the Baſe of the Figures which is alſo drawn through the Centers of all the Circles; therefore the Center of the begat muſt needs be in the Concourſe or Meeting of both.

PROP. XXXI.

We have the Ratio of the Solid begat from the Fig. of the half Cycloid turned about the Axis to the Cylinder of the ſame Baſe and Altitude; let all be as above, the half Cycloid, the Fig. CBG turned about the Axis BC; that begat from it is to the Cylinder of the ſame Baſe and Altitude, as the Solid under the Altitude CB, proceeding by the

Squares CG, OS, AF, KN, &c. to the Parallelipiped under the Altitude CB, and of the Baſe from the Quadrate CG; for as the Parallelipiped is homogene to the Cylinder, ſo the aforeſaid Solid, proceeding by the Squares homogene to that begat from CBG: But the Ratio is known of the aforeſaid Solid to the Parallelipiped by Coroll. 20. of Prop. 22. therefore from thence is had the Ratio of the aforeſaid begat to the Cylinder, which we ſhall prove by Calculations beneath.

COROL. I.

Hence are had many Solids begat, being made by the Revolution about BC, to wit, that begat from the Triline BHG, alſo that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG, by taking away that begat from BEC the double of that begat from BDC; likewiſe that begat from CAFG; for if from the Parallelipiped under the Altitude CA, and of the Baſe from the Square of CG be taken away twice, the Iſoparallel under the Triline Baſe IGF, and the Altitude CG, leſs by the Solid under the Altitude IG, proceeding by the Squares under FI, ST, and from other Differences of the Arches and Sines, the Remainer will proceed by the Squares CG, OS, AF, and theſe are known by Coroll. 20. of Prop. 22. And it is as the aforeſaid Parallelipiped to the aforeſaid Remainer, ſo the Cylinder begat from CI, to another Solid, this it ſelf will be that begat from CAFG, which is homogene to the aforeſaid Remainer.

COROL.

COROL. II.

Hence is had the Center of Gravity of CBG, to wit, we have already the Diſtance of it from the Baſe CG, by Prop. 26. the Ratio of the Begetters is had which is compounded from the Ratio of the Figures which is had, and from the Ratio of Diſtances of which one is had, to wit, the half CG, therefore the other alſo is had, for from two compounded Terms being given, alſo from the two other of the compounding Ratio, ſince to the third Term of one is had, the fourth Term of the ſame, as is ſhewn above: Therefore is had the Perpendicular falling from the Center of the Fig. on the Baſe CG, alſo the Perpendicular falling from the ſame on the Axis CB; therefore is had the Concourſe of both, in which is the Center of the Fig. CBG.

COROL. III.

Alſo the Center of the Triline HBG is had, which may be demonſtrated in the ſame Manner, alſo that begat from the ſame Triline turned about GH; for we have the Ratio compounded of the Diſtances, and of the Figures, therefore of the Solids begat: Alſo we have the Center of the Trapezium CAFG, to wit, the Perpendicular falling from it on the Axis CB; therefore that begat from the ſame Trapezium turned about GI, which being taken from the Cylinder begat from GA, we have that begat from the Triline IFG.

PROP. XXXII.

The Solid begat from the Segment turned about the Axis is had, whose Base will fall in the Center of the generating Circle; (for Example) let ABF be the begat from it, it is Homogene to the Solid under the Altitude AB, proceeding by the Squares under the Compounds from the Arches and Sines, as appears; for AF is compounded from the Sine AD, and from the Arch DB; alſo KN from the Sine KL, and from the Arch LB, *&c.* And the Ratio of this Solid will be known to the Parallelipiped of the ſame Altitude and Baſe, therefore alſo of the aforeſaid begat from ABF to the Cylinder: I ſay, the aforeſaid Ratio will be known by Coroll. 7. of Prop. 14. and Coroll. 20. of Prop. 22.

COROL. I.

Hence is had the Center of Gravity of the Segment ABF, which may alſo be had in another Manner, to wit, from the Center being known of the whole Fig. CBG, and of the Trapezium CAFG; alſo that begat from BFC is had, *viz.* the double of that begat from ABF.

COROL. II.

That begat from DFB is had, to wit, by taking away the begat from ADB: And therefore the Center of the ſame DFB, alſo that begat from the Fig. contained by the Curves BDC, BFC, and the ſame Center of the Fig. Alſo that begat from the

the Triline EFB, by taking away that begat from or by AEB, the double of the begat from ADB, and the ſame Center of the Triline; alſo that begat from the Lunula contained by the Curves BEC, BFC by taking away the begat from or by BEC; alſo the Centers of the Figures are had, by the Begats being known.

COROL. III.

And from the Revolution made about HG, is had that begat from the Triline GHB, of which the Center is had; alſo from the Fig. CBG, by reaſon of the ſame Ratio; alſo that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG; alſo from the Fig. contained by the Curves BEC, BFG, and the right Line CG; alſo that begat from the aforeſaid Lunula; alſo from the Triline CGF or BHF, to wit, becauſe the Centers are known.

PROP. XXXIII. Fig. 16.

From a Solid being cut which proceeds by Rectangles under the right Sines and Complements, the Ratio of the Segments is had; let the aforeſaid Solid be ABDC proceeding by the Segments BAC, OHG, *&c.* or by Rectangles parallel to EG, which is under the right Sine and the Complement, as is ſhewn in Coroll. 2. of Prop. 13. let it be cut by the Plane OHG, ſince it is Homogene ASD, the Segments of the Solid are as the Segments of the Triang. And theſe are as the Squares of the Segments of the Baſe AB, BD, therefore by knowing the whole Solid, the Segments will be known.

COROL. I.

Hence from the Solid being cut by the Plane EG we may know the Segment intercepted by the Planes BAC, OHG, from which, if we take away the Iſoparallel under the Baſe EFA, and the Altitude AH, the Reſidue will be known, of which the Baſe is EG, and the Altitude EB.

COROL. II. Fig. 17.

Hence the Segments of the Solid AEC cut by the Plane DB will be known; to wit, the Iſoparallels had under the Baſe from the Trapezium ALKB, and the Altitude AG: But the Segment under the Baſe GIDH, and the Altitude GF may be had thus; the Solid will be known proceeding by the Sectors parallel to GID towards FE, it is Homogene to the Triangle, therefore the Half of the Iſoparallel under the Baſe from the Sector GID, and the Altitude GF; the Solid alſo will be known proceeding by Triangles parallel to GHD towards FE, for it is ſubduple of the Solid known proceeding by Rectangles under the right Sines, and of the Complements, by Coroll. 1. Therefore the Segment will be known, intercepted by the Planes AE, and BK, which being taken from the whole Solid AEC, we have the remaining Segment DBC.

COROL. III.

Hence the Solid Segment under the Baſe GHDI, and the Altitude GF, is equal to the Iſoparallel under the Baſe KLNP, and the Altitude LI.

COROL.

COROL. IV. Fig. 24.

If we return to the Fig. CBG, the Solid proceeding or growing up by Rectangles under the Sines and the Arches, that is, under AD, DF, KL, LN, *&c.* is Homogene to the aforeſaid Solid: Hence if it be cut by the Plane LN, the Ratio of the Segments will be had as above, and by adding the Solid proceeding by the Squares of the Sines AD, KL being known, the Solid is had proceeding by the Rectangles under AF, AD, KN, KL, that is, under the Sines and the Compounds from the Sines and Arches; alſo the Ratio of the Segments is had by drawing the Plane KN.

PROP. XXXIV.

From the Revolution being made about the Axis is had that begat from the Fig. contained by $\frac{1}{2}$ the Cycloid, and the right Line the Subtenſe of the ſame: Let CIG be $\frac{1}{2}$ the Cycloid, let the Fig. contained be the Curve BFG, and the right Line BG, the Solid will be had from it begat by being turned about BC, to wit, if from the Solid begat from CBFG, we take away the Cone begat from the Triang. CBG, the Reſidue will give that begat from the aforeſaid Fig.

COROL. I.

Hence is had the Center of Gravity of the aforeſaid Fig. alſo of that begat from [illegible] F B, to wit, by taking away the begat from A [illegible] B being known, from

from that begat from AFB alſo known: Hence is had that begat from ♂ F G; alſo the Center of Gravity as well of ♂ F B as of ♂ F G.

COROL. II.

If there be another Line of $\frac{1}{2}$ the Cycloid GDB, the common Subtenſe BG, the Center of both together is in AF, ſince both the Applicates are parallel to AF, and equidiſtant, they are equal therefore in the Point ♂, as appears.

COROL. III.

Hence is had that begat from both taken together, for it is to the Cylinder begat from BG in the Ratio compounded of the Figures and of the Diſtances; the Diſtances are equal, for the Center of both Figures is in ♂, therefore thoſe begat are as the Figures: The Rectang. is double of the Fig. therefore that begat from the Fig. is ſubduple of that begat from the Rectang.

COROL. IV.

Hence that begat from the aforeſaid Fig. is *ſeſquialter* of the Cone begat from the Triang. BCG, and *ſubſeſquitertium* of that begat from the Triangle BGH: Hence that begat from both the Trilines taken together, is equal to that begat from the Fig. And the begats are as the Figures, ſince the Center of both is in ♂.

COROL.

COROL. V.

If we turn the aforeſaid Fig. about CG, ſince that begat from CBG is to the Cylinder as 5 to 8, that is, as 15 to 24; that begat from the aforeſaid Fig. will be as 12 to 24, therefore that begat from the Triline BCG as 3: And that begat from the Triline GBH as 9. Hence that begat from $\frac{1}{2}$ the Fig. contained by the Curve BFG, and the right Line BG as 7; becauſe that begat from the Triang. GBH is as 16, therefore that begat from the other $\frac{1}{2}$ as 5: Hence you ſee the Proceſſion by odd Numbers, from the beginning being drawn from the Triline BCG, 3, 5, 7, 9: Laſtly, that begat from HGB as 21. Moreover, in this the two Trilines CBG, GBH agree with the Parabolicks, becauſe that begat from one is triple of that begat from the other.

SCHOLIUM. Fig. 13.

But I come to Calculations; Let the whole Cycloid be BAE, for altho' above it hath been inſtead of the Fig. of Sines, now I ſuppoſe it to be a Cycloid which is turned about BE: As CA begets a Circle, ſo alſo all the other ordinate Applicates parallels: Let CA, the Axis of the Fig. (for Example) be 14, EB the Baſe 44, according to the *Archimedean* Ratio; let the Homogene Solid ariſing or proceeding by the Squares under the Diameters of the Circles begat from the aforeſaid ordinate Applicates under the Altitude EB be called the greater Homogene; let there be another under the Altitude CE,

CE, proceeding or arifing by the Squares of the Applicates Parallels to CA, call it the leffer Homogene: This is fuboctuple of the greater, as appears; let the Ifoparallel be under the Bafe BAE, and the Altitude AI cut by the Plane EIB, the leffer Homogene is equal to the upper Fruftum; fince it is double of the half Fruftum, to wit, the Homogene will proceed by Squares, and the half Fruftum by Triangles, both under the fame Altitude CE, and every Square is double of its Triangle. Moreover, let the Center of Gravity of the Fig. BAE be in K, fo that CK be to CA as 5 to 12, the aforefaid Fruftum will be to the Ifoparallel as 5 to 12; therefore alfo the leffer Homogene as 5 to 12; therefore the greater Homogene as 40 to 12, that is, as 10 to 3. Therefore fince the Fig. EAB is equal to three Circles under the Diameter CA, the Circle under the Diameter double of CA will be to the Fig. CAB, as 4 to 3; therefore the Cylinder under the Bafe from the aforefaid Circle, and the Altitude AI to the Ifoparallel as 4 to 3, therefore the greater Homogene to this Cylinder as 10 to 4, or as 5 to 2. But this Cylinder is to that begat from the Rectang. under BE, CA turned about EB, as CA to EB, that is, as 14 to 44, therefore the greater Homogene will be to that begat from the aforefaid Rectang. as 35 to 44. But the Parallelipiped under the Bafe from the Square of the double of CA, and the Altitude CB, is to the aforefaid begat as 14 to 11, that is, as 56 to 44, and to the greater Homogene as 56 to 35, that is, as 8 to 5: Therefore the greater Homogene to that begat from the Fig. EAB as 35 to 27 $\frac{1}{2}$, that is as 14 to 11, therefore the greater Homogene to the Cylinder under the

Bafe

Baſe from the Circle of the Diameter CA, and the Altitude AI, will be as 40 to 4, that is, as 10 to 1; therefore that begat from EAB to the Cylinder under the Baſe from the generating Circle, and the Altitude CA, is as 55 to 14.

But let IY or DF be 11, CB 14, the Parallelipiped under the Baſe from the Square IY, or DF, and the Altitude IH will be 847, that drawn into three is	2541
The Cube under the Side IH, will be 343, which, drawn into three, makes	1029
The Solid proceeding or ariſing by the Squares of the Arches IY, XZ, &c. under the Altitude IH, will be 392, which, drawn into three, makes	1176
The Solid Homogene to the Sphere, proceeding by the Squares of the Sines under the Altitude IH, will be 228 $\frac{2}{3}$, which, drawn into three, makes	686
The Solid proceeding by Rectangles under the Arches and Sines, and under the ſame Altitude IH, will be 297 $\frac{1}{2}$, which, drawn into three, makes	892 $\frac{1}{2}$
The Difference of both Solids	206 $\frac{1}{2}$
This Difference being added to the laſt Solid, the Aggregate will be	1099
This Aggregate being taken from the Solid, proceeding by the Squares of the Arches, the Remainer will be	77

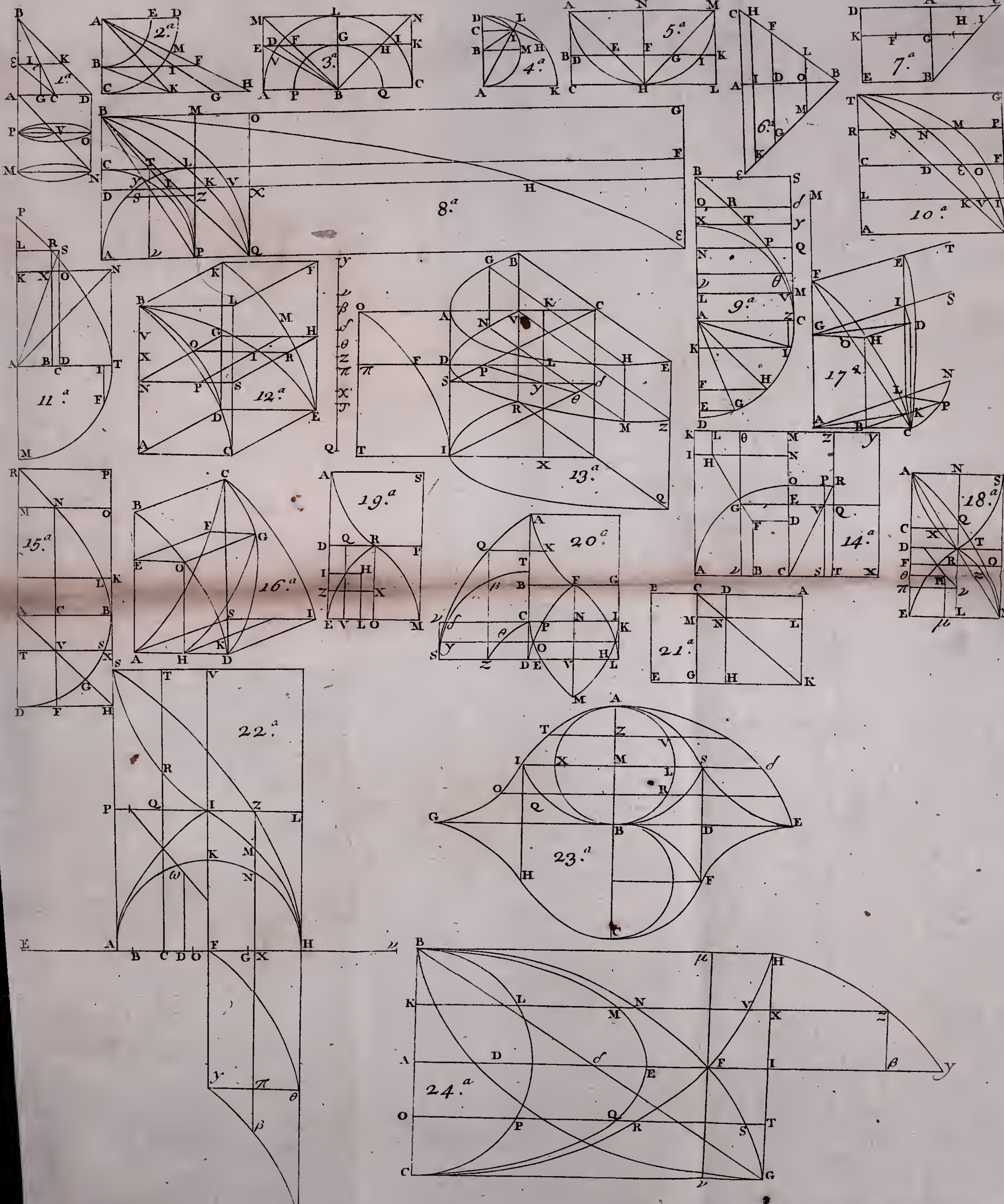
And this is the Solid proceeding or ariſing by the Squares of the Differences between the Arches and the Sines.

If from the Parallelipiped under the Square CG, and the Altitude CA, we take away twice the Isoparallel under the Base from the Triline GIF and the Altitude CG, less by the Solid proceeding by the Squares FI, ST, to wit, by the Squares of the Differences, as above, the Remainer will be the Solid, proceeding by the Squares CG, OS, AF, to wit	9779
Being added to the Solid proceeding by the Squares of the Arches; twice the Solid proceeding by the Rectangles under the Arches and Sines, less by the Solid proceeding by the Squares of the Sines, and the Aggregate proceeding by the Squares AF, KN, to wit, compounded from the Arches, and the Sines will be	2275
One being added to the other, and will be the Solid proceeding by the Squares CG, AF, &c.	12054
And the Parallelipiped under the Base from the Quadrate CG, and the Altitude CB, is	20328

Therefore the Parallelipiped is to the Solid proceeding by the Squares CG, AF, &c. that is the Cylinder begat from the Rectangle CH, to that begat from the Fig. CBG turned about BC, as 484 to 287.

Hence if we take away from that begat from the Fig. CBG, that begat from the Triangle CBG, that is $\frac{1}{3}$ of that begat from CH, the Remainer will give the begat from the Fig. contained by the Curve Line BFG, and the right Line BG, to wit $125\frac{2}{3}$; therefore

I



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therefore that begat from the other $\frac{1}{2}$ Portion 116 $\frac{1}{3}$: Therefore that begat from the Triline BCG 45, therefore that begat also from the other Triline 197.

And by taking away twice the Solid proceeding by the Squares of the Sines, from the Solid which proceeds by the Squares of CG, AF, the Remainer will be 10682; therefore if that begat from CDFG is 287, that begat from the Fig. contained by the Curves BDC, BFG, and the right Line CG will be 254 $\frac{1}{3}$; and consequently that begat from the Semicircle CDB 32; therefore that begat from CEB 130 $\frac{2}{3}$; therefore that begat from the Fig. contained by the Curves BEC, BFG, and the right Line CG, is 56 $\frac{1}{3}$; and that begat from CH turned about CB to that begat about CG, as 847 to 539, that is 484 to 308; therefore that begat from CBG about BC, to that begat from CBG about CG, as 287 to 192 $\frac{1}{2}$: That begat from the Triline BCG is had in another Manner, to wit, by taking away that begat from the Fig. contained by the Curves BFG, BPG, half of that begat from CH, from that begat from CBG, that is 242 from 287, the Remainer 45 will be that begat from the Triline BCG.

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